OPTICS 380A
Lab 5: Wave Motion

Introduction

This lab is an introduction to the study of waves and wave motion. Many optical phenomena cannot be explained by geometrical optics and rays. What is needed is a description of light as an electromagnetic (EM) wave. This picture of light as a wave does not take the place of our geometrical description of light as rays, but rather compliments it. Both are necessary for a full understanding of optics. In this lab, we will investigate the behavior of wave motion in three situations—vibrating strings, water waves and a transmission line.

SIMPLE HARMONIC MOTION

The starting point to understanding waves is to understand the principles of a body or particle moving in a repetitive, oscillatory manner known as simple harmonic motion. The following treatment was adopted from University Physics. Sears, Zemansky, and Young, 5th ed., 1977.

First, consider a particle at rest in some equilibrium position. When displaced from this position the particle experiences a restoring force proportional to the displacement. The restoring force causes the particle to undergo a back-and-forth motion past the point of equilibrium. This general type of particle motion is known as simple harmonic motion, often abbreviated **SHM.** The particle undergoing SHM is called a simple harmonic oscillator, abbreviated SHO. When the position of the particle is plotted as a function of time, the variation about the equilibrium point is observed to be sinusoidal. Equation (5.1) describes the behavior

$$y(t) = A\sin\left[\sqrt{\frac{k}{m}}t + \theta_0\right] \tag{5.1}$$

where:

$y(t)$ is the distance away from the equilibrium position at any instant in time, [m]

A is the amplitude of oscillation, equal to the maximum value of $|y(t)|$. The total range of motion is therefore equal to $2|A|$. [m]

k is a proportionality constant called the force constant that relates the restoring force to the position y. [N/m]

m is the mass of the particle, [kg]

t is the time in seconds, [sec]

θ_0 is the initial phase angle of the oscillation at time $t=0$. [radians]

Various examples display this type of oscillation—a pendulum, a mass on a spring, the balance wheel in a watch, the pistons in a gasoline engine, and the strings in a musical instrument are just a few examples.

TRAVELING WAVES

The concept of SHM may now be extended to waves. A wave may be considered to be any disturbance from an equilibrium position which travels or propagates with time from one point in space to another. Imagine a medium consisting of a large number of particles, each connected or coupled to its neighbors by elastic material. Examples of this arrangement include a pool of water or a stretched string. If one end of the medium is disturbed or displaced in any way, the displacement does not occur immediately at all other points in the medium. The original displacement gives rise to an elastic force in the material adjacent to it; then the next particle is displaced, and then the next, and so on. In other words, the displacement is propagated along the medium with a definite speed. Finally, if the original displacement is that of SHM, the motion at any other fixed point along the direction of wave travel will be that of SHM. The speed of the wave is then the speed at which a point of constant phase of the SHM moves through the medium. The relationship between the speed of the wave v its frequency f, and its wavelength λ is expressed as:

$$v = f \cdot \lambda \tag{5.2}$$

A wave that moves through space is called a traveling wave, and may be described by the following equation:

$$y(z,t) = A \sin(kz - \omega t) \qquad (5.3)$$

or equivalently as:

$$y(z,t) = A\sin\left[\frac{2\pi}{\lambda}(z - vt)\right] \qquad (5.4)$$

where:

k is called the wavenumber, equal to $2\pi/\lambda$.

y(t) is the amplitude of the wave at any point in space and time.

A is the wavelength, or distance between any adjacent points of constant phase in the wave, [m]

z is the distance coordinate along the direction of travel of the wave, [m]

t is the time, in seconds, [sec]

v is the speed of travel of the wave, [m/sec]

Note that if the direction of displacement, y, is in a plane normal to the direction of travel of the wave, the wave is said to be a **TRANSVERSE** wave. This is the nature of light waves.

STANDING WAVES

The concept of standing waves involves an understanding of the PRINCIPLE OF SUPERPOSITION of waves. This principle states that the actual displacement of a wave at any point in space and time is the algebraic sum of all the displacements of the waves existing at the same point. The usual case is to consider a wave traveling in the +z direction, being superimposed on a wave traveling in the -z direction at the same time.

Consider a wave traveling in the +z direction:

$$y_1(z,t) = A \sin(kz - \omega t) \qquad (5.5)$$

and a wave with the same wavelength and frequency traveling in the -z direction:

$$y_2(z,t) = A \sin(-kz - \omega t + \pi) \qquad (5.6)$$

The resultant disturbance at any point is the sum of these two waves:

$$y_{tot} = y_1(t) + y_2(t) = A\left[\sin(kz - \omega t) + \sin(-kz - \omega t + \pi)\right] \qquad (5.7)$$

Eq. (5.7) may be simplified to the following:

$$y_{tot}(z,t) = 2\,A\,\sin(kz)\,\cos(\omega t) \qquad (5.8)$$

This resulting wave is called a standing wave. It has a sinusoidal spatial variation with the same wavelength as the original wave, and an amplitude that varies cosinusoidally with the same frequency as the original wave. Note, however, that this wave does not travel, and is therefore called a standing wave. It is the type of wave that we will observe when looking at the vibrating string. Figure 5.1 graphically shows the formation of standing waves, and Figure 5.2 simulates a time exposure photograph of the first four modes of a vibrating string.

