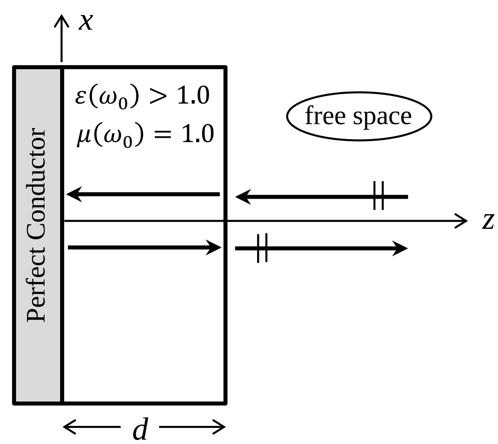
Student Code # ___________

Fall 2013 Written Comprehensive Exam
Opti 501

System of units: MKSA

A homogeneous, monochromatic plane-wave arrives at normal incidence on a perfect electrical conductor coated with a dielectric layer of thickness d, as shown. The boundary between the dielectric and the perfect conductor is the xy-plane at $z = 0$. The refractive index of the dielectric is given by $n(\omega) = \sqrt{\mu(\omega)\varepsilon(\omega)}$, where the relative permittivity $\varepsilon(\omega)$ is real-valued and greater than 1.0, while the relative permeability $\mu(\omega)$ may, as usual, be set equal to 1.0 at optical frequencies. Inside the dielectric layer, the (real-valued) E-field amplitude is given by $\boldsymbol{E}(\boldsymbol{r}, t) = E_1\hat{\boldsymbol{x}} \sin(k_1 z + \varphi_1) \cos(\omega_0 t)$.



1 Pt a) Use the relevant boundary condition at the surface of the perfect conductor to determine the value of φ_1.

2 Pts b) Use Maxwell's equation $\boldsymbol{\nabla} \times \boldsymbol{E}(\boldsymbol{r}, t) = -\partial \boldsymbol{B}(\boldsymbol{r}, t)/\partial t$ to determine the H-field amplitude $\boldsymbol{H}(\boldsymbol{r}, t)$ inside the dielectric.

2 Pts c) Use Maxwell's equation $\boldsymbol{\nabla} \times \boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{J}_{\text{free}}(\boldsymbol{r}, t) + \partial \boldsymbol{D}(\boldsymbol{r}, t)/\partial t$ in conjunction with the results obtained in parts (a) and (b) to determine the propagation constant k_1 in terms of ω_0, $n(\omega_0)$, and c, the speed of light in vacuum.

1 Pt d) Show that the remaining Maxwell equations, $\boldsymbol{\nabla} \cdot \boldsymbol{D}(\boldsymbol{r}, t) = \rho_{\text{free}}(\boldsymbol{r}, t)$ and $\boldsymbol{\nabla} \cdot \boldsymbol{B}(\boldsymbol{r}, t) = 0$, are automatically satisfied.

In the free-space region $z > d$, The E-field amplitude is $\boldsymbol{E}(\boldsymbol{r}, t) = E_0\hat{\boldsymbol{x}} \sin(k_0 z + \varphi_0) \cos(\omega_0 t)$. (This standing wave is the result of interference between the incident and reflected plane-waves.)

2 Pts e) Use Maxwell's equations as before to determine $\boldsymbol{H}(\boldsymbol{r}, t)$ in the free-space region, and also to relate the propagation constant k_0 to ω_0 and c.

2 Pts f) Invoke the relevant boundary conditions at $z = d$, the interface between free-space and the dielectric layer, to relate E_1/E_0 and φ_0 to the various parameters of the system.

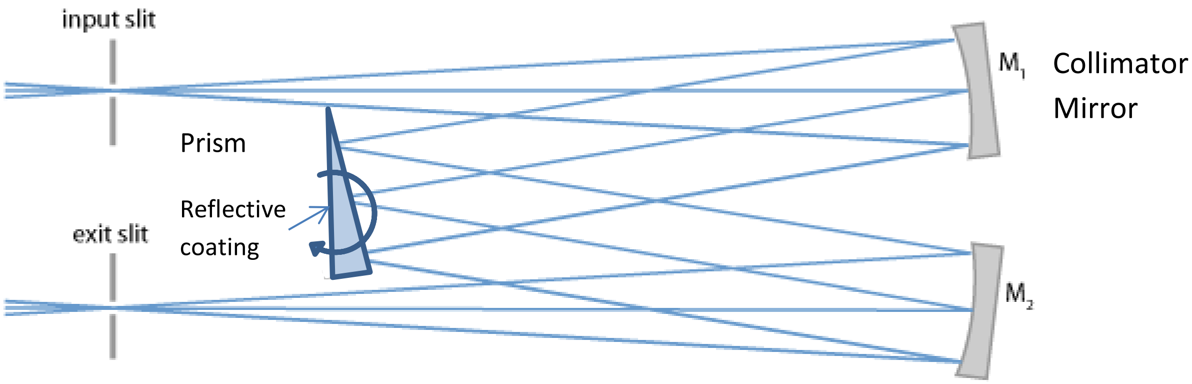
$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{z}}.$$

Student Code # ___________

Fall 2013 Written Comprehensive Exam
OPTI 502

Consider a prism spectrograph that uses the Czerny-Turner geometry as shown. The prism is tilted to select the wavelength.



Properties:
Prism aperture is 10 mm diameter, wedge angle is 0.1 radian
Prism is made of SF57, $n_d = 1.85$, $v_d = 24$
Input and exit NA = 0.1
Input and exit slit are both 2.5 µm wide

F: 486 nm
d: 587 nm
C: 656 nm

Using paraxial optics, including the small angle approximation, provide the following:

1. (1 pt) Focal length and approximate diameter for M1 and M2
2. (1 pt) Calculate the shift in the image at the exit slit for a 1 mrad rotation of the prism
3. (1 pt) Calculate the approximate $dn/d\lambda$ for PK51
4. (2 pts) Calculate the approximate spectral deviation $d\theta/d\lambda$ for the light reflected from the prism
5. (2 pts) Calculate the approximate dispersion of the light incident on the exit slit in units of microns/nm
6. (2 pts) Calculate the approximate resolving power, $\frac{\lambda}{\Delta\lambda}$, for this system
7. (1 pts) Calculate the prism rotation needed to change the output wavelength by 50 nm

Fall 2013 Comprehensive Exam
OPTI 503

An optical system has a focal length f = 100 mm and an entrance pupil diameter D_E = 25 mm. The object distance L = -1000 mm and the wavelength λ = 0.5 μm. Answer the following:

a) (1 point) Where is the image formed?

b) (1 point) What is the magnification?

