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OPTICS 380A
Lab 10: Michelson Applications

Lab 10 is an extension of the last lab, where we learned to properly align the Michelson interferometer. This lab is designed to show how a Michelson may be used to measure the wavelength of a source, the spectral separation of the sodium doublet, and the refractive index of air. Once aligned, the interferometer may be used to quickly perform all of the tasks in this lab.

MEASURING WAVELENGTH-THEORY

The equation describing the fringes is given as:

$$m\lambda = 2nd\cos(\theta) \qquad (10\text{-}1)$$

where m is the fringe <u>order number</u>, λ is the wavelength of light, d is the distance between the mirrors (2nd is the optical path difference, OPD), θ is the angle of inclination of the mth order fringe, and n is the refractive index. For the on-axis fringe, θ=0 and cos(θ)=1. Setting the refractive index equal to 1, the equation then simplifies to:

$$m\lambda = 2d \qquad (10\text{-}2)$$

In practice, when OPD is changed, a change in fringe order number is observed. To measure the wavelength of light, we could change d and count the number of fringes that move past an on-axis detector. The wavelength is then calculated:

$$m_1\lambda = 2d_1 \text{ (starting point)} \qquad (10\text{-}3)$$

$$m_2\lambda = 2d_2 \text{ (after changing the OPD)} \qquad (10\text{-}4)$$

Subtracting gives:

$$\lambda = \frac{2\cdot(d_2 - d_1)}{(m_2 - m_1)} \qquad (10\text{-}5)$$

Note that the actual values for m_1 and m_2 do not need to be known. It is only the difference that is needed. However, the relationship between the micrometer movement and the mirror movement DOES have to be known, in order to calculate $(d_2 - d_1)$. For our instrument, the mechanical linkage between the micrometer and mirror is arranged to provide the following mechanical (dis)advantage:

1 division on the micrometer = 1 micron movement of the mirror

CALIBRATING THE INTERFEROMETER-THEORY

Use a Helium-Neon laser to calibrate the actual ratio between movement of the micrometer and movement of the mirror. The He-Ne laser wavelength is known to be exactly 632.8nm. To do this, first re-write equation (10-5) as follows:

$$\left(d_2 - d_1\right) = \frac{\left(m_2 - m_1\right) \cdot \lambda}{2} \tag{10-6}$$

You will use your eye to observe changes in the on-axis fringe. After counting ($m_2 - m_1$) number of fringes, the mirror will have moved a distance ($d_2 - d_1$), having the same units that you use for λ. By recording the equivalent (much larger) distance that the micrometer moves (when counting ($m_2 - m_1$) number of fringes) you will have a calibration factor for your interferometer. Note that the accuracy of this factor increases as you count a greater number of fringes!

CALIBRATING THE INTERFEROMETER--EXPERIMENT

NOTE: In all of the following steps, DO NOT LOOK INTO THE OUTPUT OF THE INTERFEROMETER. OBSERVE THE FRINGES ON A WHITE CARD AT THE OUTPUT OF THE INTERFEROMETER.

(1) Gently move the entire interferometer until the laser spot is centered on the movable mirror. Rotate the entire interferometer about this mirror until the laser beam is returned in the vertical plane containing the input beam. Adjust the tilt screw on the laser holder to center this returned beam back down the throat of the laser. All of this may take a few iterations for proper alignment.

(2) Look at the white card. You should see two sets of spots in the horizontal plane. Adjust the fixed mirror until the two sets coincide. With careful adjustment you will begin to see fringes. Because the source is not an extended source, you will not see circular fringes yet at this point.

(3) Place the +25mm F.L. lens in the laser beam to diverge the beam into the interferometer. Adjust the lens position until the beam is centered in the beamsplitter and the two mirrors. The circular fringes will now be readily apparent, which are similar to the fringes you observed with the extended source.

(4) Record the initial micrometer reading.

(5) Move the micrometer slowly, counting at least 100 fringes.

(6) Record the final setting of the micrometer.

1 division on the micrometer = ________ micron movement of the mirror

PART A: MEASUREING WAVELENGTH

MEASURING WAVELENGTH – GREEN MERCURY SOURCE

(1) Align the interferometer to give circular fringes. Use the telescope to view the central, on-axis fringe.