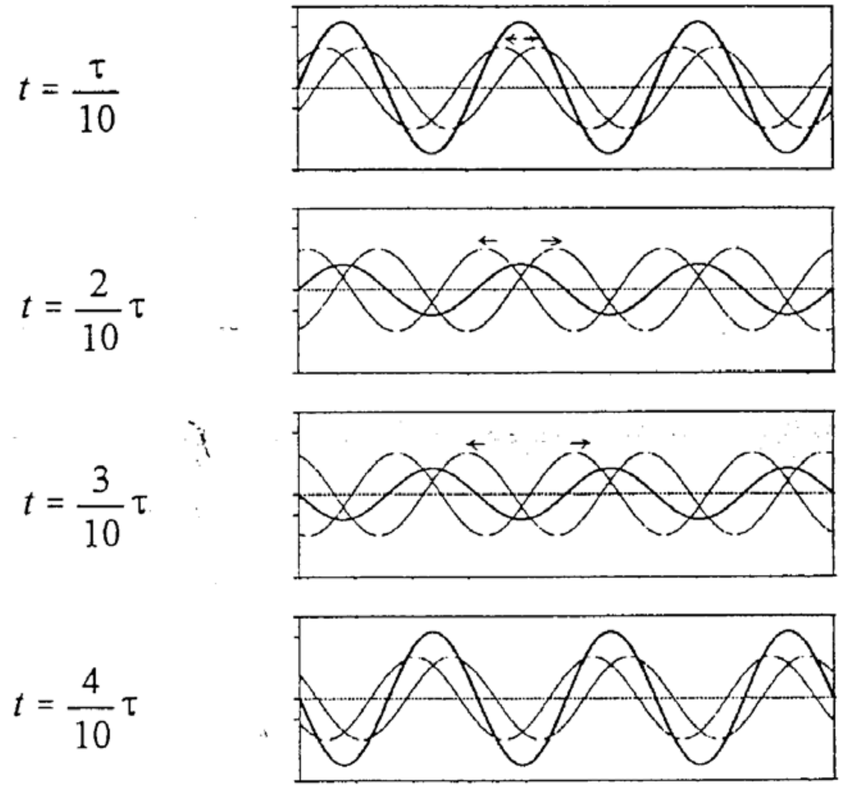


Figure 5.1. Formation of a standing wave.

In describing various kinds of wave motion, it is the particles of the medium in which the wave travels that oscillate with SHM. Water waves are a good example of this situation. Each individual element (or droplet) of water in a large body of water oscillates with SHM. However, it is important to note a significant difference for light waves. For electromagnetic waves in general, the quantities that oscillate are the electric and magnetic fields, not the particles of the medium in which the wave is traveling.

In the early investigations of light, it was thought that for light to travel through space as a wave, it must have some elastic medium ("sea of particles") to propagate through. This was called the *ether,* and was considered necessary to support wave propagation as understood at the time (all other kinds of waves were known to have some kind of medium). With the formulation of Maxwell's equations and the experimental work of Michelson and Moorley, the ether was shown not to exist. Electromagnetic waves could, and can, propagate through a vacuum, containing no particles at all. It is, in fact, the electric and magnetic fields of the wave that oscillate with SHM.