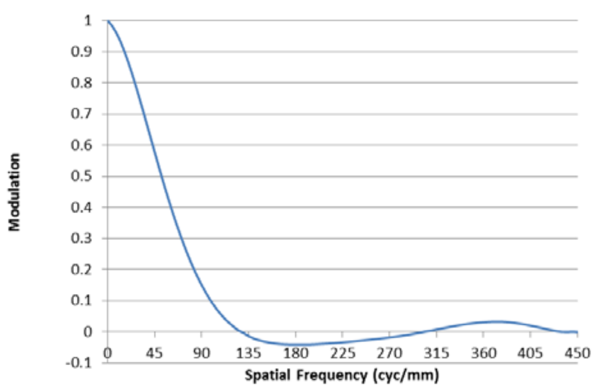
c) (1 point) What is the F-Number f/#?

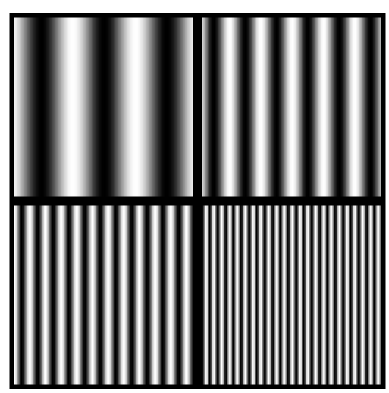
d) (1 point) What is the Working F-Number $f/\#_w$?

e) (1 point) Based on the Rayleigh Criterion, what is the resolution limit of the system?

f) (1 point) What is the cutoff frequency in image space f_c of the Optical Transfer Function (OTF) for this system?

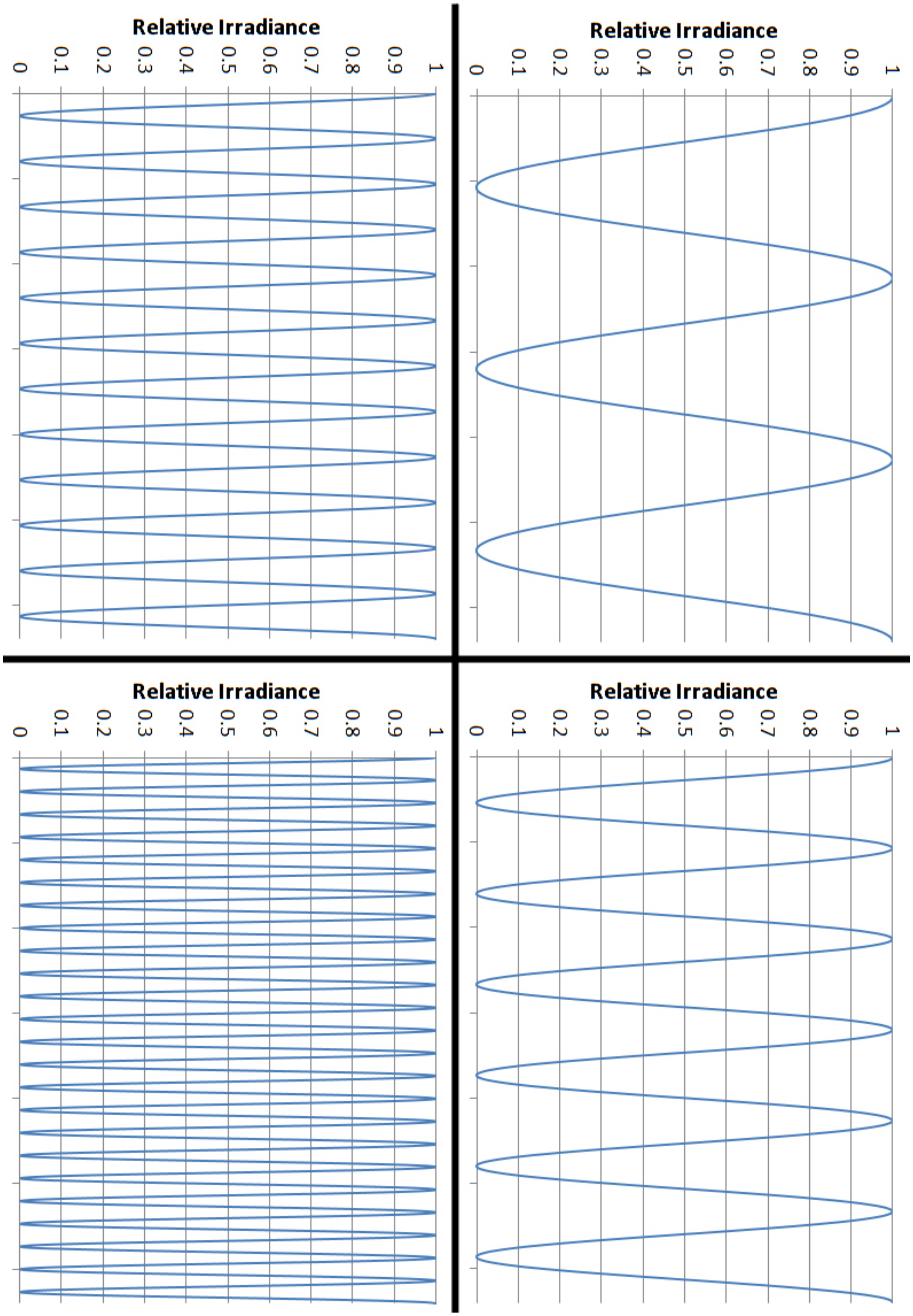
g) (4 points) The OTF for the optical system is shown below. The object (also shown below) consists of four sinusoidal patterns with 100% contrast. The image space spatial frequencies of the patterns are 90, 180, 360 and 720 cyc/mm (You can ignore edge effects and just treat these as pure sinusoids). The plots on the following page show the irradiance profile through each of the four sinusoids in the image plane. Each plot assumes a "perfect" imaging system in which the 100% contrast in each sinusoid is preserved. However, the OTF tells us that the true image is degraded. Sketch on each plot, the true irradiance profile through each sinusoidal pattern. Be sure to label important features such as period, average irradiance, maximum and minimum irradiance, etc.





