

Optical Sciences PhD Qualifying Exam:  
Instructions and Equation Sheet

Answer all four questions on the following pages. Start each answer on a new sheet of paper.

In the upper right hand corner of each sheet, write your code number (NO NAMES PLEASE) and the problem number (course number) that you are answering. Attach together all sheets for a given problem in order.

When finished, insert your answers into the envelope provided. Also insert all scratch paper, this equation sheet, the exam problems, and any other items that were distributed to you in the envelope.

The following are some helpful items:

$$h = 6.625 \times 10^{-34} \text{ J} \cdot \text{s} = 4.134 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/K}^4 \cdot \text{m}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$$

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$$\cos 2A = 1 - 2 \sin^2 A$$

$$\sin^2\left(\frac{A}{2}\right) = \frac{1}{2}(1 - \cos A)$$

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$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$$

$$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$$

$$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$$

$$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$$

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

$$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$$

$$\nabla \times \nabla\phi = 0$$

$$\oint_S (\mathbf{F} \cdot \mathbf{n}) da = \int_V (\nabla \cdot \mathbf{F}) d^3x$$

$$\oint_C \mathbf{F} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} da$$

$$\oint_S \phi \mathbf{n} da = \int_V \nabla\phi d^3x$$

$$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n}) da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}] d^3x$$

$$\oint_S (\mathbf{n} \times \mathbf{F}) da = \int_V (\nabla \times \mathbf{F}) d^3x$$

**PhD Qualifying Exam, Fall 2020**  
**Opti 501 Day 1**

**System of units: MKSA**

1) A monochromatic plane-wave, having frequency  $\omega$  and wave-vector  $\mathbf{k}$ , propagates in free space. For all practical purposes, one may assume that  $\omega$  is a real-valued scalar, while  $\mathbf{k}$  is a complex-valued vector, that is,  $\mathbf{k} = k_0 + i\mathbf{k}''$ . Let the scalar and vector potentials associated with this plane-wave be written as  $\psi(\mathbf{r}, t) = \psi_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$  and  $\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ , respectively.

- a) Write the differential equation relating the scalar and vector potentials in the Lorenz gauge, then derive the relation among  $\psi_0$ ,  $\mathbf{A}_0$ ,  $\mathbf{k}$  and  $\omega$  assuming the aforementioned plane-wave satisfies the Lorenz gauge.
- b) Find expressions for the  $E$ - and  $B$ -fields of the plane-wave in terms of  $\psi_0$ ,  $\mathbf{A}_0$ ,  $\mathbf{k}$  and  $\omega$ .
- c) Write the differential form of Maxwell's equations, then obtain the constraints on  $\psi_0$ ,  $\mathbf{A}_0$ ,  $\mathbf{k}$  and  $\omega$  that ensure the above plane-wave is a solution of Maxwell's equations.
- d) Specify the condition(s) under which the plane-wave is evanescent (i.e., inhomogeneous).
- e) Specify the condition(s) under which the plane-wave is homogeneous and linearly polarized.
- f) Specify the condition(s) under which the plane-wave is homogeneous and circularly polarized.
- g) In terms of  $\mathbf{A}_0$ ,  $\psi_0$ ,  $\mathbf{k}$  and  $\omega$ , find the time-averaged rate-of-flow of electromagnetic energy (per unit area per unit time) for the plane-wave.