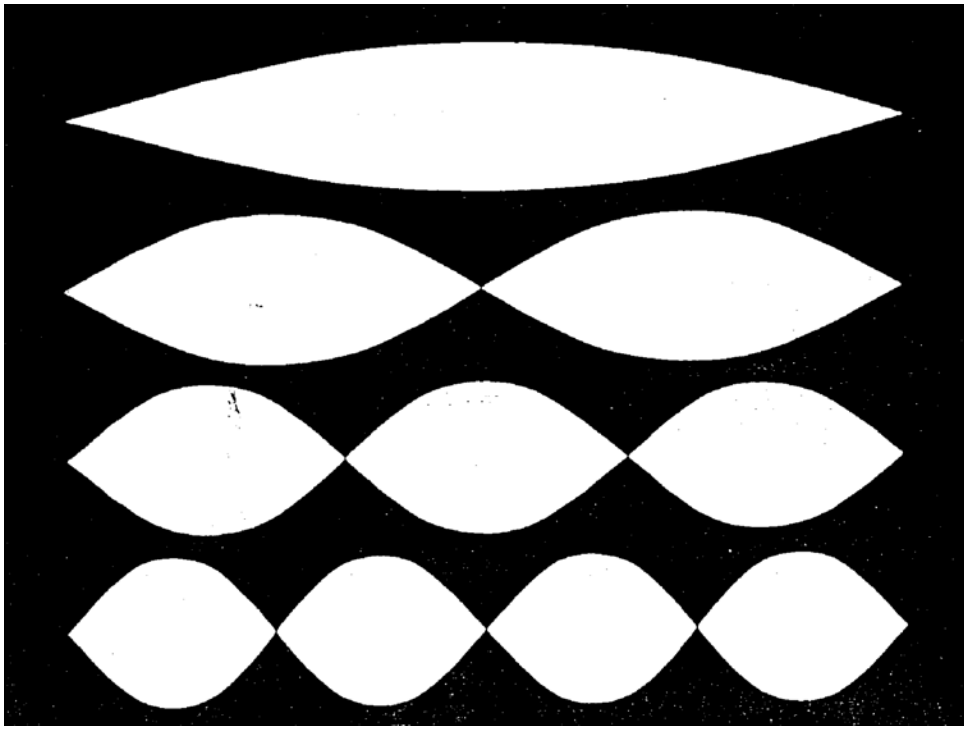


Figure 5.2. Simulation of a time exposure of a vibrating string.

Part A: WATER WAVES IN A RIPPLE TANK

Use the ripple tank to create water waves. With reference to Appendix A: Ripple Tank Experiments, perform Experiment 1 (Reflection) and Experiment 2 (Refraction) and answer the questions in the two experiments. Please consider the additional procedures listed below for this portion of the lab:

1.) Experiment 1, Part 1: Reflection using a straight barrier

  a.) Disregard the Setup section on the bottom of page 11 in Appendix A. The setup has been done for you.

  b.) Add the following steps to the Procedure on page 12 of Appendix A:

    7.) Measure the distance between wavefronts of the incident wave.

    8.) Turn off the strobe and observe dark lines parallel to the barrier. Measure the perpendicular distance between these lines.

2.) Experiment 1, Part 2: Reflection using a curved barrier

  a.) It may be easier to see wave effects without the strobe in this portion of the experiment.

3.) Experiment 2: Refraction

  a.) Disregard the Setup section on the bottom of page 15 in Appendix A. The setup has been done for you.

  b.) A careful adjustment of water level is required to observe the effects properly. Ask your TA to help if you are having difficulty.

  c.) Use a source frequency of about 15Hz.

  d.) In Part 2, use the flat barriers to form an aperture for the lenses.

  e.) Again, it might be easier to see some of the wave effects if the strobe is turned off.

Part B: STANDING WAVES ON A VIBRATING STRING

Consider a string of length L fixed at both ends so they cannot move. A driving force exhibiting SHM of frequency f is applied to the string. The question to ask is how does the string respond as the frequency of the driving force is changed?

Start by referring to Figures 5.1 and 5.2. We define a node as being any point along the string that has a (constant) amplitude of zero. The ends of the string, being fixed, are therefore always nodes. At the same time, we define the points along the string having the largest amplitude as antinodes. It may be seen from the figures that the distance between nodes or antinodes is one-half the wavelength, $\lambda/2$, of the standing wave on the string. Note that at the lowest frequency of vibration, called the fundamental frequency, 2 nodes and 1 antinode exist on the string. The string will vibrate at higher frequencies, called overtones, such that an integer number of half-wavelengths of the standing wave exist on the string.

The experimental setup provides you with a string of a given fixed length fixed at both ends. Our setup uses a wire-wound guitar string, held fixed at one end and draped over a pulley at the other end. A weight at the end of the string keeps the string pulled tight over the pulley. Both ends are effectively held fixed, as the string cannot vibrate at either end. A very strong permanent magnet is located at a point along the string. The magnet faces are vertical, with the string passing in-between them. As a result, a magnetic field is applied in the horizontal plane, perpendicular to the string. Finally, our generator of SHM is a variable audio oscillator, driving an audio power amplifier. The amplifier output is connected in series with the (conductive!) guitar string, in turn connected to an 8 ohm resistor.

The physics of why the string vibrates is quite simple. An alternating electric current flows along the guitar string, in step with the driving voltage from the audio oscillator. This alternating current is sinusoidal in nature, and interacts with the magnetic field according to:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{5.9}$$

where:

$\vec{F}$ is the force exerted on an electron

q is the charge of an electron flowing in the wire

$\vec{v}$ is the velocity of the electrons

$\vec{B}$ is the magnetic field

More appropriately, this may be written as:

$$\vec{F} = \vec{I}\ L \times \vec{B} \tag{5.10}$$

where:

$\vec{F}$ is the force exerted on an electron (and hence the string)

$\vec{I}$ is the current flowing through the string

L is the length of the string (over which the magnetic field exists)

$\vec{B}$ is the magnetic field

The result is that the string experiences a force alternating up and down, and the string vibrates in the vertical plane. Vary the frequency of the driving force and observe how the string responds to the answer the following questions:

1. How does the string vibrate as a function of frequency? (qualitative answer)
2. What is the fundamental frequency of the string?
3. Make sketches of what you observe as a function of frequency.
4. Derive an equation that relates the allowed wavelengths of the standing wave to the length L of the string.
5. Derive an equation that relates the allowed frequencies of the standing wave to the length L of the string.
6. How does the number of nodes relate to the number of half-wavelengths in the standing wave on the string? (quantitative answer)
7. What was the maximum number of nodes that you observed?

