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OPTI 380A
Intermediate Optics Lab 12: Diffraction

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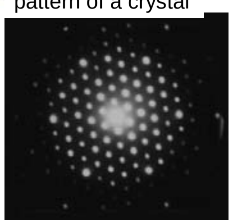
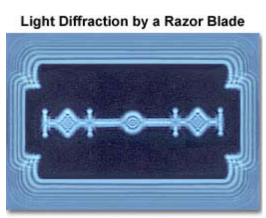
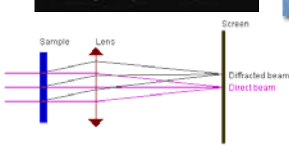
Examples of diffraction

electron diffraction pattern of a crystal

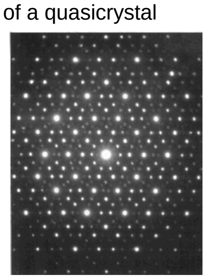




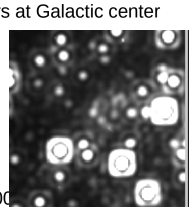
silver lining occurs when light is diffracted by cloud droplets along cloud's outer edge

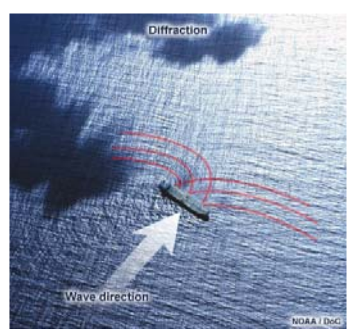


X-ray diffraction pattern of a quasicrystal



Telescope image showing diffraction rings (Airy rings) around stars at Galactic center

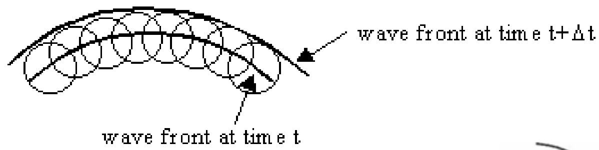




wave energy is bended around corner of ship

Diffraction From a Circular Aperture

- Huygens Principle of Diffraction

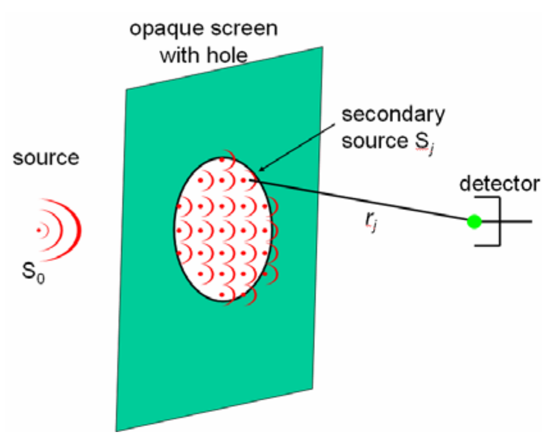


Assume new wavefront is combination of spherical waves from the first wavefront.

Christiaan Huygens, *Treatise on Light* (1690)

Diffraction From a Circular Aperture

- Interpretation for a circular aperture



Common Sense:

- In-phase radiators produce *constructive interference* (bright spot)
- Out-of-phase radiators produce *destructive interference* (dark spot)
- Relative phase is determined by path length from *S*.

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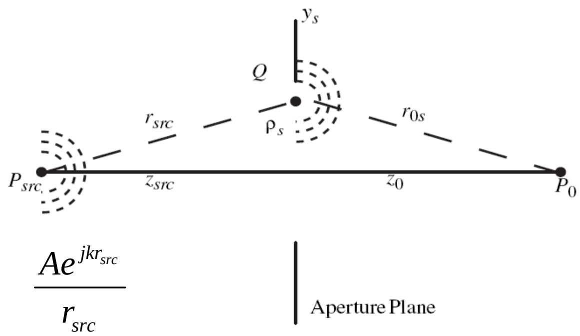
Diffraction From a Circular Aperture

- Diffraction geometry (on-axis source and observation)



$$\frac{Ae^{jkr_{src}}}{r_{src}}$$

Phase of light from radiator Q at P_0 relative to on-axis:

$$\phi = \frac{2\pi}{\lambda}\left[r_{src} + r_{0s} - \left(z_{src} + z_0\right)\right]$$
$$= \frac{2\pi}{\lambda} OPD$$

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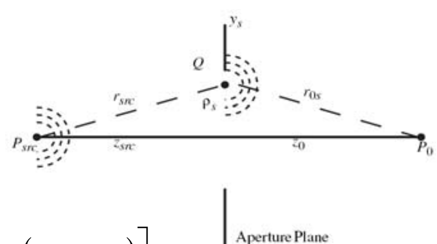
Diffraction From a Circular Aperture