(2) Move the micrometer to give a bright or dark fringe on-axis. Record the micrometer reading.

(3) Move the micrometer slowly, counting at least 50 fringes.

(4) Record the final setting of the micrometer.

(A) What is the wavelength, in nanometers, of the source?

(B) Suppose you made a mistake in counting by one fringe. Given the number of fringes that you did count, what is the error in calculating the wavelength?

(C) To know the value of wavelength to within 0.1 nm, how many fringes would you need to count? (Assume the same error of one fringe when counting).

MEASURING WAVELENGTH-LOW PRESSURE YELLOW SODIUM SOURCE

(1) Replace the green source with the sodium source. Make sure the source has been turned on for some time, so that it is hot. Center the beams on both mirrors and the beamsplitter.

(2) Move the micrometer until the fringes appear darkest.

(3) As above, count at least 50 fringes and record the readings of the micrometer.

(D) What is the wavelength, in nanometers, of the source?

(E) In step (2) above, what measure of fringe quality are we referring to when adjusting for darkest fringes? How does this quantity change as the fringes become darkest?

(F) Look at the fringes and move the micrometer through a large range of its travel. Describe what you see.

MEASURING SPECTRAL LINE SEPARATION-THEORY

The sodium source actually contains two spectral lines separated in wavelength by a very small amount. The term spectral line is used because if we plot the light intensity of this type of source vs. wavelength across the entire visible spectrum, we would see light output only at very discrete wavelengths on such a scale. In other words, the spectral width of each color that the source emits is very much narrower than the resolution of our graph. The result would look something like the following:

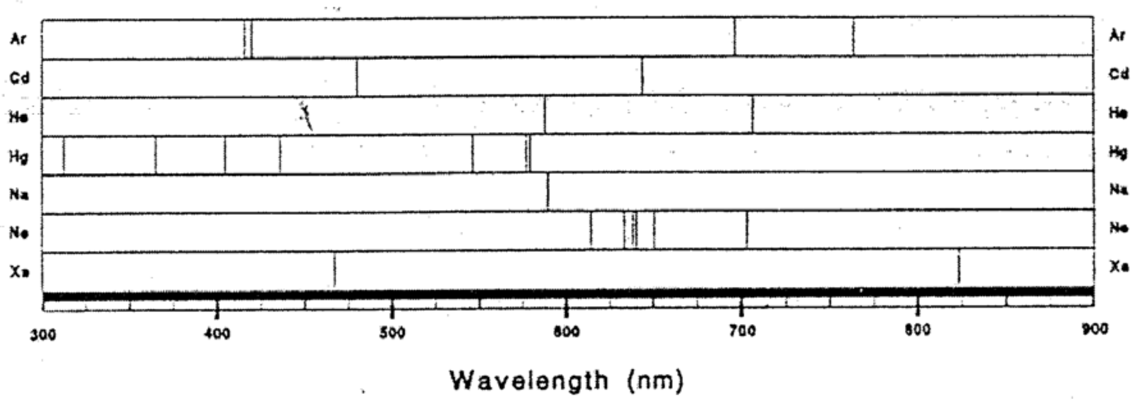

Fig. 10.1. Low-pressure spectral line lamps.

The purpose of this next part is to measure the wavelength separation of the sodium doublet with the Michelson.

The observations made when moving the micrometer through a large range of travel give evidence that two spectral lines exist, not just one. This may be seen by considering that each wavelength will produce its own set of circular fringes with an on-axis intensity given by the standard expression for two-beam interference:

$$
\begin{aligned}
I_1 &= 2I\left[1+\cos\left(\frac{4\pi}{\lambda_1}\cdot d\right)\right] \\
I_2 &= 2I\left[1+\cos\left(\frac{4\pi}{\lambda_2}\cdot d\right)\right]
\end{aligned}
\tag{10-7}
$$

where λ_1 and λ_2 are the two wavelengths of the doublet and d is the mirror separation. The intensity one sees when looking into the interferometer, on-axis, is given by the algebraic sum of these two intensities:

$$I_{tot} = I_1 + I_2 \tag{10-8}$$

When this sum is written out and simplified, the intensity becomes:

$$I_{tot} = 4I\left[1 + \left(\cos\frac{4\pi}{\lambda}\cdot d\right)\cdot\cos\left(\frac{2\pi\Delta\lambda}{\lambda^2}\cdot d\right)\right] \quad , \tag{10-9}$$

where the following simplifications have been used:

$$\left(\lambda_1 + \lambda_2\right) = 2\lambda \quad ; \quad \lambda_1\cdot\lambda_2 = \lambda^2 \ . \tag{10-10}$$

