OPTI 380A
Intermediate Optics Lab 7:
Waveplates and Stokes Vectors

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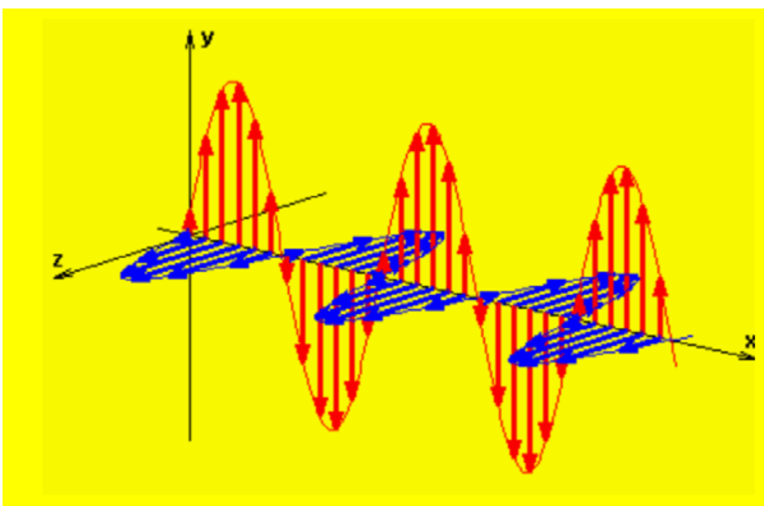
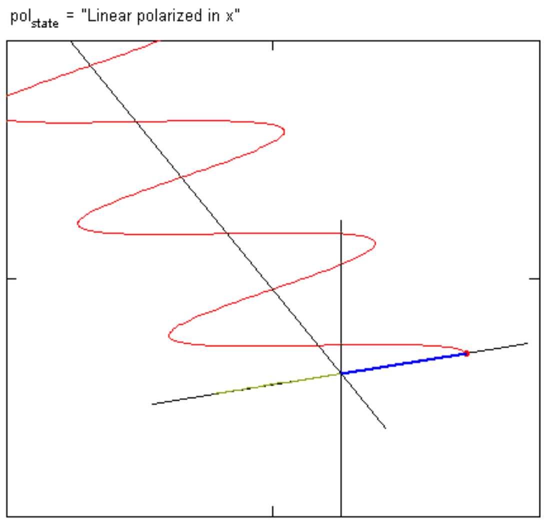
Linearly Polarized EM Wave



Various States of Polarization



Simulation by Bernd Geh of Zeiss

Polarization

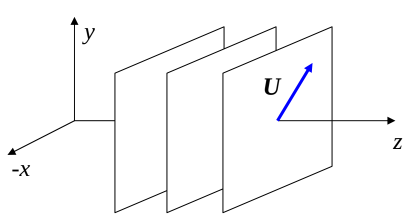
Consider the case of an EM plane wave traveling in the +*z* direction.

E oscillates perpendicular to *z*.

General solution:

$$\boldsymbol{U}(z,t) = \boldsymbol{U}_0 e^{j(kz-\omega t)} = \left[A_x \hat{\boldsymbol{x}} + A_y e^{j\phi} \hat{\boldsymbol{y}} \right] e^{j(kz-\omega t)} \quad A_x, A_y \in \mathrm{Re}\{\ \}$$

We will now trace the tip of the electric vector for several special cases.



We will look at both:

fixed $z = z_0$ with t = variable

fixed $t = t_0$ with z = variable

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Polarization – Linear

$\phi = 0$

Special case where $A_x = A_y = A_0$:

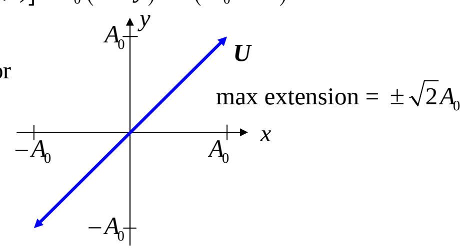
$$\boldsymbol{U}(z,t) = A_0\left(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\right)e^{j(kz-\omega t)}$$

$$\mathrm{Re}\left[\boldsymbol{U}(z,t)\right] = A_0\left(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\right)\cos\left(kz-\omega t\right)$$ ← This is the physical wave.