PART C: ELECTRO-MAGNETIC (E-M) STANDING WAVES ON A TRANSMISSION LINE

A direct analogy to standing waves on a string, when talking about light waves, is the study of E-M radio waves on a transmission line. Radio waves have the nice properties that their wavelengths are large enough to be directly measurable, and they are easily generated. Just like the vibrating string mechanical analog, E-M standing waves can be generated on a transmission line. In this case, however, we refer to the change in amplitudes of the electric and magnetic fields along the transmission line instead of the mechanical displacement of the string along its length.

The generation of E-M standing waves on a transmission line is directly analogous to the generation of mechanical standing waves on a string. A driving force at one end of the transmission line generates traveling waves that move down the line. The driving force is typically a radio transmitter. Upon reaching the far end of the transmission line, the E-M wave can encounter one of two situations.

The first is that the line is terminated in an electrical impedance equivalent to the impedance of the line itself. This usually takes the form of an antenna or a resistor. In this case, the antenna completely radiates all of the energy in the traveling wave, or the resistor absorbs all of the energy. Either way, no part of the wave is reflected back down the transmission line and therefore no standing wave is set up. (This is the desired situation when broadcasting radio waves!)

The second situation is what we are interested in for our lab. In this case, the line is terminated with a resistor, but of a different impedance than the line itself. The resistor will absorb some of the energy, but the remainder is reflected back down the line in the form of a traveling wave heading back towards the source. It is this reflected wave that adds to the forward traveling wave (by linear superposition) to create the standing wave along the line.

Like the standing wave on the vibrating string, the E-M standing wave exhibits the same relationship between nodes and wavelength. The distance between nodes along the standing wave is one-half the wavelength of the two traveling waves. (Note that unlike the string, nodes may or may not form at the ends of the transmission line.)

Figure 5.3 shows a diagram of the experimental setup. The transmission line consists of two parallel wires spaced a fixed distance apart. The impedance of this line is calculated to be approximately 400 ohms. At one end of the line is a radio transmitter operating at a frequency of 145.00 MHZ, and at the other end is a 200 ohm resistor. The transmitter emits a sinusoidal E-M wave of constant frequency and amplitude.

A detector is used to measure the amplitude of the standing wave along the transmission line. The detector consists of a coil of wire, wound around a split ferrite core, as shown in Figure 5.4. This design allows the coil to be clamped around one of the wires along the line and slid back and forth along its length. The coil acts as a step-down transformer, sampling a small portion of the current at any point along the transmission line. Diode D1 rectifies the sampled current and R1 and C1 act to filter the alternating current providing a DC voltage at the output. In effect, the detector circuit is a relative radio frequency (RF) ammeter. Its output voltage is directly proportional to the amplitude of the electric field of the standing wave existing along the transmission line.

A plot of the output voltage of the detector vs. position along the line will yield the “shape” or amplitude variation of the standing wave. Familiarize yourself with the operation of the detector, and record the output vs. length over a distance of 3-4 nodes.