**Hint:** You might find the following vector identities useful:

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

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PhD Qualifying Exam

Fall 2020

OPTI 502 Day 1

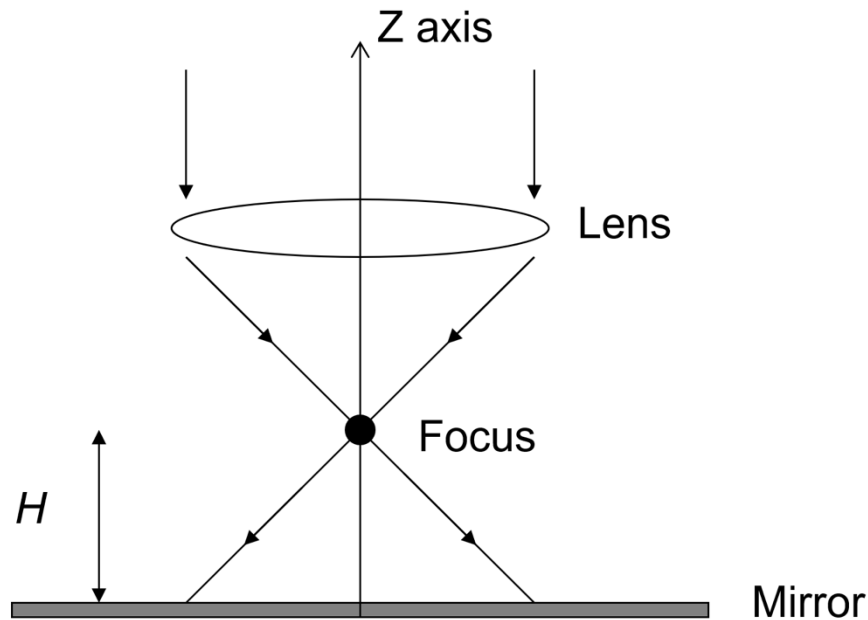
Consider a thin lens in air with a focal length of 5 mm. It operates at an F/# of 2.8 and the full field of view is  $\pm 33$  degrees. The stop aperture is at the lens. Except for part E assume the object is at infinity.

- A) Determine the image size corresponding to the full field of view.
- B) Determine the diameter of the first zero ring in the Airy pattern produced by the lens at a wavelength of 555 nm.
- C) The lens is to be used with a sensor that has a pixel size of 0.005 mm. Determine the hyperfocal distance for the system.
- D) Determine the Lagrange invariant.
- E) Two of these lenses are placed next to each other with parallel but displaced optical axes. The lenses are spaced by 10 mm center-to-center, and their image planes are coplanar. Both lenses image the same object which is very small in size. The image of the object cast by the first lens falls on the optical axis of the first lens. However, the image of the object cast by the second lens falls 0.1 mm, off-axis, from the optical axis of this second lens. Determine the distance to the object.
- F) Draw one of these lenses, the optical axis, the marginal ray, and the chief ray.

OPTI 505R Day 1

A lens is used to focus a  $\lambda = 0.5 \mu\text{m}$  laser beam so that the focus point is  $H = 1 \mu\text{m}$  above a perfect electrical conductor (PEC) mirror. The full cone angle of the focus beam is  $90^\circ$ . Ignore any effects due to diffraction. State any assumptions that you make.

- (2 pts) Mark the y axis intercepts of bright fringe centers in the pattern between the focus and the mirror.
- (2 pts) What is the distance between adjacent y-axis intercepts?
- (2 pts) What are the geometric shapes of the fringes?
- (2 pts) Make a sketch of the interference pattern generated between the incident laser beam and the reflected beam between the focus and the mirror. Use simple lines to indicate centers of BRIGHT fringes. Do NOT shade to show variation in brightness.
- (1 pt) How many fringes can be observed in the region between the focus and the mirror?
- (1 pt) Label the fringe order numbers of the bright fringe centers corresponding to the number of waves of OPD between the focus point and the virtual focus point.



A thin 1 mm long glass cell contains a dilute gas of hydrogen atoms with number density  $N = 1 \times 10^{10} \text{cm}^{-3}$ . The atoms are in thermal equilibrium with the room temperature cell. At time  $t = 0$  a monochromatic laser beam of the form  $\vec{E} = \hat{z} \frac{1}{2} E_o e^{-i\omega t} e^{ikx} + c.c.$  is turned on, where  $E_o$  is a constant. In this problem you can assume the dipole approximation is valid. The laser frequency is on resonance with the  $n = 1$  to  $n = 3$  transition. The on-resonance absorption cross section for this transition is  $\sigma(\nu_0) = 5 \times 10^{-9} \text{cm}^2$ . See the bottom of this page for additional information you may find useful.

(a) [2 point] The value of the on-resonance absorption cross-section  $\sigma(\nu_0)$  listed above is significantly less than the maximum possible value due to line broadening mechanisms. For this room temperature gas cell, name 2 line broadening mechanisms that may be contributing to this.

(b) [2 point] Assuming the laser intensity  $I$  is very low compared to the saturation intensity  $I_{sat}$  (ie  $I \ll I_{sat}$ ), calculate the transmission through the cell.