- Simplify *OPD*



$$OPD = \left[r_{src} + r_{os} - \left(z_{src} + z_0\right)\right]$$
$$= \left[\sqrt{\rho_s^2 + z_{src}^2} + \sqrt{\rho_s^2 + z_o^2} - \left(z_{src} + z_0\right)\right]$$
$$\approx \frac{\rho_s^2}{2L}$$

$$L = \frac{z_{src} z_0}{z_{src} + z_0}$$

OPD increases quadratically as radius increases.

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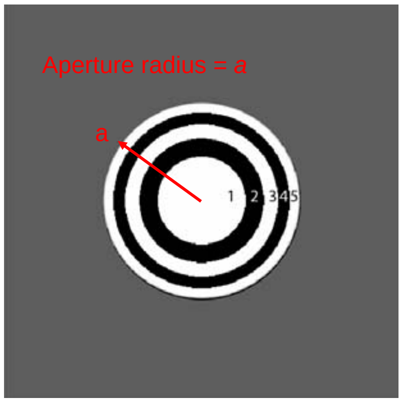
Diffraction From a Circular Aperture

- Definition of Fresnel Zones



- *Fresnel zones* specified by λ/2 *OPD* boundaries in the aperture.
- Changes with z_{src}, z_0 , a and λ.
- N_f = *Fresnel Number* = Total number of zones

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Diffraction From a Circular Aperture

- Some relationships
  - $N_f = \dfrac{a^2}{\lambda L}$
  - Area of each zone = $A_f = \pi\lambda L$
  - Zone radius = $\rho_m = \sqrt{m\lambda L}$

Important!!! – This is the ***Fresnel Number***

Diffraction From a Circular Aperture

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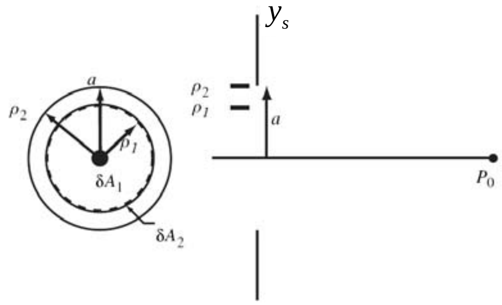
- Consider the case where $N_f = 2$



- Differential areas dA_1 and dA_2 have equal area and are π out of phase.
- Destructive interference occurs between dA_1 and dA_2 , so a dark spot at P_0 occurs for light from these two sub-zones.
- The next pair of sub-zones exhibit the same behavior, as do all the corresponding sub-zones.
- The on-axis irradiance at *P* is DARK.

Diffraction From a Circular Aperture

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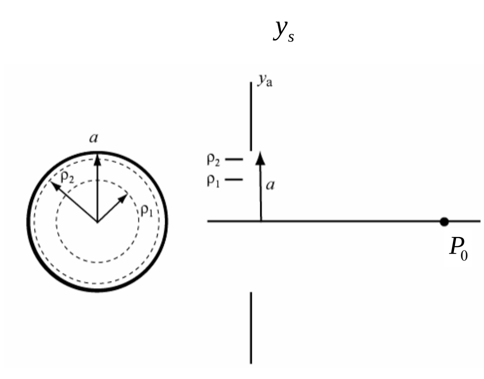
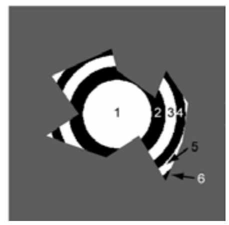
- Move *P* toward the aperture.



- The third zone enters the aperture.
- Light from the first and second zones still cancel.
- Net effect is an increase in irradiance at the on-axis observation point.
- A maximum BRIGHT spot is obtained when zone 3 boundary is at the aperture radius.
- Further movement of *P* toward the aperture decreases on-axis irradiance.