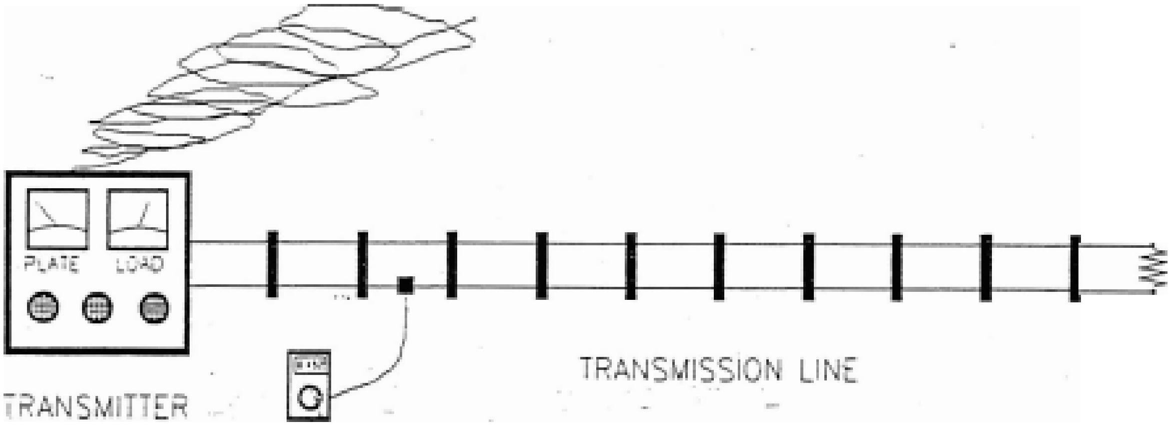


Figure 5.3. E-M standing waves--experimental setup

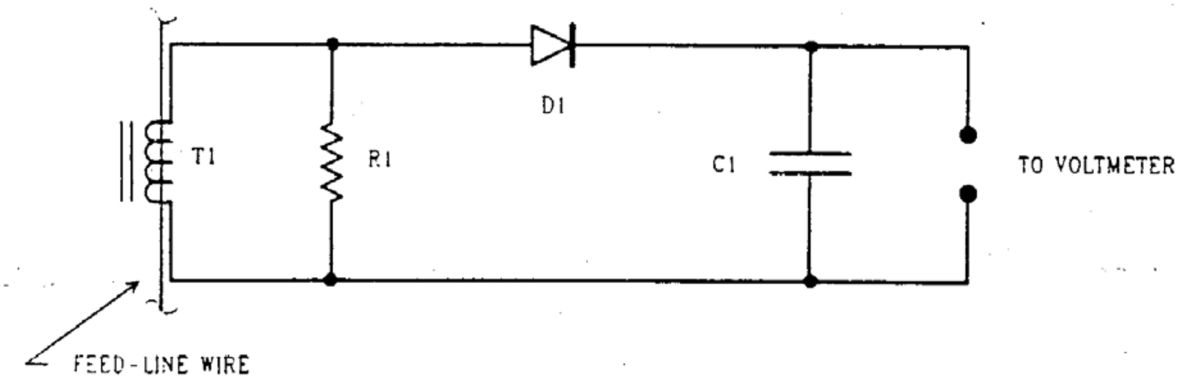


Figure 5.4. Schematic diagram of the detector.

1. Record the output **vs.** length over a distance of 3-4 nodes.
2. Plot the detector probe output [mV] vs. distance [cm] along the transmission line.
3. From this plot, what is the measured wavelength of the source?
4. What is the calculated value of f_{source} x □$_{\text{source}}$?
5. What should this be equal to and how does it compare?
6. What is the wavelength of the standing wave compared to the wavelength of the two traveling waves?
7. What is the wavelength of radio waves emitted by KNST, the UA Wildcats AM sports station operating at 790 kHz?
8. What is the wavelength of radio waves emitted by your favorite FM station?
9. Cell phone transmits in the 824.040 to 848.970 MHz range. What are the wavelengths?

APPENDIX A: RIPPLE TANK DETAILS

Experiment 1: Reflection

Equipment from Ripple Tank System

Ripple Tank
Long Barrier
Curved Barrier
Ripple Generator and Light Source
Plane Wave Dipper
Ruler

Other Equipment and Materials

Large Rod Stand (ME-8735)
90-cm Rod (ME-8738)
Paper (about 40 cm by 40 cm)
Protractor
Drawing compass

Purpose

The purpose of this activity is to study the reflection of a plane wave from different shaped barriers: a long straight barrier and a curved barrier.

Theory

A ray is a line that indicates the direction of motion of a plane wave. Wave fronts are perpendicular to the ray. When a wave reflects from a surface, the angle of incidence is the angle between the incoming (or incident) ray and the normal (a line perpendicular to the surface). The angle of reflection is the angle between the outgoing (reflected) ray and the normal.

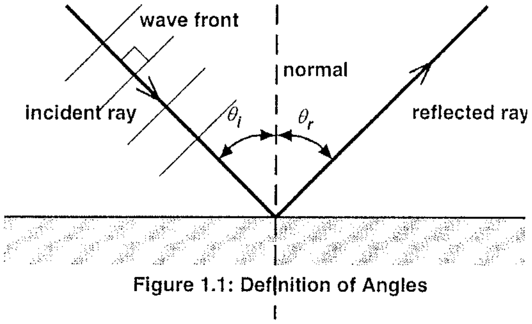
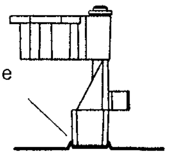


Figure 1.1: Definition of Angles

Setup

1. Mount the light source onto its rod at the back edge of the ripple tank.
2. Pour a small amount of water into the tank and adjust the feet on the legs of the tank to level the tank.
3. Place the long straight barrier in the middle of the tank and add about 800 mL of water to the tank (or enough water so that the water level is about halfway up the long straight barrier.)
4. Use a rod and base to support the ripple generator and position the generator over the midpoint of one side of the ripple tank. Plug the light source into the ripple generator and connect the ripple generator to its power adapter.
5. Connect the plane wave dipper to the ripple arms. Adjust the ripple generator until the bottom of the plane wave dipper is barely in contact with the surface of the water.
6. Place a sheet of paper directly under the ripple tank so you will be able to sketch the images of the waves that are projected onto the sheet by the light source.