Student Code # ___________

Fall 2013 Comprehensive Exam
OPTI 503 (Continued)



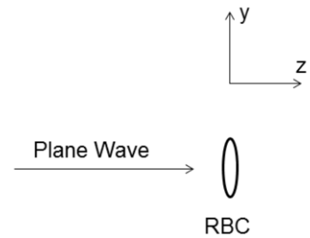
Student Code # ___________

Fall 2013 Written Comprehensive Exam
OPTI 505

A red blood cell (RBC) can be approximated by a phase circle that is D = 10μm in diameter. It is transparent at the 650nm wavelength. Transmission of the RBC is modeled by

$$t_{RBC}(x_s, y_s) = \exp\left[j\phi(x_s, y_s)\right]\mathrm{circ}\left(\frac{\sqrt{x_s^2+y_s^2}}{D}\right)$$

$$\approx \left[1 + j\phi(x_s, y_s)\right]\mathrm{circ}\left(\frac{\sqrt{x_s^2+y_s^2}}{D}\right) \quad ,$$



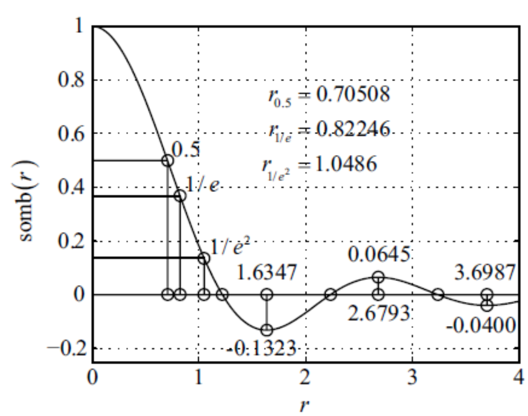
where the RBC phase is weakly scattering. The medium surrounding the RBC is uniform and transparent at this wavelength. You may use the relationships listed below the problem. State any assumptions that you make. (Each section is worth 2.5pts.)

a) The RBC is illuminated by a unit-amplitude, on-axis plane wave at λ=650nm. What is the nearest distance can we assume that just light diffracted from the RBC is in the Fraunhofer zone?
b) Using Babinet's principle, write an expression to represent the total field transmitted through the plane of the RBC.
c) Assume $\phi(x_s, y_s) = C$, where C = constant. Find an expression that is proportional to irradiance in the Fraunhofer plane of the RBC diffraction, where you may assume that the plane wave-portion of the expression found in (b) propagates with simply $\exp(jkz_0)$.
d) Assume that the area over which coherent illumination is necessary in the Fraunhofer zone is 600μm diameter and that the distance from the source to the Fraunhofer zone is 100mm. What is the maximum diameter quasimonochromatic incoherent extended source that can be used for illumination?

Useful Relations:

$$\mathbf{F}_\eta \mathbf{F}_\xi\left[\mathrm{circ}\left(\frac{\sqrt{x^2+y^2}}{D}\right)\right] = \frac{\pi D^2}{4}\mathrm{somb}\left(\sqrt{\xi^2+\eta^2}\right)$$

Table B.3 Zeros of the somb(r) function	
Zero	r
1st	1.2196
2nd	2.2331
3rd	3.2383



Fall 2013 Comprehensive Exam
OPTI 507

Consider an optical transition (vertical in k-space) in GaAs. For simplicity, assume the conduction and valence band to be parabolic and isotropic with equal effective electron and hole masses, $m_e = m_h = 0.1m_0$ (where m_0 is the electron mass in vacuum) . Assume you want to excite the state at $k = 0.1\text{A}^{-1}$ (A=Angstrom), and your laser frequency is $\hbar\omega = 1.8\text{eV}$. The bandgap varies approximately linearly with temperature, with $E_g(300\text{K}) = 1.43\text{eV}$ and $\left.\frac{dE_g}{dT}\right|_{300\text{K}} = -5\times10^{-4}\frac{\text{eV}}{\text{K}}$. Determine the temperature you need to make the desired optical transition.

(You may use $\hbar^2 / m_0 = 7.62 \text{ eV A}^2$.)

(10 points)

Fall 2013 Written Comprehensive Exam
OPTI 510

Comparison of a Fabry-Pérot filter and a diffraction grating

We want to build a spectrometer using either a Fabry-Pérot filter or a diffraction grating. The spectrometer will operate from 400nm to 700nm, with the center wavelength of our signal λ =550nm. The filter is made of two identical mirrors of amplitude reflectance of 0.993 separated by an air gap of 1.5mm. The grating has a groove density of 1800mm^{-1} and a dimension of 1cm x 1cm. We are using the grating's second diffraction order.

(a) What is the free spectral range, ν_F, of the Fabry-Pérot filter? (2 points)

(b) What is the spectral width, $\delta\nu$, of the filter, if the finesse of the filter is 223? (2 points)

(c) Write down the grating equation and define the parameters. (2 points)

(d) What is the resolution, $R=\lambda/\Delta\lambda$, of the grating? The resolution is defined by minimum detectable separation $\Delta\lambda$ between two spectral peaks. You can assume the entire grating is illuminated by the signal. (Hint: R is proportional to the number of lines illuminated by the signal) (2 points)

(e) If you want to build a spectrometer with the highest resolution, which component will you use, the Fabry-Pérot filter or the grating? Assume a single pass configuration for both components. (2 points)

Fall 2013 Written Comprehensive Exam
Opti 511R

Student Code # ___________

A laser beam is sent through a 4 cm long cell containing hydrogen atoms in thermal equilibrium with the room temperature cell walls. The atomic density in the cell is $N = 10^{11}/\text{cm}^3$.The laser is resonant with the 1S→2P transition of the hydrogen atom. Assume the absorption cross-section on resonance is $\sigma(\omega_o) = 10^{-12}$ cm^2 and the on resonance saturation intensity is $I_{sat} = 5\text{W/cm}^2$.

(a) Do you expect the transition absorption profile to be homogeneously or inhomogeneously broadened? Please state why in 1 or 2 sentences.

(b)Determine the laser wavelength (to 3 significant digits).

(c) An external magnetic field is applied to the atoms along the direction of the laser beam. To ensure only the m=1 sub-level of the 2P state is excited, should the laser polarization be linear or circular?

(d) Assuming the initial intensity of the laser is much less than the saturation intensity of the transition, calculate the percentage of power transmitted through the gas cell.

(e) Calculate the intensity of the laser needed to saturate the absorption coefficient to 70% of its small signal value.

(f) If the intensity of the laser is turned up sufficiently high, is it possible to have a steady-state population inversion between the 1S and 2P energy levels? Please state why in 1 or 2 sentences.