The intensity is seen to go through successive maxima and minima for mirror separations Δd given by:

$$\Delta d = \frac{\lambda^2}{2\cdot\Delta\lambda} \tag{10-11}$$

Solving for the wavelength separation gives:

$$\boxed{\Delta\lambda = \frac{\lambda^2}{2\cdot\Delta d}} \quad . \tag{10-12}$$

The variation of intensity with mirror separation d is due to the fact that two wavelengths are present in the source. Physically, this variation is the spatial beat frequency between the two interference patterns from the two wavelengths. The second cosine term in (10-9) is this beat frequency modulation. In the language of optics, this is a sinusoidal variation in the fringe visibility. In the language of electrical engineering, this is a sinusoidal amplitude modulation of the carrier frequency (the fringe pattern itself).

PART B: MEASURING SPECTRAL LINE SEPARATION-EXPERIMENT

(1) The interferometer should already be aligned to produce circular fringes with the sodium source. If not, align it.

(2) Move the micrometer to the end of its scale which is closest to 0. DO NOT FORCE THE MICROMETER AGAINST THE END OF ITS TRAVEL. GENTLY APPROACH THE LIMIT.

(3) Observe the on-axis fringe, and begin to move the micrometer through its full travel. Stop when the fringes disappear for the first time. This null is broad and occurs over a range of motion of the micrometer. Estimate the point of sharpest null (minimum fringe visibility) as best you can. Record the micrometer setting.

(4) Observe the fringe pattern and move the micrometer through 2 more nulls, recording the micrometer position at each null.

(5) Use the micrometer readings to get the distance ($2 \cdot \Delta d$) between successive nulls.

(G) What is the calculated wavelength separation of the sodium doublet, in nanometers? (Use the value of wavelength that you measured in the previous section as the `average' wavelength for the sodium doublet.)

(H) How well does your number compare to the accepted separation of 0.6nm? (quote % error).

(I) Derive equation (6-9), the intensity for the combined fringe pattern. Use the following trig identity:

$$\cos(a)+\cos(b)=2\cdot\cos\left(\frac{a+b}{2}\right)\cdot\cos\left(\frac{a-b}{2}\right) \quad . \qquad (10\text{-}13)$$

HINT: The expression will contain various combinations of λ_1 and λ_2. Set $(\lambda_1+\lambda_2)=2\lambda$ and $\lambda_1\cdot\lambda_2=\lambda^2$.

(J) Derive equation (10-12), the expression for $\Delta\lambda$. This is easily done by considering when the second cosine term in equation (10-8), the modulation term, goes through a zero. These are the point of minimum fringe visibility.

MEASURING THE REFRACTIVE INDEX OF AIR-THEORY

The Michelson interferometer can be used to measure the refractive index of gases as well as solids. In this experiment, we will measure the refractive index of a mixture of gases, namely air.

The refractive index of air depends not only on temperature, as does a solid, but also on the atmospheric pressure, relative humidity, and of course wavelength. It also depends on the particular mixture of gases in the air when measured, but this will be assumed to be the "standard atmosphere"; 78% N_2, 20.9% O_2, 0.93% Ar, and 0.03% CO_2. We will ignore the effects of relative humidity, but we will correct our data for the measured temperature in the lab.

By definition, the refractive index of vacuum is 1. As the pressure of the air increases, so does the refractive index. Physically, the refractive index varies linearly with the density of a gas, and therefore varies linearly with pressure as well. A plot of index vs. pressure will be a straight line, having a positive slope and a y-intercept of 1. In this lab, we will measure the slope of this line, and use it to calculate the refractive index of air at local atmospheric pressure.