Tip: Make sure that the plane wave dipper is in contact with the water evenly over its length.



Water appears to curve upward on the sides of the plane wave dipper

Part 1: Reflection Using a Straight Barrier

Procedure

1. Arrange the long barrier in the middle of the tank so the barrier is at an angle to the plane wave dipper (see Figure 1.2).
2. Turn on the ripple generator and the light source. Set the light source to 'STROBE'. Set the ripple generator frequency to 20 Hz. Set the amplitude to slightly less than half of maximum.
3. On the paper below the tank, place the ruler parallel to the plane waves that are incoming to the barrier. Make a line to show the incoming wave front.
4. Place the ruler parallel with a reflected wave and again make a line to show the outgoing (reflected) wave front.
5. Trace the position of the straight barrier.
6. Turn off the ripple generator and light source.

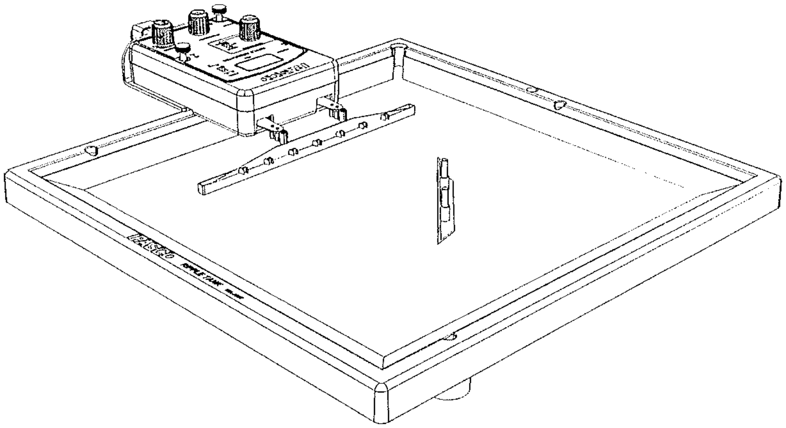


Figure 1.2: Position of Straight Barrier

Tip: Adjust the amplitude as necessary to make a clear pattern of plane waves.

Data Analysis

1. Draw a line that is perpendicular to the incoming wave front and extend the line to the outline of the straight barrier. This represents the incoming ray, so draw an arrow on it pointing to the barrier.
2. Draw a line from the point where the incoming ray intersects the straight barrier so it crosses the reflected wave front at a right angle. This represents the reflected ray, so draw an arrow on it pointing away from the barrier.
3. Draw the normal (perpendicular) line at the point of reflection on the outline of the straight barrier.
4. Measure the angle of incidence and the angle of reflection and record the measurements in the table.
5. Repeat the procedure with the barrier at a different angle.

Table 1.1: Reflection Results

	Trial #1	Trial #2
Angle of Incidence		
Angle of Reflection		

Question

1. What is the relationship of the angle of incidence and the angle of reflection?

Part 2: Reflection Using a Curved Barrier

Procedure

1. Replace the straight barrier with the curved barrier and position the curved barrier so it is aligned 'parallel' to the plane wave dipper as shown in Figure 1.3.

2. Turn on the light source. Trace the position of the curved barrier on the paper below the ripple tank.

3. Turn on the ripple generator.

4. Mark the position on the paper where the waves that reflect from the curved barrier appear to converge. Turn off the ripple generator.

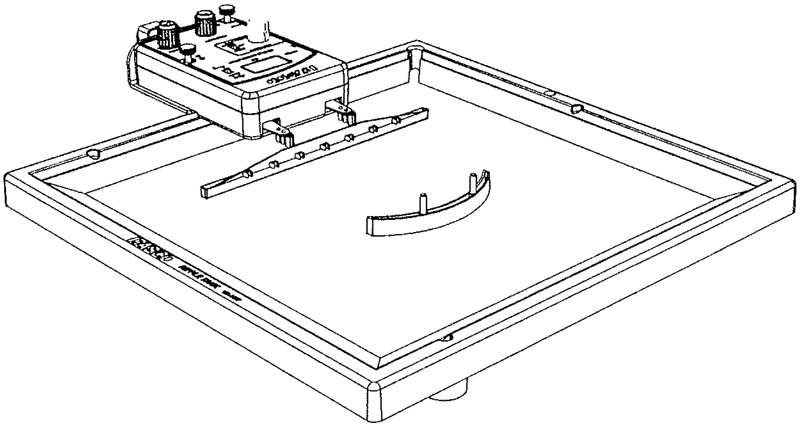


Figure 1.3: Position of Curved Barrier

5. Use the pipette to drop a single droplet of water at the position in the ripple tank where the waves converged. Describe the shape of the waves that reflect from the curved barrier.