(g) A second tunable probe laser is directed through the gas cell in the opposite direction of the first laser in order to make a measurement of the absorption coefficient versus detuning from ω_o. The probe laser intensity is much less than I_{sat}. Assuming the saturating laser beam from part (e) is still on, make a plot of the measured absorption coefficient versus frequency. Be sure to label the small-signal absorption coefficient, α_o, and the center of the atomic resonance, ω_o, on the axes of the plot.

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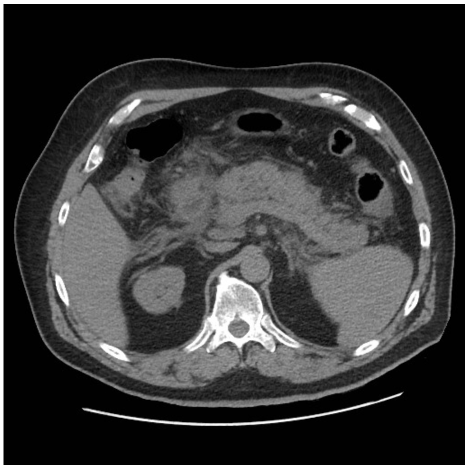
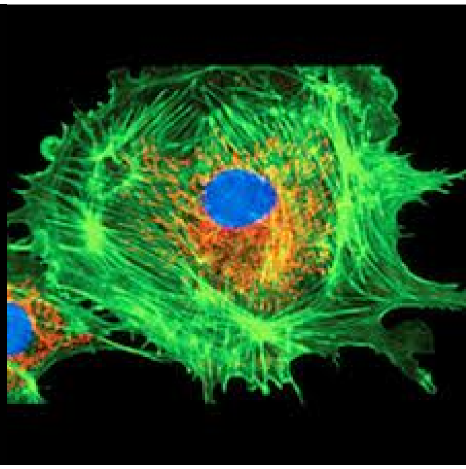
Fall 2013 Comprehensive Exam

OPTI 536



(1)



(2)

(3)

a) Identify which 2D image above (1, 2, or 3) is the fluorescence confocal microscope image, which is the pulse echo ultrasound image, and which is the x-ray CT image.
b) For each of the three imaging techniques, describe what form of energy is used to “illuminate” the object.
c) For each of these three imaging techniques, describe what is being imaged in the object. In other words, what physical characteristic of the object causes contrast in the image?
d) Each of these images represents a 2D slice from a 3D object. For each describe how one probes the object selectively in a 2D slice. In other words, how is the image content constrained to be coming from within a thin slice of a 3D object?
e) Each of these three imaging techniques requires some form of scanning process to generate a single 2D image. Briefly describe the scanning process. What is being “scanned” and how is the scanning accomplished?
f) Which technique requires an inverse operator from data to reconstructed image? Describe the inverse operation in simple mathematical terms, or alternatively by a description of what computational steps are required.
g) From your knowledge of the use of these techniques, provide a best guess as to the spatial field of view covered in each of the three images.

Fall 2013 Written Comprehensive Examination
OPTI 537

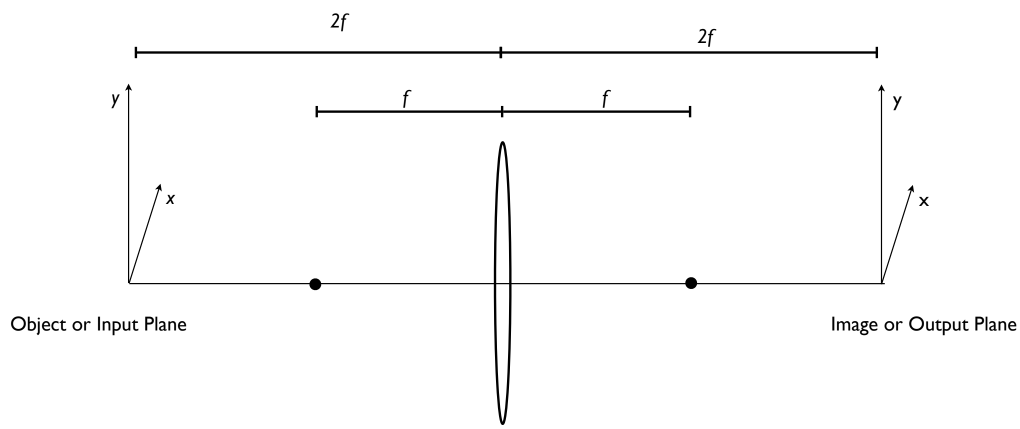
Consider imaging systems that are imaging coherent, monochromatic objects with wavelength λ. Fresnel diffraction for a system described by an ABCD matrix is given by:

$$u_{out}(\vec{r}) = \frac{-i}{B\lambda}\int_{-\infty}^{\infty} d^2r_0\, u_{in}(\vec{r_0})\exp\left(\frac{i\pi}{\lambda B}\left[A|\vec{r_0}|^2 + D|\vec{r}|^2 - 2\,\vec{r}\cdot\vec{r_0}\right]\right) \qquad Eq\ (1)$$

where both $\vec{r}$ and $\vec{r_0}$ are 2 vectors in the x-y plane. (A selection of ABCD matrices is given in the table at the bottom of the page.)

(a) Show that Equation 1 above simplifies to standard Fresnel transform when considering free-space propagation.

(b) Now consider the simple imaging system below,



Compute the ABCD matrix for this system and describe the difficulties of using this result with Equation 1.

(c) Now, move the object plane to the left focal point and the image plane to the right focal point. Compute the ABCD matrix for this new system and derive an expression for the Fresnel diffraction for this imaging problem. Be sure to comment on the final form of your expression.

(d) Use the ABCD matrices given in the table below to derive the ABCD matrix for a thick lens with radii of curvature R1 and R2, and center thickness t. Assume that the lens is in air. Describe the convention for the signs of R1 and R2.

(e) For a convex lens, assume that |R1| = |R2|, and t=0 (to approximate a thin lens). Derive an expression for the focal length (or 1/f if you prefer) using the expression you determined above.

ABCD Matrices:

Propagation in a medium of constant refractive index	$\begin{bmatrix}1 & d\\0 & 1\end{bmatrix}$	d = distance along optical axis
Refraction at a flat interface	$\begin{bmatrix}1 & 0\\0 & \frac{n_1}{n_2}\end{bmatrix}$	n1 = initial refractive index n2 = final refractive index
Refraction by a thin lens	$\begin{bmatrix}1 & 0\\-\frac{1}{f} & 1\end{bmatrix}$	F = focal length (f>0 means convex lens)
Refraction from a curved interface	$\begin{bmatrix}1 & 0\\(n_1-n_2)/Rn_2 & n_1/n_2\end{bmatrix}$	R = radius of curvature n1 = initial refractive index n2 = final refractive index

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Fall 2013 Written Comprehensive Exam
OPTI 544

Consider a gas of two-level atoms with ground and excited states $|1\rangle$, $|2\rangle$, transition frequency ω_0, and a (real-valued) transition dipole matrix element $\vec{p}$. The atoms interact with a monochromatic, classical light field $\vec{E}(t) = \vec{\varepsilon} E_0 \cos(\omega t)$, where $\vec{\varepsilon}$ is a (real-valued) unit polarization vector. Note: throughout this problem we mark vectors with arrows and operators with hats.)

