

# OPTI 380A Intermediate Optics Lab 8: Interference

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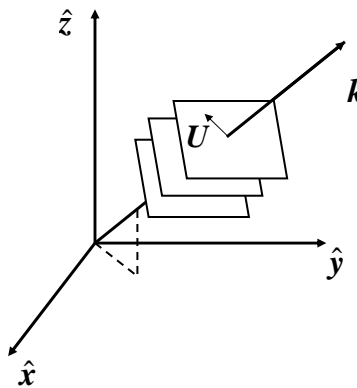
## Plane Waves

$$U(\mathbf{r}, t) = U_0 e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

Wave fronts  $\perp \mathbf{k}$ .

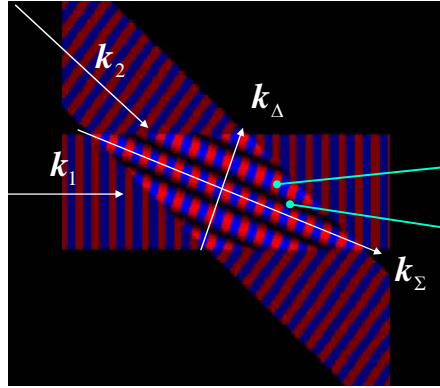
Max separation between planes =  $\lambda$ .

$U_0 \perp \hat{\mathbf{k}}$  in linear, isotropic media



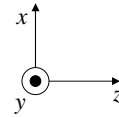
## Two Vector Plane Waves - Dynamics

(Time Domain Dynamics)



Red = negative field amplitude  
Blue = positive field amplitude

- For  $\lambda_1 = \lambda_2$ ,  $\omega_\Delta = 0$  and modulation is stationary.
- For  $\lambda_1 = \lambda_2$ ,  $\mathbf{k}_\Delta \perp \mathbf{k}_\Sigma$
- Power flow is along *constructive* lines.
- Total energy is conserved due to *destructive* lines.



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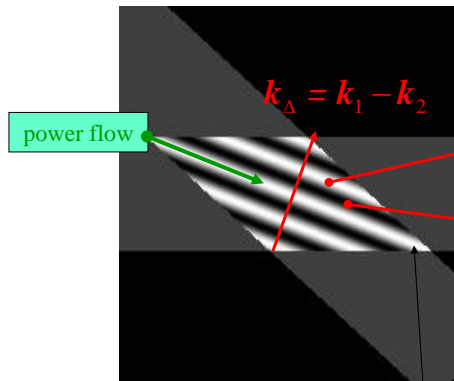
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## Two Vector Plane Waves - Irradiance

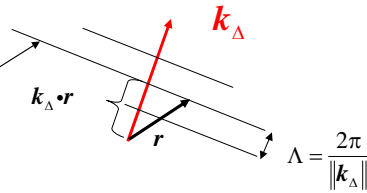
(Time Averaged Irradiance)

(We will drop the y subscript for now and assume  $\beta=0$ .)

$$I(\mathbf{r}) = CA^2 [1 + \cos(\mathbf{k}_\Delta \cdot \mathbf{r})]$$



- When  $(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} = 2\pi m$ , a *bright* fringe occurs from *constructive* interference.
- When  $(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} = 2\pi(m + 1/2)$ , a *dark* fringe occurs from *destructive* interference.



fringe field is collection of planes of constant  $\mathbf{k}_\Delta \cdot \mathbf{r}$ .

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## Two Vector Plane Waves - Visibility

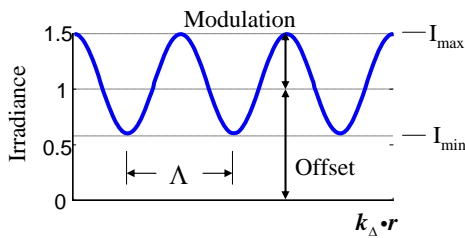
Consider the combination of two unequal amplitude plane waves. Assume  $\theta = 0$ .