Data Analysis

1. Use a drawing compass to complete the traced circular shape of the curved barrier. Mark the center of the circle and measure the radius.

Question

1. What is the shape of the wave fronts that reflect from the curved barrier when you dropped the droplet of water into the ripple tank?

2. How is the radius of the circle related to the distance between the curved barrier and the point where the reflected plane waves from the plane wave dipper appeared to converge?

Extension

Turn the curved barrier around by 180 degrees so that it 'curves away' from the plane wave barrier as shown in Figure 1.4. Repeat the procedure as before, but trace the shape of the reflected waves as well as the outline of the curved barrier.

After sketching the reflected waves, draw at least three rays perpendicular to the reflected waves. Extend the rays until they intersect and mark the point of intersection. Measure the distance from the outline of the curved barrier to the point of intersection, and compare this distance to the radius of the traced circular shape of the curved barrier.

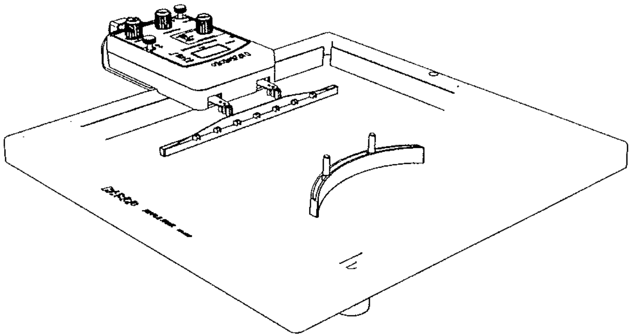


Figure 1.4: Reverse the Curved Barrier

Experiment 2: Refraction

Equipment from Ripple Tank System

Ripple Tank

Trapezoidal Refractor

Concave Refractor

Convex Refractor

Ripple Generator and Light Source

Plane Wave Dipper

Ruler

Other Equipment and Materials

Large Rod Stand (ME-8735)

90-cm Rod (ME-8738)

Paper (about 40 cm by 40 cm)

Purpose

The purpose of this activity is to show how waves change direction as they pass from one region to another where the wave speed is different.

Theory

As a wave travels from one medium to another where the wave speed is different, the wave bends to a new direction. If the wave slows down, the wave will bend toward the normal of the interface between one medium and the other as shown in Figure 2.1. This bending is called refraction.

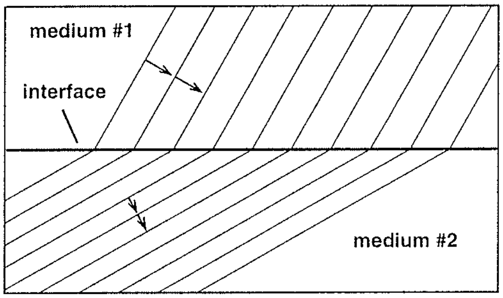
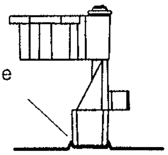


Figure 2.1: Refraction

Setup

1. Mount the light source onto its rod at the back edge of the ripple tank.
2. Pour a small amount of water into the tank and adjust the feet on the legs of the tank to level the tank.
3. Place the trapezoidal refractor in the middle of the tank and add about 700 mL of water, or enough water so that the water level is at the top edge of the refractor.
4. Use a rod and base to support the ripple generator and position the generator over the midpoint of one side of the ripple tank. Plug the light source into the ripple generator and connect the ripple generator to its power adapter.
5. Connect the plane wave dipper to the ripple arms. Adjust the ripple generator until the bottom of the plane wave dipper is barely in contact with the surface of the water.
6. Place a sheet of paper directly under the ripple tank so you will be able to sketch the images of the waves that are projected onto the sheet by the light source.

Tip: Make sure that the plane wave dipper is in contact with the water evenly over its length.



Part 1: Refraction Using a Straight Barrier

Procedure

1. Arrange the trapezoidal refractor in the water in the middle of the tank so the rectangular end of the refractor is parallel to the plane wave dipper and about 5 cm from the dipper (see Figure 2.2).

2. Add just enough water to the tank so that the refractor is evenly covered by less than 1 mm of water.

3. Turn on the ripple generator and the light source. Set the light source to 'STROBE'. Set the ripple generator frequency to 15 Hz or less. Set the amplitude to slightly less than half of maximum and adjust it as necessary to make a clear pattern of plane waves.

4. On the paper below the tank, trace the outline of the trapezoidal refractor.

5. Place the ruler parallel to the plane waves that are incoming to the refractor. Sketch lines to show the incoming wave fronts.