(a) Write down the matrix for the atom Hamiltonian $\hat{H}$ in the $\{|1\rangle, |2\rangle\}$ basis, expressing it in terms of the resonant Rabi frequency χ, the optical frequency ω, and the detuning Δ. Indicate how these parameters are defined. (15%)

(b) Expressing the atomic state vector as $|\psi(t)\rangle = a_1(t)|1\rangle + a_2(t)|2\rangle$, write down the time-dependent Schrödinger equation in terms of the probability amplitudes. Introduce slow variables $c_1(t)$, $c_2(t)$ and make a Rotating Wave Approximation to obtain a new set of equations without rapidly oscillating terms. (45%)

(c) Based on your physical intuition about driven 2-level systems, guess the solution to the above equations for $\Delta = 0$ and $c_1(0) = 1/\sqrt{2}$, $c_2(0) = i/\sqrt{2}$. Test that it is valid by plugging it into the equations. Then plot the populations in states $|1\rangle$, $|2\rangle$ as function of time, being careful to indicate the timescale on which they change and by how much. (30%)

(d) The equations for $c_1(t)$, $c_2(t)$ in the RWA from (b) above constitute a Schrödinger equation in the interaction picture. Write down the matrix for the corresponding Hamiltonian $\hat{H}_{RWA}$ in the $\{|1\rangle, |2\rangle\}$ basis. What do we call the eigenstates of this Hamiltonian? (10%)

Fall 2013 Written Comprehensive Exam
Opti 546

Consider the situation where an initial field $E(x', y', 0)$ at $z = 0$ propagates in vacuum along the z-axis. Then for $z > 0$ the field may be expressed as the following diffraction integral based on the Huygens-Fresnel principle

$$E(x, y, z) = \frac{1}{i\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx'dy'E(x', y', 0)\left(\frac{e^{ikr}}{r}\right)\left(\frac{z}{r}\right), \qquad (1)$$

where $k = 2\pi/\lambda$, and $r = \sqrt{(x - x')^2 + (y - y')^2 + z^2}$.

(a - 3pts) List the key ideas and approximations that underly the reduction of the above diffraction integral to its form in the Fresnel approximation

$$E(x, y, z) \approx \frac{e^{ikz}}{i\lambda z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx'dy'E(x', y', 0)e^{\frac{ik[(x-x')^2+(y-y')^2]}{2z}}. \qquad (2)$$

(b - 2pts) Building upon part (a) list the approximations that underly the reduction of the Fresnel approximation in Eq. (2) to its form in the Fraunhofer region

$$E(x, y, z) \approx \frac{e^{ikz}}{i\lambda z}e^{\frac{ik(x^2+y^2)}{2z}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx'dy'E(x', y', 0)e^{-\frac{ik(xx'+yy')}{z}}. \qquad (3)$$

(c - 2pts) For an initial field at $z = 0$ we hereafter use the example of an incident plane-wave that is truncated by a circular aperture of radius 'a' that is centered on the origin. Then based on your analysis from part (b), show that the Fraunhofer region arises for $z >> z_0$ where the Rayleigh range is given by $z_0 = ka^2/2$.

(d - 2pts) Sketch the variation of the on-axis intensity $|E(0, 0, z)|^2$ versus propagation z distance past the uniformly illuminated circular aperture, assuming $|E(0, 0, 0)|^2 = 1$. You should indicate the propagation regions corresponding to the Fresnel and Fraunhofer regions with reference to your results from part (c).

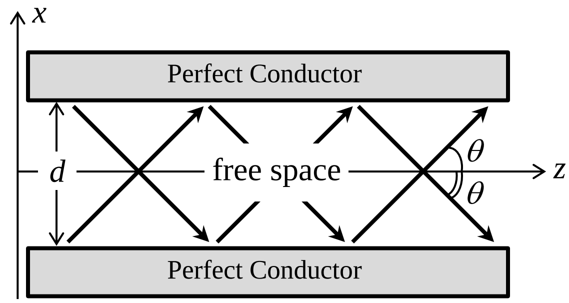
(e - 1pt) *Explain* what is meant by the term 'Airy disk' as applied to the present problem in the Fraunhofer region?

Student Code #___________

Fall 2013 Written Comprehensive Exam
Opti 501

System of units: MKSA

In the free-space region between two perfectly electrically conducting parallel plates, a guided electromagnetic wave propagates along the *z*-axis, as shown. The plates are separated by a distance *d* along the *x*-axis, and the guided wave is single-mode, monochromatic, and *p*-polarized. The (real-valued) magnetic field of the guided mode is $\boldsymbol{H}(\boldsymbol{r},t) = H_0\hat{\boldsymbol{y}}\cos(k_x x)\sin(k_z z - \omega_0 t)$.



2 Pts a) Use Maxwell's equation $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) + \partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t$ to determine the *E*-field profile of the guided mode.

2 Pts b) Use Maxwell's equation $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$ to determine the relationship among k_x, k_z, ω_0, and c, the speed of light in vacuum.

2 Pts c) What values of k_x are admissible if the guided mode is to satisfy the boundary conditions at the inner surfaces of the perfect conductors?

3 Pts d) Find the surface charge and current densities $\sigma_s(x=\pm\tfrac{1}{2}d,y,z,t)$ and $\boldsymbol{J}_s(x=\pm\tfrac{1}{2}d,y,z,t)$ at the inner surfaces of the perfect conductors.

1 Pt e) Show that σ_s and $\boldsymbol{J}_s$ obtained in part (d) satisfy the charge-current continuity equation.

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

$$\boldsymbol{\nabla}\times\boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{z}}.$$

Fall 2013 Written Comprehensive Exam
OPTI 502

An optical system in air is comprised of two thin lenses. The lens focal lengths are f_1=20mm and f_2=60mm. An aperture stop is placed between the two lenses so that the system is telecentric in both object space and image space. A circular object with a radius of 2mm is placed at a distance of Z_0=-15mm from the first lens. The slope of the paraxial marginal ray in object space is u_0=0.2 and the numerical aperture of the edge field is the same as that of the on-axis point. A circular detector is placed at the image plane to capture the image.