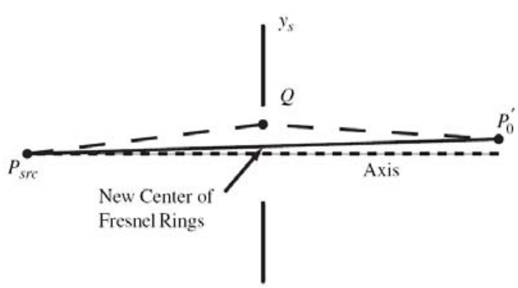
Diffraction From a Circular Aperture

- Move *P* toward the aperture.
  - Observe an oscillatory on-axis irradiance at *P* is moved toward the aperture
  - BRIGHT spot when there is an ODD number of zones.
  - DARK spot when there is an EVEN number of zones.
  - Procedure for determining irradiance:
    1.) Divide open part of aperture into Fresnel zones based on *OPD*
    2.) Subtract even and odd zone areas
    3.) Result^2 is proportional to irradiance
    4.) Works for oddly-shaped apertures



Diffraction From a Circular Aperture

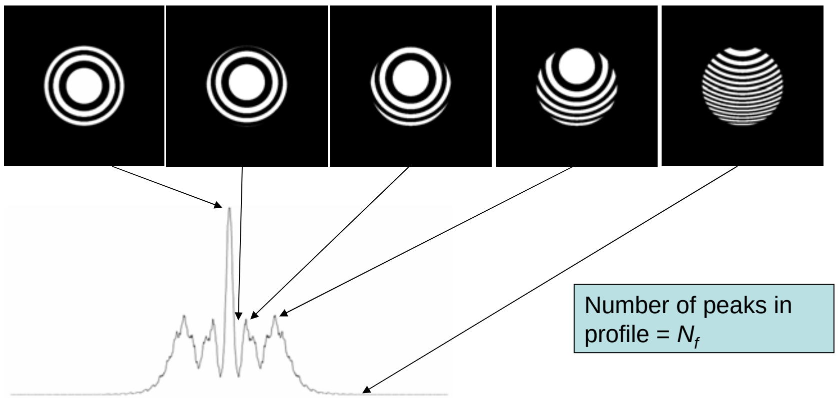
- Off-axis behavior understood with shifted zones in the aperture.



- Shift of zones due to shift of symmetry line for optical path length.
- Entire zone pattern shifted up or down, depending on observation point.
- Use geometric technique to determine irradiance by subtracting even and odd zone areas.

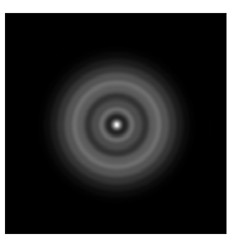
Diffraction From a Circular Aperture

- Shifted patterns for N_f = 5 yield profile.

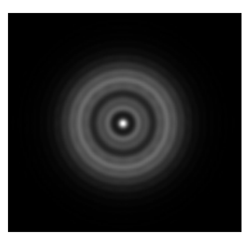


Diffraction From a Circular Aperture

- Rotate profile to get two-dimensional geometrical distribution (N_f = 5).



Geometric Calculation



Angular Spectrum Calculation

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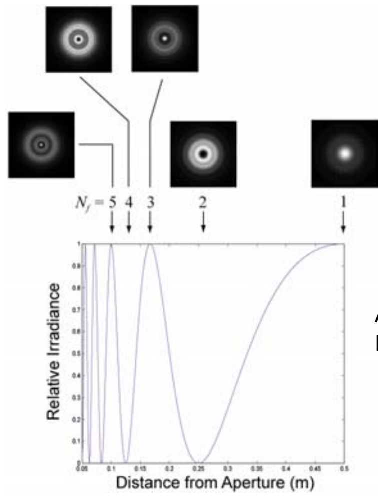
Diffraction From a Circular Aperture

- Collimated illumination

$$z_{src} \rightarrow \infty$$
$$L \rightarrow z_0$$
$$\lambda = 0.5\mu\text{m}$$
$$a = 0.5\text{mm}$$



Axial Irradiance

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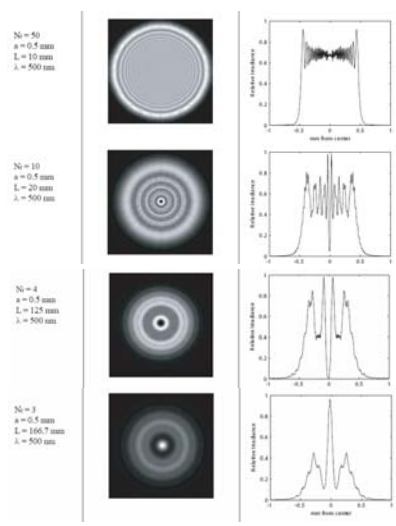
Diffraction From a Circular Aperture