$$I(\mathbf{r}) = C[A_1^2 + A_2^2 + 2A_1A_2 \cos(\mathbf{k}_\Delta \cdot \mathbf{r})]$$

interference term

$$= C[A_1^2 + A_2^2 + 2A_1A_2 \cos(\mathbf{k}_\Delta \cdot \mathbf{r})] = I_1 + I_2 + 2\sqrt{I_1I_2} \cos(\mathbf{k}_\Delta \cdot \mathbf{r}) \quad (p_{12} = 1)$$

where  $I_1$  and  $I_2$  are irradiance values for waves 1 and 2, respectively. If  $I_1 \neq I_2$ , the total irradiance across a section of the observation plane might look like:



$$\text{visibility} = V = \frac{\text{Offset}}{\text{Modulation}}$$

$$= \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$= \frac{2\sqrt{I_1I_2}}{I_1 + I_2} p_{12}$$

$$0 \leq V \leq 1$$

## Scalar Spherical Waves

A solution to the wave equation in spherical coordinates is:

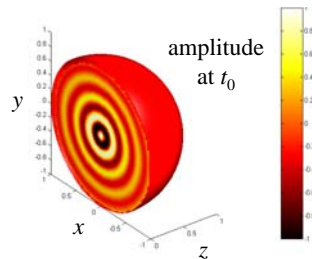
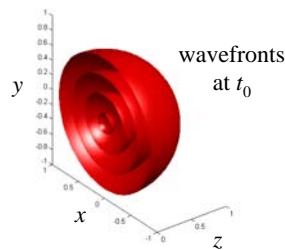
$$U(r, t) = \frac{1}{r} e^{j(kr - \omega t)}$$

where

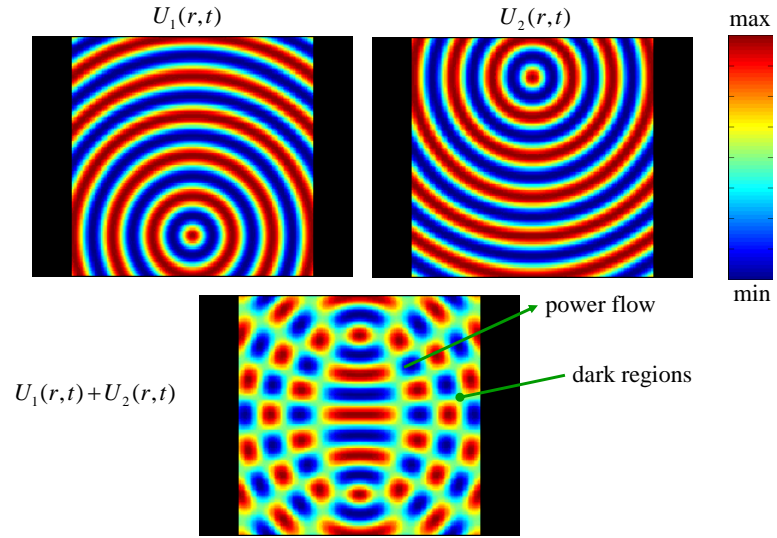
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\text{and } \psi = kr - \omega t.$$

Wave crests (also called *wavefronts*) at  $\psi = \text{constant}$  expand outwardly from the source point as a function of time. Wavefronts are perpendicular to  $r$ .