(a) Draw a neat diagram showing the optical axis, the two lenses, the object, the stop, and the detector. On the same diagram, trace the chief ray and marginal ray through the system. (2 points)

(b) Determine the separation t_1 between the lens 1 and stop and the separation t_2 between the stop and the lens 2. (1 point)

(c) Determine the transverse magnification of the system. (1 point)

(d) Determine the slope, u_i, of the marginal ray in image space. (1 point)

(e) Determine the distance, Z_I, of the image position from the lens 2 and the minimum diameter of the image detector. (2 points)

(f) Determine the diameter, D_s, of the aperture stop, the minimum lens diameters D_1 for lens 1 and D_2 for lens 2 which avoid vignetting. (2 points)

(g) If the object is displaced from its initial location to the left by a distance of Δz=-2mm, determine the direction and the amount you need to move the detector to stay in focus. (1 point)

Student Code #___________

Fall 2013 Comprehensive Exam
OPTI 503

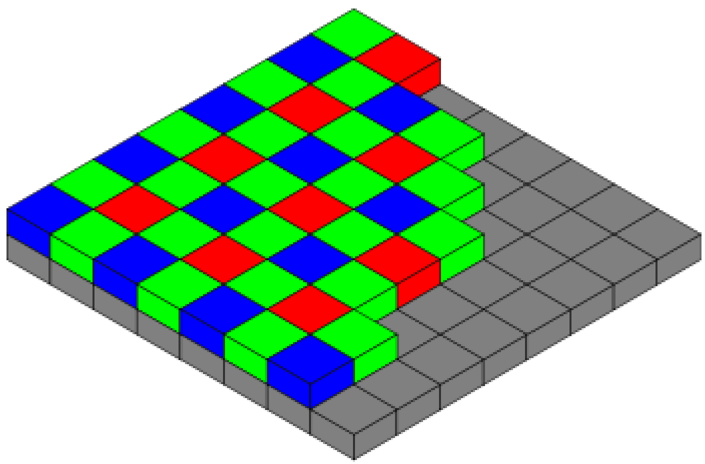
A diffraction-limited imaging system is operating at 0.5μm at a working F/# of 5.

A) (2 pts) What is the maximum pixel pitch that will guarantee alias-free sampling?

B) (3 pts) Assume that the focal plane is composed of 5 micron pixels with 100% duty cycle (in other words, the pixels are square, 5-microns on a side, and have no space between them). Sketch the *combined* MTF of the optics and the FPA at 0.5μm.

C) (3 pts) What happens to the system performance as the duty cycle of the pixels decreases (pixel pitch remains constant)?

D) (2 pts) Now assume that the FPA has an integrated Bayer filter pattern of red, green, and blue pixels as shown below. The object has a spectrum that only excites the *green* pixels, and its image is of the form $\cos\left(2\pi\xi_0\left(\cos\theta_0 x + \sin\theta_0 y\right)\right)$, where ξ_0 = 0.1 cycles/micron. Determine the spatial Fourier transform of the sampled image for θ_0 = 0° and θ_0 = 45°. Show the Nyquist domain.



http://en.wikipedia.org/wiki/Bayer_filter

Student Code #___________

Fall 2013 Written Comprehensive Exam
OPTI 505

a.) (2pts) Two waves are described by:

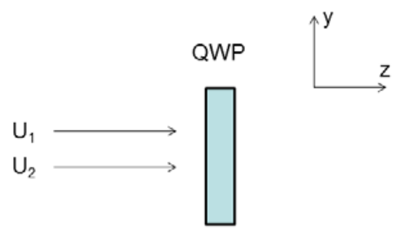
$$\boldsymbol{U}_1(x,y;z) = A\exp\left[j\left(\frac{2\pi}{\lambda}z - \omega t + \phi(x,y)\right)\right]\hat{\boldsymbol{x}}$$

$$\boldsymbol{U}_2(x,y;z) = A\exp\left[j\left(\frac{2\pi}{\lambda}z - \omega t\right)\right]\hat{\boldsymbol{y}}$$

What is the fringe visibility?

b.) (2pts) Write the 2x2 Jones matrix description for an unrotated quarter-wave plate (QWP). Normalize the (1,1) term to unity, assume the leading constant term has a magnitude of unity.

c.) (2pts) The crystal fast and slow axes of the QWP from (b) are aligned to the (*x*,*y*) Cartesian directions, and the QWP is placed in the path of the two waves described in (a), as shown below. What is the fringe visibility on the right-hand side of the QWP?



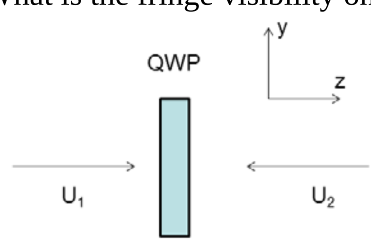
d.) (2pts) The QWP in (c) is rotated around the z axis by 45°. What is the fringe visibility on the right-hand side of the QWP?

e.) (2pts)Wave 2 is reversed, so that

$$\boldsymbol{U}_1(x,y;z) = A\exp\left[j\left(\frac{2\pi}{\lambda}z - \omega t + \phi(x,y)\right)\right]\hat{\boldsymbol{x}}$$

$$\boldsymbol{U}_2(x,y;z) = A\exp\left[j\left(-\frac{2\pi}{\lambda}z - \omega t\right)\right]\hat{\boldsymbol{y}}$$

The waves illuminate the QWP in the rotated (d) configuration, as shown below. What is the fringe visibility on the right-hand side of the QWP?