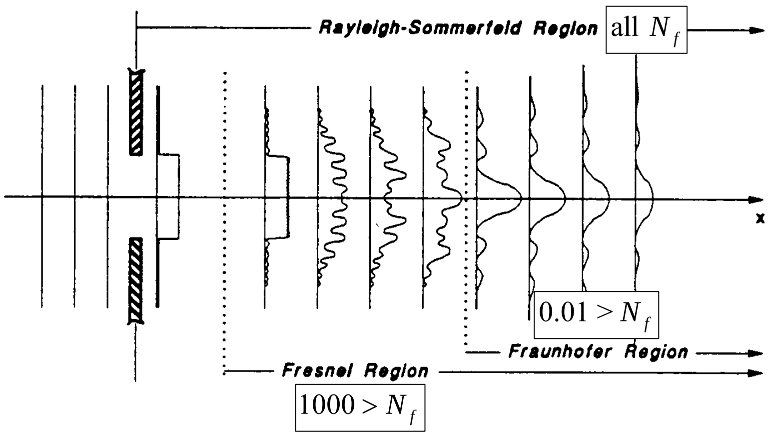
- Collimated illumination

$$z_{src} \rightarrow \infty$$
$$L \rightarrow z_0$$
$$\lambda = 0.5\mu\text{m}$$
$$a = 0.5\text{mm}$$



Diffraction Regions

Rayleigh-Sommerfeld: everything right of aperture
Fresnel: near field, assume spherical wave profile
Fraunhofer: far field, wave pattern far from the obstruction

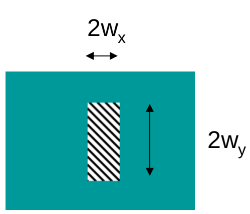


Diffraction by a single slit

Fraunhofer formula

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \iint U_A(\xi,\eta) e^{-j2\pi(f_x\xi + f_y\eta)} d\xi d\eta$$

2D Fourier Transform



$$U_A(\xi,\eta) = rect\left(\frac{\xi}{2w_x}\right) rect\left(\frac{\eta}{2w_y}\right)$$

aperture function

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \int_{-w_y}^{w_y} \int_{-w_x}^{w_x} e^{-j2\pi(f_x\xi + f_y\eta)} d\xi d\eta$$

far field pattern

$$U(x,y) = \frac{4w_x w_y e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \operatorname{sinc}\left(\frac{2\pi w_x x}{\lambda z}\right) \operatorname{sinc}\left(\frac{2\pi w_y y}{\lambda z}\right)$$

Sinc Function

definition

$$\mathrm{sinc}\,(x) \equiv \begin{cases} 1 & \text{for } x = 0 \\ \dfrac{\sin x}{x} & \text{otherwise,} \end{cases}$$

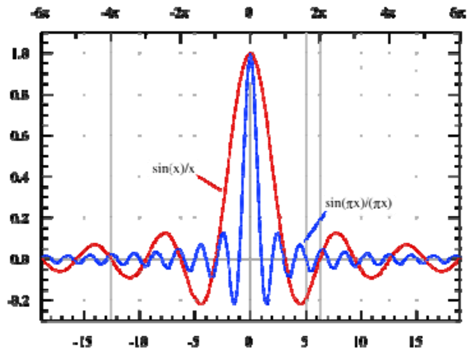
normalization

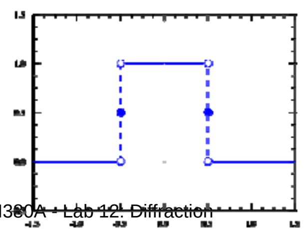
$$\int_{-\infty}^{\infty} \mathrm{sinc}\,(x)\, d\, x = \pi.$$

Fourier Transform

$$\int_{-\infty}^{\infty} \mathrm{sinc}(t)\, e^{-2\pi i f t} dt = \mathrm{rect}(f)$$

$$\mathrm{rect}(x) = \sqcap(x) = \begin{cases} 0 & \text{if } |x| > \frac{1}{2} \\ \frac{1}{2} & \text{if } |x| = \frac{1}{2} \\ 1 & \text{if } |x| < \frac{1}{2} \end{cases}$$

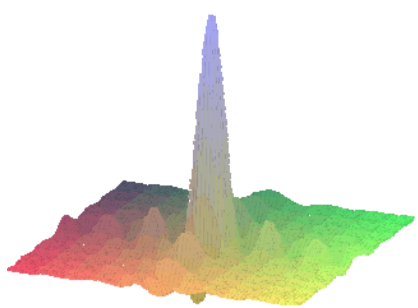




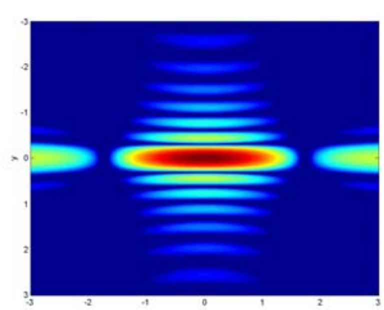
extrema of sinc and sinc^2 at
±1.4303π
±2.4590π
±3.4707π, ..

zeros at ±π, ±2π, ..