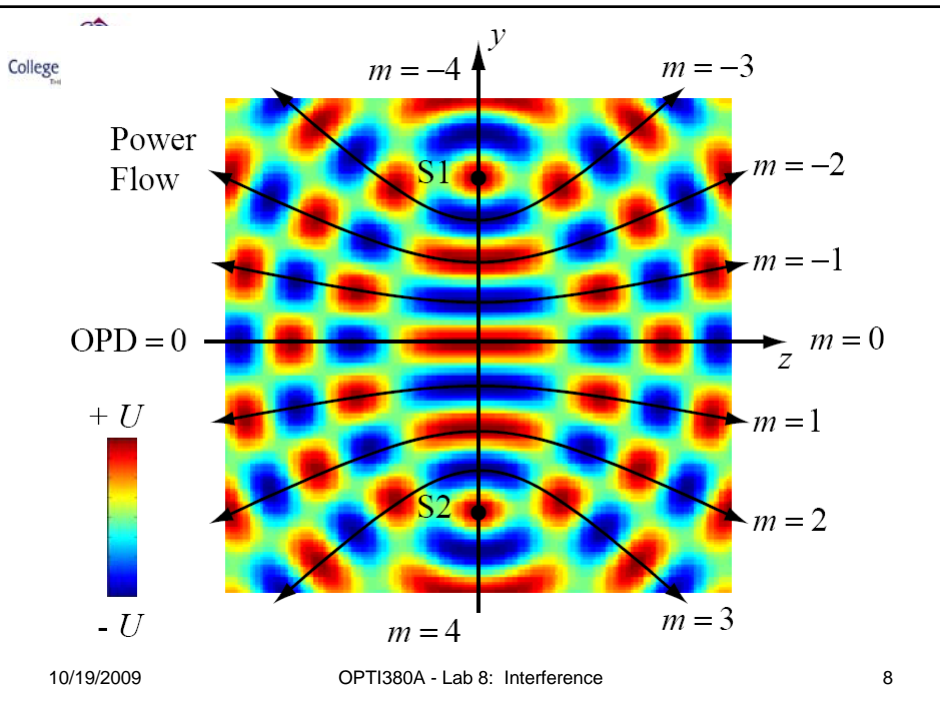
## Two Spherical Waves



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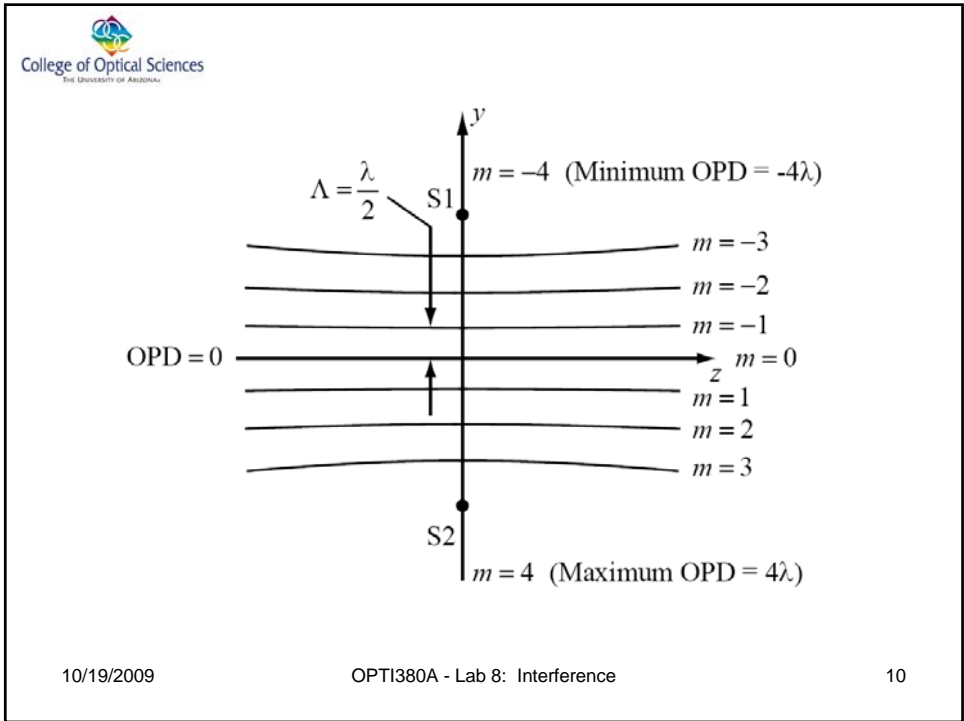
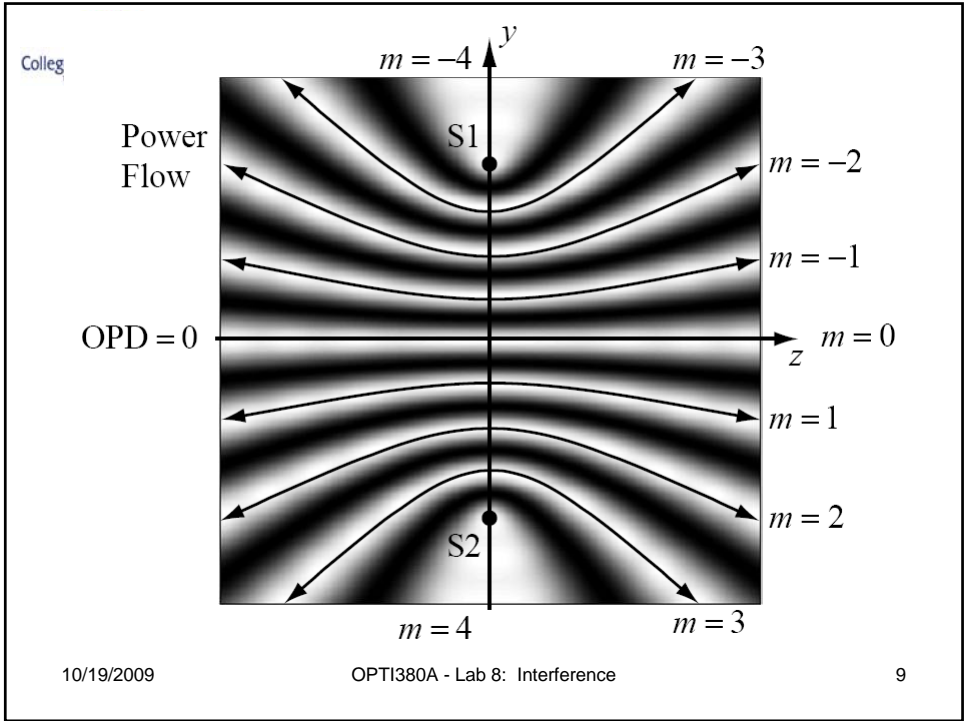
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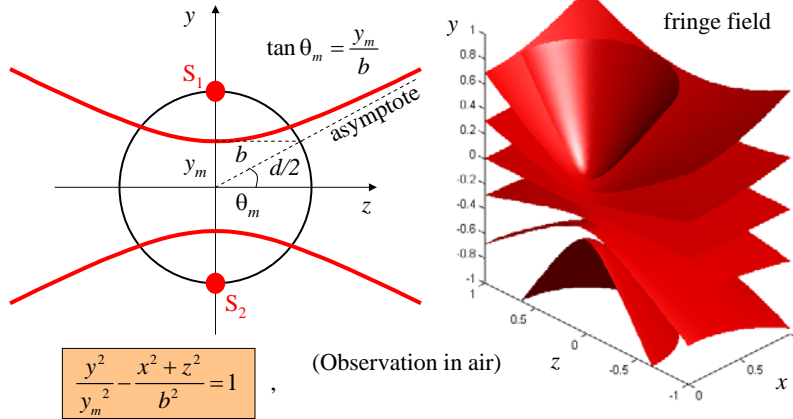
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## Two Spherical Waves – Fringe Surfaces