The equation describing fringes in a Michelson has already been given in (10-9). If somehow the pressure in one of the arms of the interferometer could be changed, the index of refraction would change as just described. Likewise, the OPD would change and, according to this equation, so should the order number of the fringe at a fixed location within the field of view. In other words, as the pressure is changed in one of the arms, the fringes should move just the same as if one of the mirrors was translated. All of this is easily accomplished using a small cell with optical windows that is inserted in one of the arms of the interferometer. A small hand pump is used to change the pressure within the cell.

Consider two pressures, P_1 and P_2. The corresponding refractive indices are n_l and n_2. The change in OPD between the two pressures is given by:

$$2n_2D - 2n_1D = m_2\lambda - m_1\lambda \qquad (10\text{-}14)$$

Where m_2 and m_l are order numbers of two different fringes, λ is the wavelength of light, and D is the length of the gas cell, EXCLUDING THE END WINDOWS. Equation (10-14) may be solved for the change in refractive index. Dividing by the change in pressure gives the slope of the curve we wish to plot:

$$SLOPE \equiv \frac{\Delta n}{\Delta P} = \frac{n_2 - n_1}{P_2 - P_1} = \frac{(m_2 - m_1)\lambda}{2D(P_2 - P_1)} \qquad (10\text{-}15)$$

The change in fringe order number, m_2-m_l is the number of fringes counted as the pressure is changed.

PART C: MEASURING THE REFRACTIVE INDEX OF AIR-EXPERIMENT

(1) Set up the Michelson to produce circular fringes using the He-Ne laser.

(2) Position the gas cell between the beam splitter and movable mirror. The light beam should pass through the cell with no vignetting. Make any mirror adjustments to obtain a clear set of circular fringes (none should really be needed, just so the central fringe is visible).

(3) Slowly pump the air out of the cell and observe the fringes. Stop pumping and watch the pressure gauge. If there are no leaks, the needle should remain constant. If the needle slowly moves, the system has a leak and needs to be checked before the next step.

(4) Release air into the cell by rotating the small valve on the hand pump. The needle should be fully clockwise.

(5) Slowly pump air out of the cell and stop when the needle is on one of the marks of the cm Hg scale. Record this pressure.

(6) Slowly continue to pump the air out, counting the number of fringes that pass through the center of the pattern. Stop when you have counted 20 fringes. Record the pressure reading on the cm Hg scale.

(7) Release air into the cell, remove the cell from the interferometer, and use the calipers to carefully measure the length D of the cell.

(8) Record the temperature of air in the lab.

(K) What was the measured change of refractive index with pressure? (Quote pressure as cm Hg).

(L) Plot the refractive index of air (y-axis) vs. pressure (x-axis) using an appropriate scale to show the slope. Make the maximum value of the x-axis 100cm Hg. Note that atmospheric pressure at sea level is 76cm Hg.

(M) What is the measured value of the refractive index of air at the lab temperature? You will need to look up the atmospheric pressure in Tucson for the day that you perform this experiment. (1" Hg = 33.8 millibar may be a useful conversion to know).

(N) What would be the refractive index of air at 0°C? This is the standard temperature for quoting the index. Use the following formula to do the calculation:

$$\frac{\left(n(T=0)-1\right)}{\left(n(T)-1\right)}=1+(0.00367)T \tag{10-16}$$

where T is the lab temperature in degrees Centigrade.

(O) Based on Eq. (10-16), how would the index change with temperature?

(P) Compare your results at 0˚C with those taken from the Handbook of Chemistry and Physics (page E-393). Give the accepted value and the % error in your measurement. Use the following equation to calculate the accepted value of index at 0.6328 microns wavelength and at 0˚C:

$$\left(n-1\right)\cdot 10^{7}=a+\frac{b}{\lambda^{2}\cdot 10^{-8}}+\frac{c}{\lambda^{4}\cdot 10^{-16}} \tag{10-17}$$

where a = 2875.66, b = 13.412, c = 0.3737, and X is in angstroms (1 angstrom A = 0.1 nm).