Student Code #___________

Fall **2013 Written Comprehensive Exam**
OPTI506

I'm planning on taking some photographs with a lens that has a 25 mm focal length. The active area of the sensor is 24 mm x 36 mm. The exit pupil of the lens is 22 mm from the back focal plane, 3 mm from the rear principal plane and it is round with a diameter of 9 mm. I'm taking the pictures outside, so you may assume that the sunlight results in 1 kW/m^2 on the scene. You should assume that the scene is very large. Since you don't know what the scene is, you should also assume that it is a uniform, Lambertian scatterer that reflects 18% of the incident power. You may assume that the lens is ideal and lossless. If you make any other assumptions, you must clearly state what they are. Please answer the following questions:

a) (40%) For an in focus object 1 km from the lens, what is the irradiance at the center of the sensor?
b) (15%) If I decrease the aperture size so that the exit pupil now has a diameter of 4.5 mm, what will the irradiance at the center of the sensor be?
c) (15%) Write the exact integral (this means no approximations other than light is a ray and the ones listed above) that you would need to solve to determine the total amount of power on the sensor when the aperture has a 9 mm diameter. Make sure that it is clear which limits of integration go with which variables of integration. The correct answer for this part is an integral that could be solved, but you do <u>not</u> need to solve the integral.
d) (15%) I now want to take a close-up photograph. I want to take a picture where the magnification is -1 and the exit pupil diameter is 9 mm. If I position my lens the correct distance from the object, but the focus of the camera is still set for part (a), what will the irradiance be at the center of the sensor?
e) (15%) If I now adjust the focus of my camera so that the object is in focus and the field of view has the same size as the sensor, what will the irradiance be at the center of the sensor?

Fall 2013 Comprehensive Exam
OPTI 507

Consider acoustic phonons (assumed to have linear dispersion) and optical phonons (assumed to be dispersionless) in GaAs.

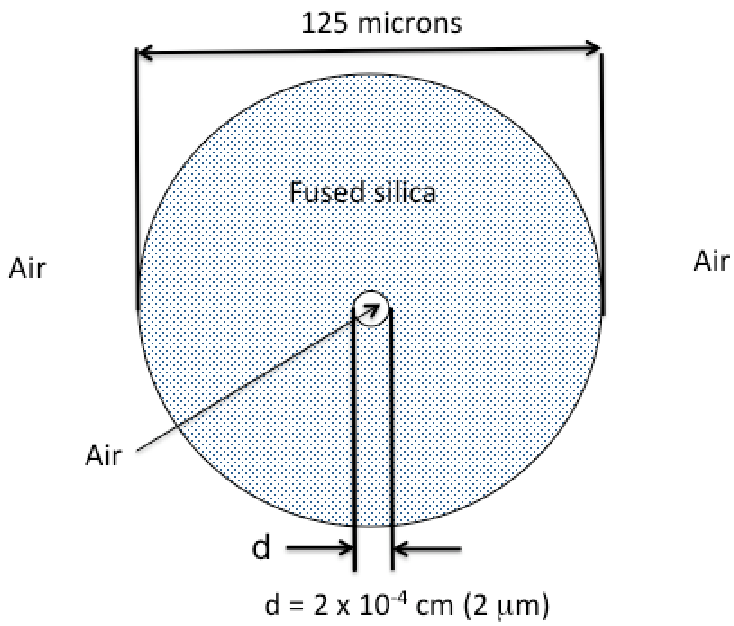
Sketch the dispersion together with the dispersion of light inside GaAs (assuming the refractive index to be $n = 3.6$). Assume the sound velocity to be $c_s = 10^{-5} c$ (where c is the light velocity in vacuum) and the optical phonon frequency to be $\hbar\Omega_{opt} = 36\text{meV}$ (your sketch does not need to be to scale). Clearly label all dispersions and indicate the point of possible strong light absorption. For that point, determine the value of the k-vector and the corresponding wavelength (inside the medium).

Finally, sketch the corresponding phonon polariton dispersion. No calculations regarding polaritons are requested, but the sketch needs to be qualitatively correct. Indicate on the vertical axis the position of 36meV.

(10 points)

Fall 2013 Written Comprehensive Exam
OPTI 510R

This problem concerns the optical fiber shown below. It is a hollow core fiber with a core diameter of 2 μm and a fused silica cladding with a diameter of 125μm as shown (not to scale). The hollow core is initially filled with air at atmospheric pressure. Light propagates at 1550nm in this fiber, where the fused silica cladding has a refractive index of 1.4440.



We want to fill the fiber with a mixture of two liquids that will allow it to propagate light in a single mode. The two liquids are carbon disulfide (CS_2) with n_{1550nm} = 1.590 and bromotrichloromethane ($CBrCl_3$) with n_{1550nm} = 1.507. An approximate expression for the refractive index at λ_0=1550nm of a mixture that had X volume fraction CS_2 and (1-X) volume fraction $CBrCl_3$, where $0 < X < 1$, is given by

$$n_{1550nm}^{mixture} = 1.507 + 0.083X$$

a) What is the maximum value of X for which the fiber stays single-mode at 1550nm? (4 points)

b) Draw a sketch that compares the mode shape for the value of X determined in a) with the mode shape for X = 0 (pure $CBrCl_3$). (2 points)

c) In conventional all-glass single-mode optical fiber (i.e. SMF-28), birefringence induced by anisotropic stresses is often a problem. Do you expect the liquid core fiber we have developed to have lower or higher birefringence than standard single-mode optical fiber? Explain your answer. (2 points)

d) For X = 0 (pure $CBrCl_3$) determine the transmission loss in dB per meter at 1550nm, if the absorption coefficient of $CBrCl_3$ is given by 1.5 m^{-1} at 1550nm. For the purposes of this question assume that all of the guided light is in the core. (2 points)

Fall 2013 Written Comprehensive Exam
Opti 511R

Student Code #___________

Assume a particle of mass m is trapped in a three-dimensional harmonic oscillator potential centered at the position (x, y, z)= (0,0,0). The potential strength in each direction is characterized in the usual way by the oscillation frequencies ω_x, ω_y, and ω_z. For this problem, let $\omega_x = \omega_y = 10\omega_z$.

(a) Write out the Hamiltonian associated with this problem.

(b) Write an expression for the eigenvalues of the Hamiltonian.

(c) What is the ground state energy of the system expressed in terms of ω_z?

(d) How much energy must be added to the system to excite the particle from the ground state of the potential well to the first excited state?

The solutions to the Time-Independent Schrödinger Equation (TISE) may be written as the product state $\psi(x, y, z) = \psi_x(x) \cdot \psi_y(y) \cdot \psi_z(z)$. **For the rest of the problem, assume the particle is in the first excited state of the potential well**.

(e) Make a plot of $\psi_x(x)$ versus x.

(f) Given that here $\psi_y(y) = Ae^{-y^2/(2\sigma^2)}$, where $A = (\sigma\sqrt{\pi})^{-1/2}$, use the TISE to express the constant σ in terms of quantities already defined. If you know the expression for σ already, be sure you still use the TISE to prove this.