Far Field Pattern of Rectangular Aperture

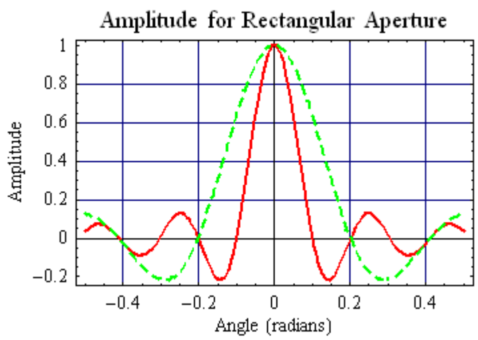


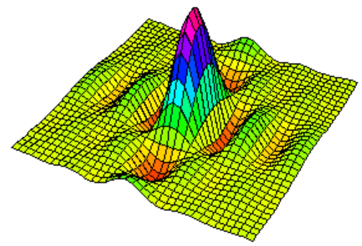
sinc(x) * sinc(y)

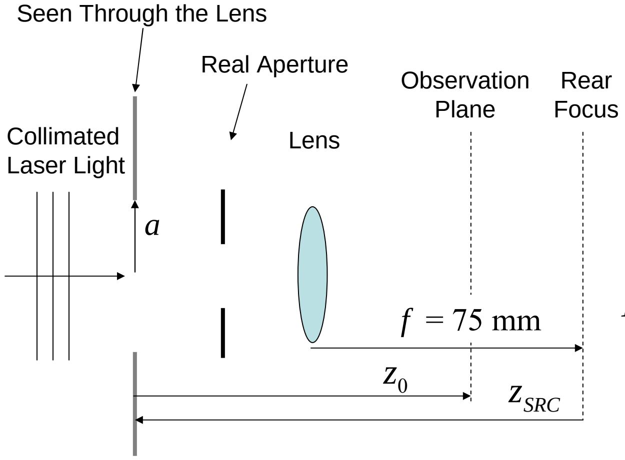


Fun With Applets

- www.falstad.com
- From http://wyant.optics.arizona.edu/math.htm

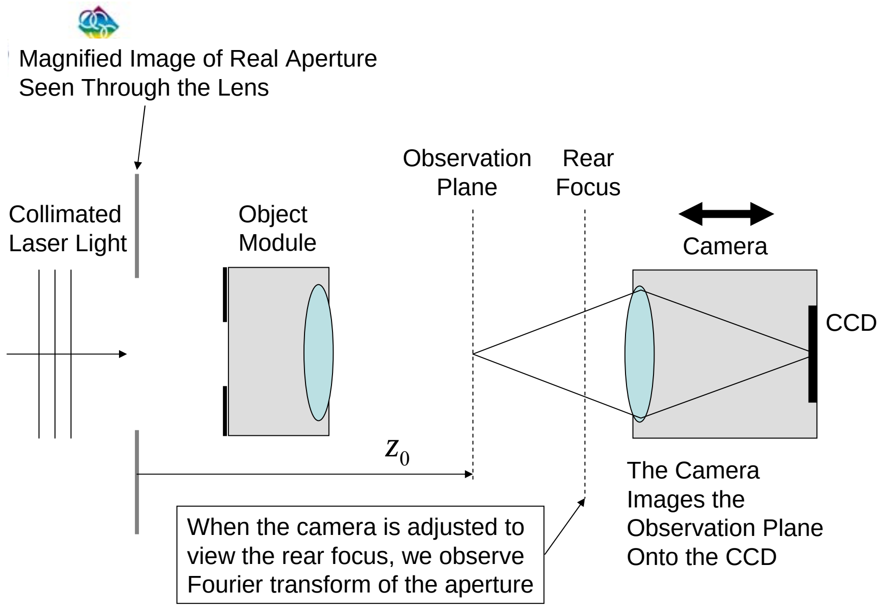






$$N_f = \frac{a^2}{\lambda L}$$

$$L = \frac{z_0}{1 + z_0 / z_{SRC}}$$



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Lab 12 Preparation

- We will be using some features of the camera that allows us to save data onto a hard drive.
- We will be using Matlab to process the data.
- Although lab computers are available, you can bring your own laptop, if you prefer. The lab computers tend to be slow.
- If you use your own laptop, have Matlab loaded and download the camera driver (Lopitech Quickcam) from http://www.logitech.com/index.cfm/435/254&hub=1&cl=us,en