Consider $z = z_0$, where $\mathrm{Re}\left[\boldsymbol{U}(z_0,t)\right] = A_0\left(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\right)\cos\left(kz_0-\omega t\right)$

Trace the tip of the electric vector in the (x,y) plane in time:

This is true for *all* z_0.



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Polarization – Left Circular

$A_x = A_y = A_0$
$\phi = \pi/2$

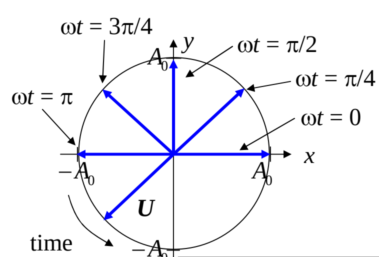
$$\boldsymbol{U}(z,t) = A_0\left[\hat{\boldsymbol{x}} + e^{j\pi/2}\hat{\boldsymbol{y}}\right]e^{j(kz-\omega t)}$$

Consider $z = 0$, where

$$\mathrm{Re}\left\{\boldsymbol{U}(0,t)\right\} = A_0\left[\cos(\omega t)\hat{\boldsymbol{x}} + \cos(\pi/2-\omega t)\hat{\boldsymbol{y}}\right] = A_0\left[\cos(\omega t)\hat{\boldsymbol{x}} + \sin(\omega t)\hat{\boldsymbol{y}}\right]$$

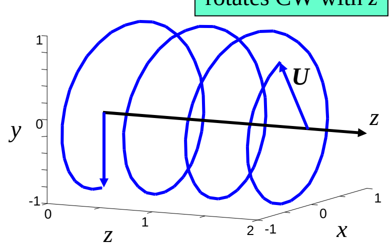
Consider $t = 0$, where

$$\mathrm{Re}\left\{\boldsymbol{U}(z,0)\right\} = A_0\left[\cos(kz)\hat{\boldsymbol{x}} + \cos(\pi/2+kz)\hat{\boldsymbol{y}}\right] = A_0\left[\cos(kz)\hat{\boldsymbol{x}} - \sin(kz)\hat{\boldsymbol{y}}\right]$$



rotates CCW with time

rotates CW with z



Polarization – Right Circular

$A_x = A_y = A_0$
$\phi = -\pi/2$

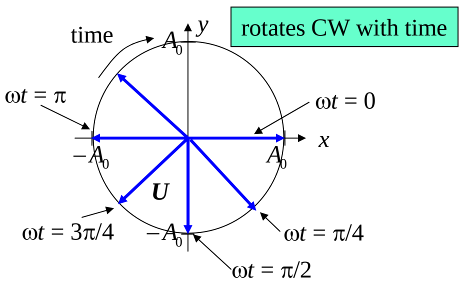
$$\boldsymbol{U}(z,t) = A_0\left[\hat{\boldsymbol{x}} + e^{-j\pi/2}\hat{\boldsymbol{y}}\right]e^{j(kz-\omega t)}$$

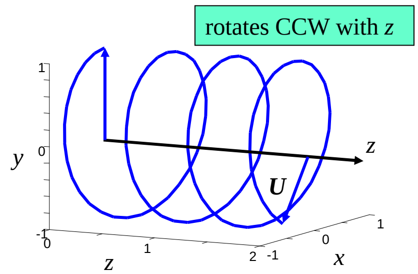
Consider $z = 0$, where

$$\mathrm{Re}\{\boldsymbol{U}(0,t)\} = A_0\left[\cos(\omega t)\hat{\boldsymbol{x}} + \cos(-\pi/2-\omega t)\hat{\boldsymbol{y}}\right] = A_0\left[\cos(\omega t)\hat{\boldsymbol{x}} - \sin(\omega t)\hat{\boldsymbol{y}}\right]$$