(c) [3 points] The field strength of the laser is now turned up to  $E_o \approx 3 \times 10^3 [\frac{J}{C \cdot m}]$ . The laser is turned on at time  $t = 0$  and then turned off at time  $t_1$ . Calculate the time  $t_1$  such that the maximum number of atoms are excited to the 3P ( $m=0$ ) state. Provide a numeric answer. Note: ignore the fine structure and hyperfine structure of the hydrogen atom energy levels due to electron and proton spin in this problem.

(d) [3 points] Into what states can the 3P ( $m=0$ ) state spontaneously decay? Given this, calculate the total radiative lifetime of the 3P ( $m=0$ ) state.

The following expressions may be useful for these calculations:

Between any two levels with an energy difference  $\delta E = \hbar\omega_o$  and a dipole matrix element  $\vec{\rho}$ , the Einstein A coefficient is:

$$A = \frac{\omega_o^3 |\vec{\rho}|^2}{3\pi\epsilon_0 \hbar c^3} = \left| \frac{\delta E}{E_1} \right|^3 \times \frac{|\vec{\rho}|^2}{e^2 a_0^2} \times 2.35 \times 10^9 \text{ s}^{-1}$$

$E_1 = -13.6 \text{ eV}$  for the hydrogen atom.

$$a_o = 0.5 \times 10^{-10} (m)$$

$$e = 1.6 \times 10^{-19} (C)$$

$$\hbar = 1.05 \times 10^{-34} (J \cdot s)$$

Cartesian Components of Angular Matrix Elements

$$\langle l = 0, m = 0 | \hat{r} | l = 1, m = 0 \rangle = (0, 0, \sqrt{\frac{1}{3}})$$

$$\langle l = 0, m = 0 | \hat{r} | l = 1, m = 1 \rangle = (-\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0)$$

$$\langle l = 0, m = 0 | \hat{r} | l = 1, m = -1 \rangle = (\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0)$$

Radial Coordinate Matrix Elements for Atomic Hydrogen

$$\langle 1S | r | 2P \rangle = 1.29 a_0$$

$$\langle 1S | r | 3P \rangle = 0.517 a_0$$

$$\langle 2S | r | 3P \rangle = 3.07 a_0$$

$$\langle 2P | r | 3S \rangle = 0.95 a_0$$

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**PhD Qualifying Exam, Fall 2020**  
**Opti 501 Day 2**

**System of units: MKSA**

2) A linearly-polarized light pulse of duration  $T$ , frequency  $\omega$ ,  $E$ -field amplitude  $E_0$ , and cross-sectional area  $A$ , propagates in free space. (The pulse is long enough and broad enough that one can ignore its spectral content and treat it simply as a section from a plane wave.) The pulse arrives at normal incidence at the flat surface of a linear, isotropic, homogeneous, semi-infinite material of refractive index  $n$ , where  $n$  is real-valued and greater than unity. You may assume that the transparent dielectric material is non-magnetic [i.e.,  $\mu(\omega)=1$ ] and non-dispersive (i.e.,  $n$  does *not* vary with frequency  $\omega$  within the bandwidth of the light pulse).

- a) What is the total energy content of the light pulse in free space?
  - b) Describe the properties of the reflected light pulse (e.g., frequency, wavelength, duration, polarization state, total optical power).
  - c) Find the  $E$ -field and  $H$ -field amplitudes of the light pulse that enters the glass medium. Describe the properties of the light pulse that propagates within the glass medium.
  - d) Show that the total energy of the pulse is conserved upon reflection/transmission at the glass surface.
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PhD Qualifying Exam

Fall 2020

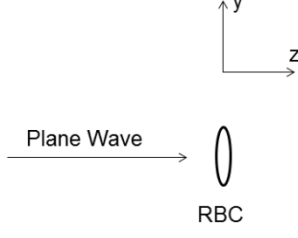
OPTI 502 Day 2

The two thin lenses of a double-telecentric system are separated by 250 mm. The image is  $\frac{1}{4}$  the size of the object and the separation between the object and the image is 500 mm. The image-space working f-number is  $f/\#_w = 4$ . The object size is  $\pm 10$  mm. Provide the complete design of the system, including the minimum required lens element diameters for this double-telecentric system to function correctly at the maximum object size. Provide a drawing of the system.