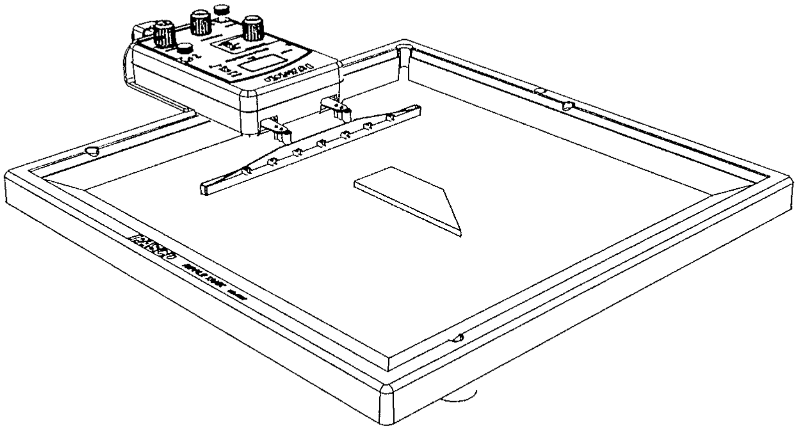


Figure 2.2: Position of Trapezoidal Refractor

6. On the outline of the refractor, trace the shapes of the refracted waves to show the bending of the refracted waves as they travel over the refractor.

7. After sketching the waves, *reverse* the trapezoidal refractor so that the triangular end of the refractor points toward the plane wave dipper and repeat the procedure.

8. Turn off the ripple generator and light source.

Tip: Adjust the frequency as needed to make a clear wave pattern as the waves move over the refractor. The lower the frequency, the more pronounced the refraction.

Data Analysis

1. Draw a line that is perpendicular to the incoming wave front and extend the line to the outline of the trapezoidal refractor. This represents the incoming ray, so draw an arrow on it pointing to the refractor.

2. At the point where the line representing the incoming ray meets the outline of the refractor, draw a new line that is perpendicular to the wave fronts of the refracted waves as they pass over the trapezoidal refractor.

Questions

1. What happens to the direction of the wave fronts as they move over the trapezoidal refractor?

2. As the plane wave from the deep water moves through the shallower water over the refractor, does the plane wave speed up or slow down?

Part 2: Refraction Using Curved Refractors

Procedure

1. Replace the trapezoidal refractor with the convex refractor, placing it in the middle of the tank with the straight side parallel to the plane wave dipper and about 5 cm from the dipper as shown in Figure 2.3.

2. Turn on the ripple generator and light source. Trace the position of the convex refractor on the paper below the ripple tank.

3. Trace the pattern of plane waves as they move from the plane wave dipper over the convex refractor.

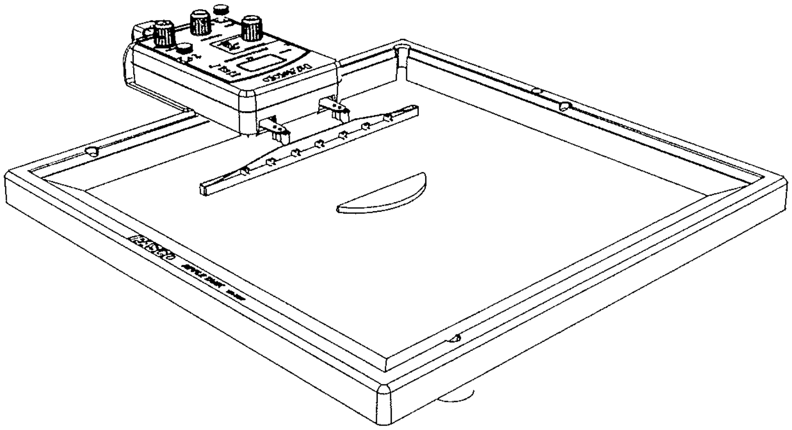


Figure 2.3: Position of Convex Refractor

4. Use the ruler to measure the focal length of the convex 'lens'. This is the distance from the center of the lens to the point where the refracted plane waves appear to converge (come to a focus).

5. Replace the convex refractor with the concave refractor and trace the new pattern of the plane waves as they move from the dipper over the refractor.

Data Analysis

1. Use the ruler to sketch three rays that represents the direction of motion for three sections of the plane waves from the dipper as they pass over the convex refractor. Draw one ray for the wave fronts that move over the upper third of the convex refractor; draw a second ray for the wave fronts that move over the center third of the refractor, and draw a third ray for the wave fronts that move over the lower third of the refractor.

2. Repeat the sketching of rays for the wave pattern of the waves moving over the concave refractor. Draw one ray for the wave fronts that move over the upper third of the concave refractor; draw a second ray for the wave fronts that move over the center third of the refractor, and draw a third ray for the wave fronts that move over the lower third of the refractor.

Questions

1. What happens to the direction of the rays for the wave fronts of the plane waves as they move over the concave refractor?

2. Do the refracted waves from the concave refractor appear to converge or diverge?