Consider $t = 0$, where

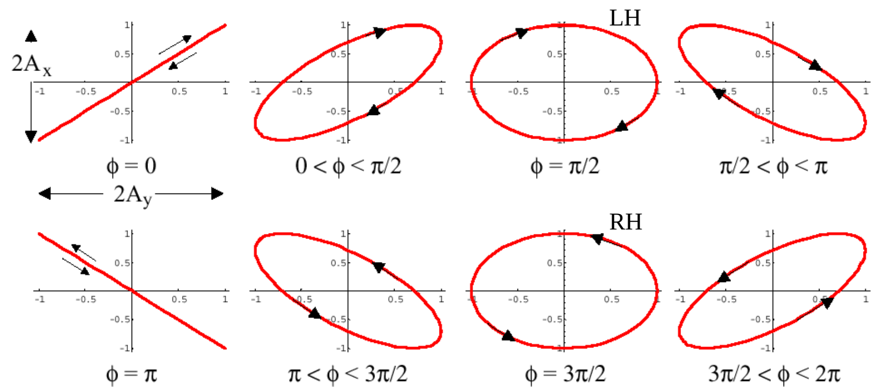
$$\mathrm{Re}\{\boldsymbol{U}(z,0)\} = A_0\left[\cos(kz)\hat{\boldsymbol{x}} + \cos(-\pi/2+kz)\hat{\boldsymbol{y}}\right] = A_0\left[\cos(kz)\hat{\boldsymbol{x}} + \sin(kz)\hat{\boldsymbol{y}}\right]$$





Polarization – Elliptical

(Arrows show trace in *z* direction)



This slide from Jim Wyant, 2000.

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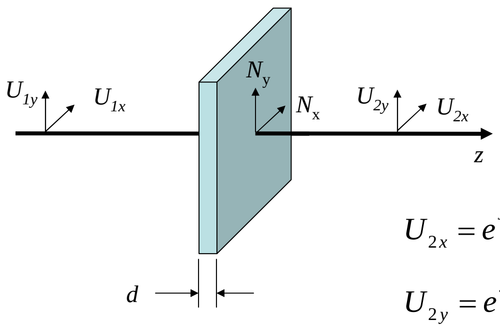
Polarization – Waveplates

Consider optical elements with polarization-dependent refractive index.



$$N_x = n_x + j\kappa_x$$
$$N_y = n_y + j\kappa_y$$

$$U_{2x} = e^{j\frac{2\pi}{\lambda}n_x d} e^{-\frac{2\pi}{\lambda}\kappa_x d} U_{1x}$$
$$U_{2y} = e^{j\frac{2\pi}{\lambda}n_y d} e^{-\frac{2\pi}{\lambda}\kappa_y d} U_{1y}$$

Retardation plate: $n_x \neq n_y,\ \kappa_x \approx \kappa_y \approx 0$

Polarizer plate: $n_x \approx n_y,\ \kappa_x \neq \kappa_y$

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Half-Wave Retardation Plate

The thickness *d* is fabricated so that the phase difference between *x* and *y* polarization components through the plate is π.

$$U_{2x} \approx e^{j\frac{2\pi}{\lambda}n_x d} U_{1x}$$
$$U_{2y} \approx e^{j\frac{2\pi}{\lambda}n_y d} U_{1y}$$

Change in phase through the plate:

$$\Delta\phi_{2x} = \frac{2\pi}{\lambda} n_x d$$
$$\Delta\phi_{2y} = \frac{2\pi}{\lambda} n_y d$$

Note that one axis introduces more phase than the other, because the wave velocity is slower. Therefore, it is called the “slow” axis. The other axis is the “fast” axis.

$$\Delta\phi_{2x} - \Delta\phi_{2y} = \frac{2\pi}{\lambda}\left(n_x - n_y\right) d = \pi$$