A red blood cell (RBC) can be approximated by a phase circle that is  $D = 10\mu\text{m}$  in diameter. It is transparent at the 650nm wavelength. Transmission of the RBC is modeled by

$$t_{RBC}(x_s, y_s) = \exp[j\phi(x_s, y_s)] \text{circ}\left(\frac{\sqrt{x_s^2 + y_s^2}}{D}\right)$$

$$\approx [1 + j\phi(x_s, y_s)] \text{circ}\left(\frac{\sqrt{x_s^2 + y_s^2}}{D}\right),$$


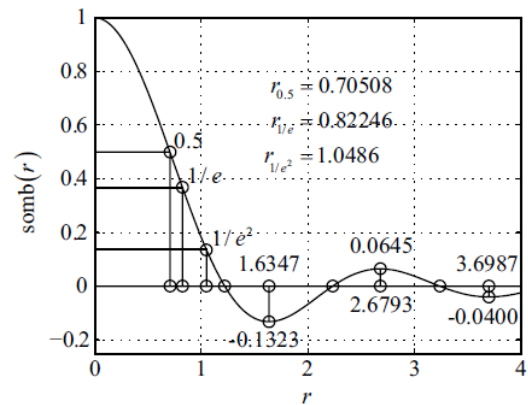
where the RBC phase is weakly scattering. The medium surrounding the RBC is uniform and transparent at this wavelength. You may use the relationships listed below the problem. State any assumptions that you make.

- (3 pts) The RBC is illuminated by a unit-amplitude, on-axis plane wave at  $\lambda=650\text{nm}$ . What is the nearest distance can we assume that just light diffracted from the RBC is in the Fraunhofer zone?
- (3 pts) Using Babinet's principle, write an expression to represent the total field transmitted through the plane of the RBC.
- (4 pts) Assume  $\phi(x_s, y_s) = C$ , where  $C = \text{constant}$  and  $C \ll 1$ . Find an expression that is proportional to irradiance in the Fraunhofer plane of the RBC diffraction using the expression from (b), where you may assume that the plane-wave portion of the expression found in (b) propagates with simply  $\exp(jkz_0)$ .

Useful Relations:

$$\mathbf{F}_\eta \mathbf{F}_\xi \left[ \text{circ}\left(\frac{\sqrt{x^2 + y^2}}{D}\right) \right] = \frac{\pi D^2}{4} \text{somb}\left(\sqrt{\xi^2 + \eta^2}\right)$$

Zero	r
1 <sup>st</sup>	1.2196
2 <sup>nd</sup>	2.2331
3 <sup>rd</sup>	3.2383



PhD Qualifying Exam  
Fall 2020

OPTI 511/544/570 Day 2

1. Write down the time-dependent Schrodinger equation for the wave function  $\Psi(x,t)$  describing the quantum motion of a particle of mass  $m$  in a one-dimensional harmonic oscillator (HO) potential of frequency  $\omega$  and which is centered on  $x=0$ . (1 pt)
2. Give an expression for the energy eigenvalues  $E_n$  for the quantum HO. (1 pt)
3. Sketch the form of the ground and first excited energy eigenstates  $\psi_n(x)$ ,  $n=0,1$  for the quantum HO (discard any overall phase factors and take the energy eigenstates to be real). (2 pts)
4. Consider an initial wave function of the form  $\Psi(x,t=0) = c_0\psi_0(x) + c_1\psi_1(x)$ , with  $c_0$  and  $c_1$  real and greater than zero,  $c_0^2 + c_1^2 = 1$ , and  $c_0 \gg c_1$ . Given the form of the ground and first excited energy eigenstates  $\psi_n(x)$ ,  $n=0,1$  you sketched in part (3) argue that this initial wave function is like the ground state but with its center displaced slightly to one side with respect to the origin. (2 pts)
5. Given the initial wave function above write down an expression for the wave function  $\Psi(x,t)$  for  $t > 0$  that involves the frequency  $\omega$ . (1 pt)
6. Using your solution from part (5) describe how the probability density profile is changed with respect to the initial profile for times  $t=\pi/\omega$  and  $t=2\pi/\omega$ . (2 pts)
7. Based on what you have learned above describe how the probability density profile changes with time  $t=[0,2\pi/\omega]$ . (1 pt)