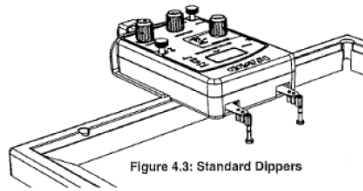
$$\frac{y^2}{y_m^2} - \frac{x^2 + z^2}{b^2} = 1$$

(Observation in air)

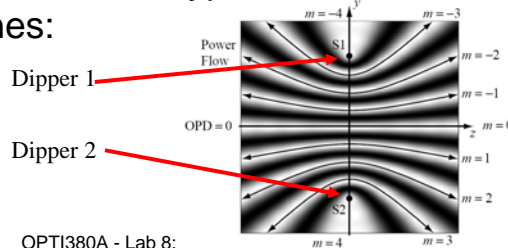
where  $y_m = -\frac{1}{2}m\lambda$  and  $b^2 = (d/2)^2 - y_m^2$ . These are *hyperboloidal sheets* in 3D

## Two-Point Interference with a Ripple Tank (Part C of This Week's Experiment)

- Each actuator (dipper) acts like a point source



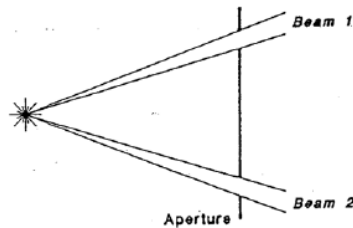
- We would expect to see hyperboloidal interference lines:



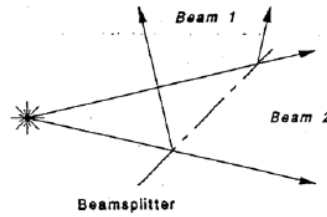
# Interferometers

- Division of Wavefront
  - Samples wavefront at different points
- Division of Amplitude
  - Creates copies of the wavefront