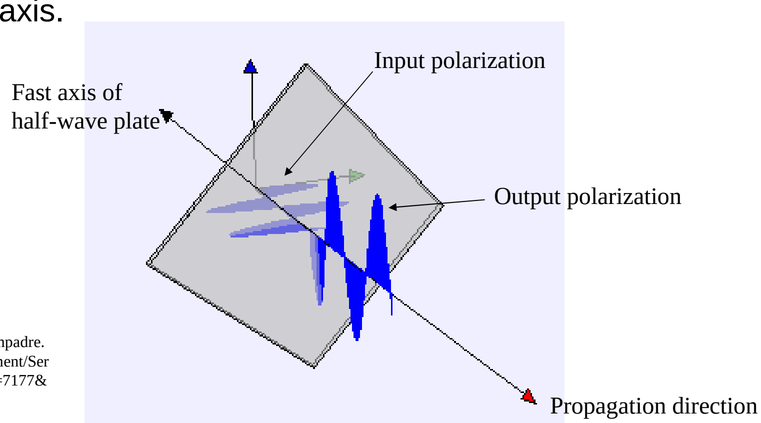
Usually, crystal axes are called “ordinary” n0 and “extraordinary” ne instead of nx and ny.

Solving for *d*: $$d = \frac{\lambda}{2\left|n_x - n_y\right|}$$

Example: Quartz
$n_0 = n_x = 1.544$
$n_e = n_y = 1.553$
at $\lambda = 500$ nm

Rotated Half-Wave Plate

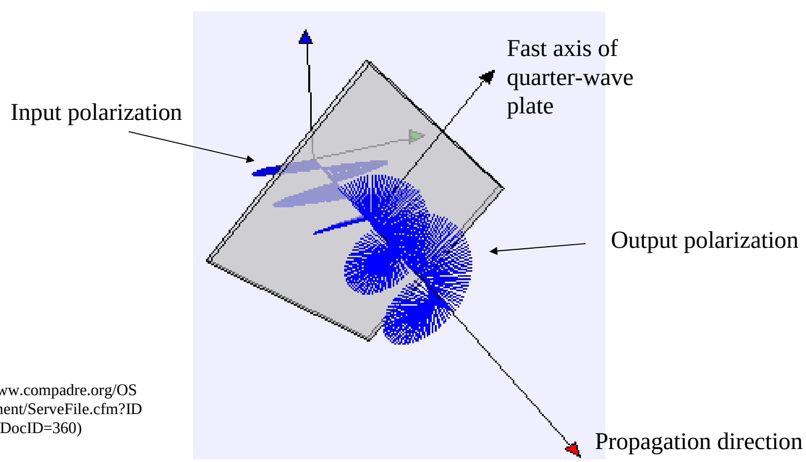
- A half-wave plate rotates the output state of polarization by twice the angle between the input polarization and the fast axis.



(From http://www.compadre.org/OSP/document/ServeFile.cfm?ID=7177&DocID=361)

Quarter-Wave Plate at 45°

- A quarter-wave plate rotated at 45° changes linear polarization to circular polarization.



(From http://www.compadre.org/OSP/document/ServeFile.cfm?ID=7176&DocID=360)

Stokes Vectors (I)

- The preceding analysis is good for laser beams, where there is an exact relationship defining the electric field vector in time.
- Real light sources (like light bulbs and LEDs) do not emit such nicely defined light waves.
- However, transmission through polarizers, waveplates and reflections from surfaces can partially polarize the light.
- The Stokes vector is a way to describe the state of polarization for light other than laser beams.

Stokes Vectors (II)

- The four Stokes elements are:
  - S0 is proportional to the amount of light in the beam being measured.
  - S1 is proportional to the tendency of the light to be linearly horizontally (+) or vertically (-) polarized.
  - S2 is proportional to the tendency of the light to be linearly +45° (+) or -45° (-) polarized.
  - S3 is proportional to the tendency of the light to be RHC (+) or LHC (-).

Stokes Vectors (III)

- We determine the Stokes vector by measuring power in the beam and using a series of three filters:
  - I0 is measurement of the beam power.
  - I1 is measurement of power transmitted through a horizontal linear polarizer.
  - I2 is measurement of power transmitted through a linear polarizer oriented at +45°
  - I3 is measurement of power transmitted through a polarizer opaque to LHC.

Stokes Vectors (IV)

- The Stokes vector is calculated from:
  - S0 = I0
  - S1 = 2*I1 – I0
  - S2 = 2*I2 – I0
  - S3 = 2*I3 – I0