Day 2 Exam

Comprehensive Exam, Fall 2013
Opti 536

Student Code #___________

Parts a-e collectively are worth 7 points. Part f is worth 3 points

Consider an ideal thin lens with focal length 50 mm and a square 2 mm X 2 mm aperture in the lens plane. This lens is used to image a planar self-luminous object at 1:1 magnification onto an image plane. The object is perpendicular to the optical (z) axis, and it emits light in a narrow wavelength range around 500 nm In Cartesian coordinates, the object and image will be functions of x and y and the edges of the aperture are parallel to the x and y axes.

a. Explain qualitatively the concept of Modulation Transfer Function (MTF). What is meant by the word modulation? Of what variables is MTF a function? What is being 'transferred'?

b. Discuss the MTF of the imaging system described above, and illustrate your discussion with appropriate sketches. Remember that MTF is a function of two variables, so a 1D sketch will not suffice. Label the axes in your sketches carefully.

c. Compute the cutoff spatial frequencies in the x and y directions; call them ξ_{max} and η_{max} , respectively. Give the numerical value for the absolute highest spatial frequency that can be passed by the system.

d. Consider a cosinusoidal object with radiant exitance given by $M(x,y) = M_0[1 + \cos 2\,\pi\,\xi_0\,x]$. Compute the modulation (contrast) of this object. What is the modulation in its image if $\xi_0 \ll \xi_{max}$? Sketch the image in this case; a 1D sketch as a function of x will suffice.. Repeat the modulation calculation and sketch for $\xi_0 = ½\,\xi_{max}$.

e. Now let $\xi_0 = 1.1\,\xi_{max}$. Compute the modulation and sketch the image. How does the modulation in the image change in this case if the cosinusoidal object is rotated by 45° around the optical axis?

f. Now suppose we place a detector array, such as a CCD or CMOS camera, in the image plane. The detector elements are squares of size $\epsilon \times \epsilon$ with edges aligned with the x and y axes. There are no gaps between detector elements. What must the dimension ϵ be to avoid aliasing? What is the effect of the finite detector size on the MTF?

Student Code #___________

Fall 2013 Written Comprehensive Examination
OPTI-537

Answer the following questions related to semiconductor detectors. All parts weighted as indicated.

(a) (10%) What is the definition of a Bravais lattice? What is a primitive lattice vector? Write an expression for a translation vector **R** that spans the equivalent locations in a real-space (3D) Bravais lattice.

(b) (10%) Write the expression that defines the relationship between the direct (**R**) and reciprocal (**G**) lattice vectors for a perfect crystal. Define all terms.

(c) (10%) Write the 3D time-independent Shrödinger equation for a single electron in a potential with periodicity defined by a Bravais lattice. Show where the periodicity is expressed in the Hamiltonian and explain what assumptions are necessary to invoke the time-independent form of the Shrödinger equation to describe the electronic structure and related properties of crystals.

(d) (10%) What is the significance of the reciprocal lattice when, for example, one is writing a Fourier representation for a function that obeys the periodicity of the lattice. An example is the potential energy, which obeys $V(\mathbf{r}) = V(\mathbf{r}+\mathbf{R})$. Write such a Fourier representation for $V(\mathbf{r})$.

(e) (10%) What is Bloch's theorem? What is a Bloch function? Show that a Bloch function can be alternatively written as

$$\psi(\mathbf{r}) = \sum_{\mathbf{G}} u_{\mathbf{G}}(\mathbf{k})\, e^{i(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}}.$$

(f) (20%) What is the quantum mechanical operator that corresponds to momentum. Work out the expectation value for the momentum for an electron with the wavefunction as written in problem (e).

(g) (15%) Draw the band structure of an unbiased PN junction. Sketch the locations of donor and acceptor dopant states relative to the conduction and valence bands, and indicate where the Fermi level is at room temperature. Label which side is P and which is N and clearly indicate majority and minority carriers. Finally, clearly diagram and label the 2 electron and 2 hole currents present that exactly balance when the diode is in thermal equilibrium.

(h) (15%) Make a sketch of the diode current versus voltage (IV) curve, indicate which of the currents from (g) dominate in the reverse and forward biased regions. Explain why photodetectors in CMOS imaging sensors are normally operated in conditions of reverse bias.

Fall 2013 Written Comprehensive Exam
OPTI 544

Two separate groups of otherwise identical three-level atoms are prepared in different superpositions of the energy levels $|1\rangle, |2\rangle$, and $|3\rangle$. The groups contain equal numbers of atoms.

The atoms of the first group are prepared in the state

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + i|2\rangle\right).$$

The atoms of the second group are prepared in the state

$$|\psi_2\rangle = \frac{1}{2}\left(|1\rangle + i\sqrt{3}\,|3\rangle\right).$$

All of the atoms are then thoroughly mixed. We will assume that the mixing process does not change the state of each individual atom in the ensemble.

(a. 4 pts). Find the density matrix for the ensemble.

(b. 3 pts). Is the ensemble in a pure or mixed state? Justify your answer mathematically.

(c. 3 pts). Suppose that an unspecified observable $\hat{A}$ exists for which α is an eigenvalue and $|\phi\rangle = \frac{1}{\sqrt{5}}\left(|1\rangle + i|2\rangle + i\sqrt{3}\,|3\rangle\right)$ is the corresponding eigenstate (assume that $\hat{A}$ has a non-degenerate spectrum). If a measurement of this observable is made on all atoms of the ensemble, what fraction of atoms will give the result α?

Student Code #___________

Fall 2013 Written Comprehensive Exam
Opti 546

(a- 2pts) Name and sketch an example of a interferometer involving two or more beams that is based on division of amplitude.

(b - 1pt) Consider a two-arm interferometer with free-space path lengths L_1 and L_2 in the two arms that employs a single light source with coherence time $\tau = 3\ ps$. *Explain* under what conditions on the path lengths L_1 and L_2 the visibility of interference fringes at the output of the interferometer will become degraded. (Assume that in the limit $\tau \to \infty$ the interference fringes have unit visibility.)

(c - 2pts) Write down the Lorentz electron oscillator model in component form in the principal axis system as appropriate to a uniaxial crystal, and identify the terms that account for the presence of longitudinal and transverse electric fields.

(d - 1pt) Identify two approximations employed in obtaining the Lorentz oscillator model you gave in part (c).

(e - 2pts) *Explain* why the Lorentz oscillator model you gave in part (c) cannot describe the process of harmonic generation, and list the additional physics that must be added to allow for the treatment of harmonic generation in transparent dielectric crystals. You may consider the simplified one-dimensional model for your discussion.

(f - 2pts) For an isotropic medium give a description of the circular dichroism that can result from the magneto-optical Zeeman effect making sure to highlight the geometry involved, and list the physics that must be added to the Lorentz electron oscillator model in part (c) to describe this effect.