Division of Wavefront



Division of Amplitude

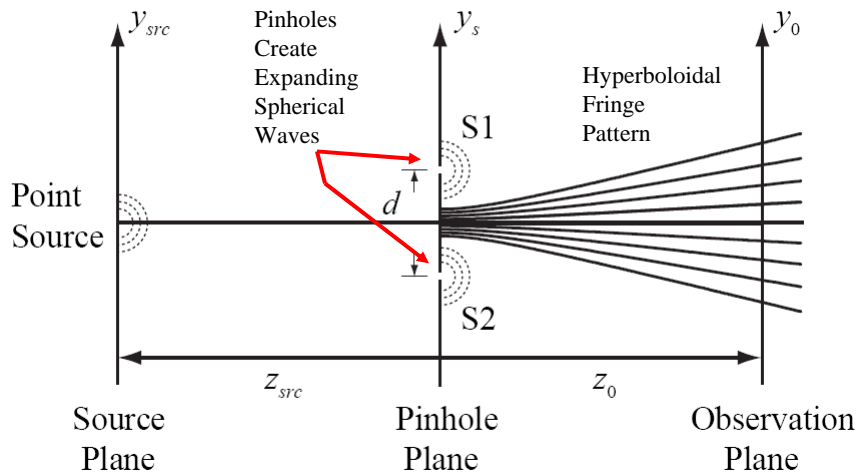


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## Part A: Young's Double Slit Interferometer (Model)

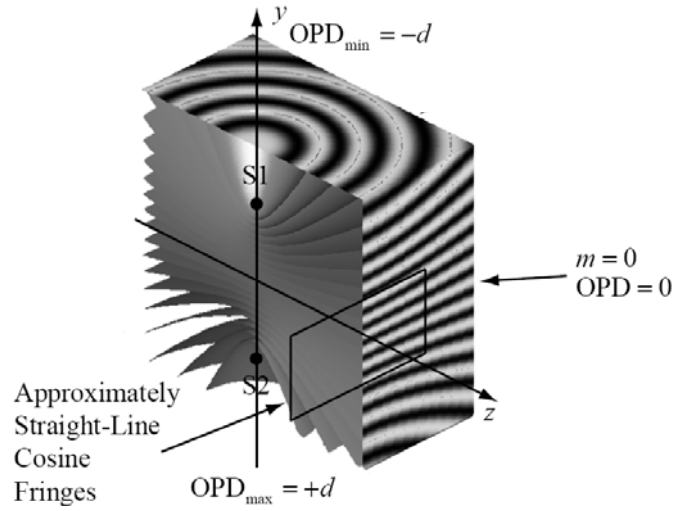


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### YDPI with $z_0$ Large



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### Young's Double Slit Interferometer (YDSI)

(Implementation)

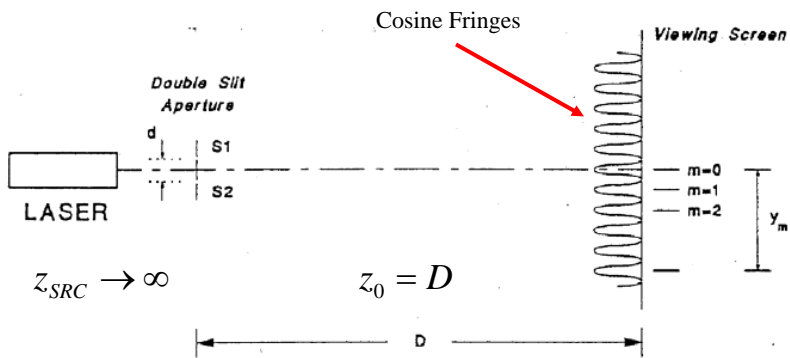


Figure 8.4. Setup for Young's Double Slit experiment (NOT TO SCALE).

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## YDSI Optical Paths

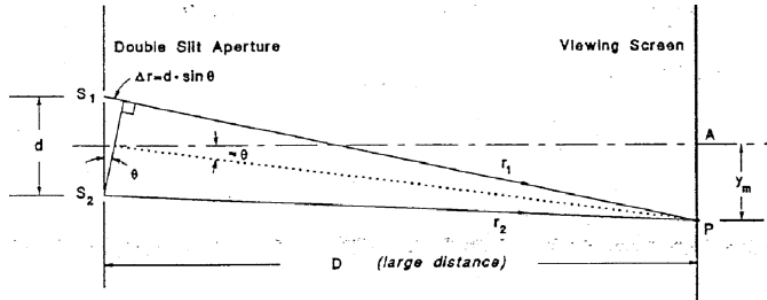
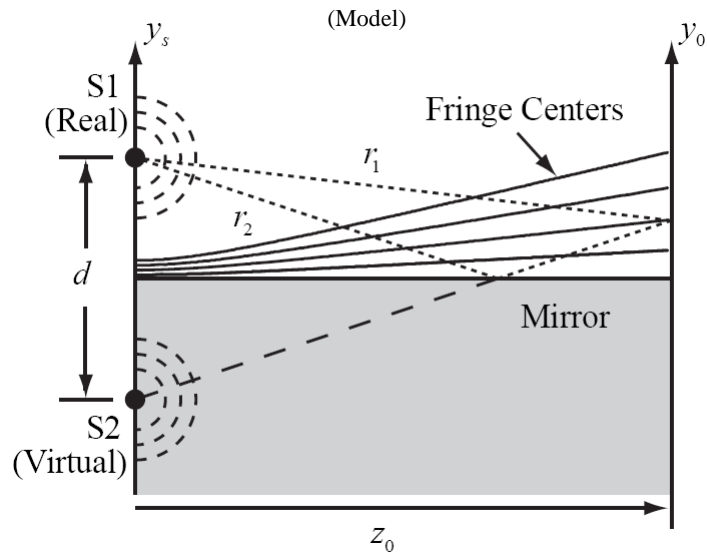


Figure 8.5. Optical paths in Young's Double Slit experiment.

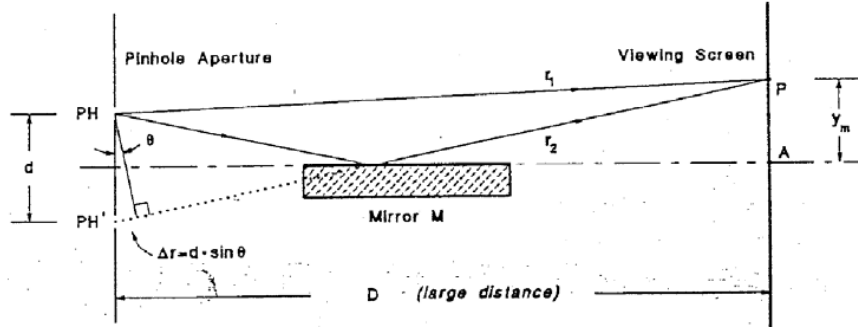
$$y_m \approx \frac{mD\lambda}{d}$$

## Part B: Lloyd's Mirror



# Lloyd's Mirror

(Optical Paths)



$$y_m \approx \frac{mD\lambda}{d}$$

(Reverse fringe contrast due to Fresnel reflection from mirror.)

# Lloyd's Mirror

(Implementation)

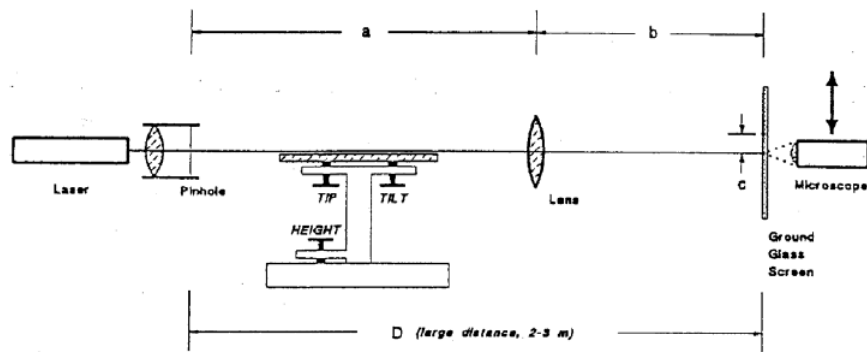
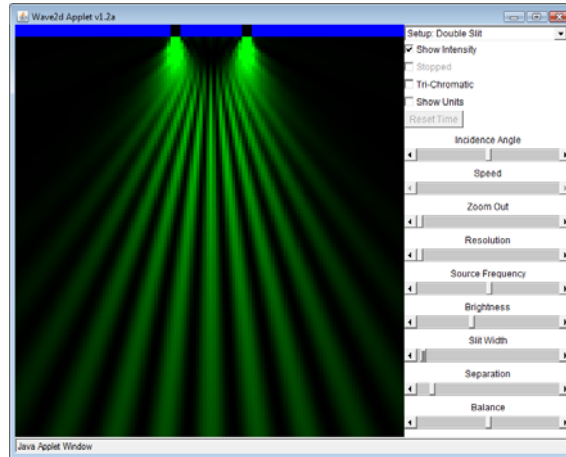


Figure 8.6. Setup used for Lloyd's Mirror (NOT TO SCALE).

## Good Website for YDSI

- <http://www.falstad.com/mathphysics.html>



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