

D-B1) Consider a simple grating, where the transmission of the grating is given by

$$t(x_1) = \frac{1}{D} \text{rect}\left(\frac{x_1}{D}\right) * \frac{1}{2D} \text{comb}\left(\frac{x_1}{2D}\right) ,$$

where  $D = 10\mu\text{m}$  and  $\lambda = 0.5\mu\text{m}$ .

- a.) Graph the angular spectrum of the transmitted wave if the illumination is an on-axis plane wave with unit amplitude.
- b.) Graph the angular spectrum of the transmitted wave if the illumination  $U(x_1)$  is a tilted plane wave, where

$$U(x_1) = e^{-j2\pi\alpha_0 x_1} ,$$

and  $\alpha_0 = 0.01$ .

- c.) Assume that a perfect lens with focal length 100 mm is placed immediately following the grating. If we assume an infinite diameter lens, graph the distribution of light in the rear focal plane for cases (a) and (b).

D-B2) Consider a simple grating, where the transmission of the grating is given by

$$t(x_1) = \frac{1}{D} \text{rect}\left(\frac{x_1}{D}\right) * \frac{1}{2D} \text{comb}\left(\frac{x_1}{2D}\right) ,$$

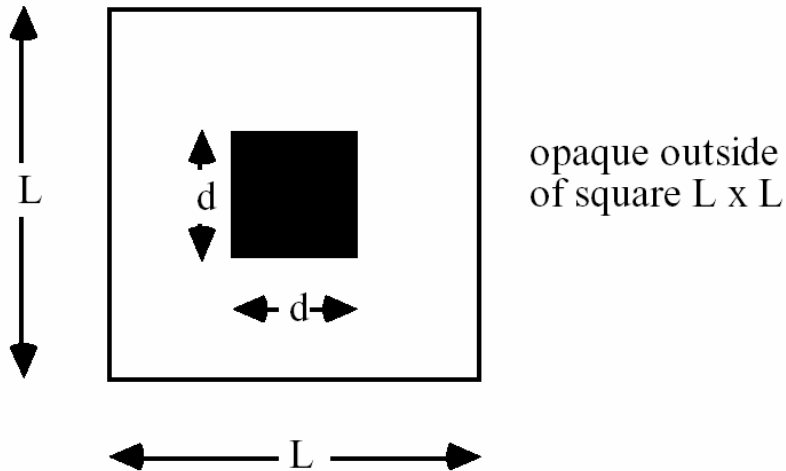
where  $D = 10\mu\text{m}$  and  $\lambda = 0.5\mu\text{m}$ . Graph the angular spectrum of the transmitted wave if the illumination is

$$U(x_1) = \frac{1}{16D^2} \text{sinc}\left(\frac{x_1}{4D}\right) \text{sinc}\left(\frac{y_1}{4D}\right) .$$

D-B3) Imagine that you are looking through a piece of square woven cloth at a point source of wavelength 500 nm a distance of 10 meters away. If you see a square arrangement of bright spots located about the point source, each separated by an apparent nearest-neighbor distance of 5 cm, how close together are the strands of cloth?

D-B4) Use the plane wave spectrum approach to look at the propagation of two plane waves between two planes separated by an unknown distance. One plane wave makes an angle of 5 degrees with respect to the  $z$  axis, and the second plane wave is along the  $z$  axis. The phase difference between the two given components of the angular spectrum changes by 30 degrees as the beam propagates between the two planes. The wavelength is 500 nm. What is the distance between the two planes?

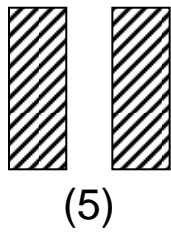
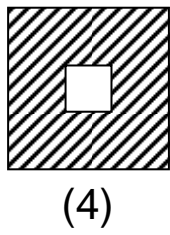
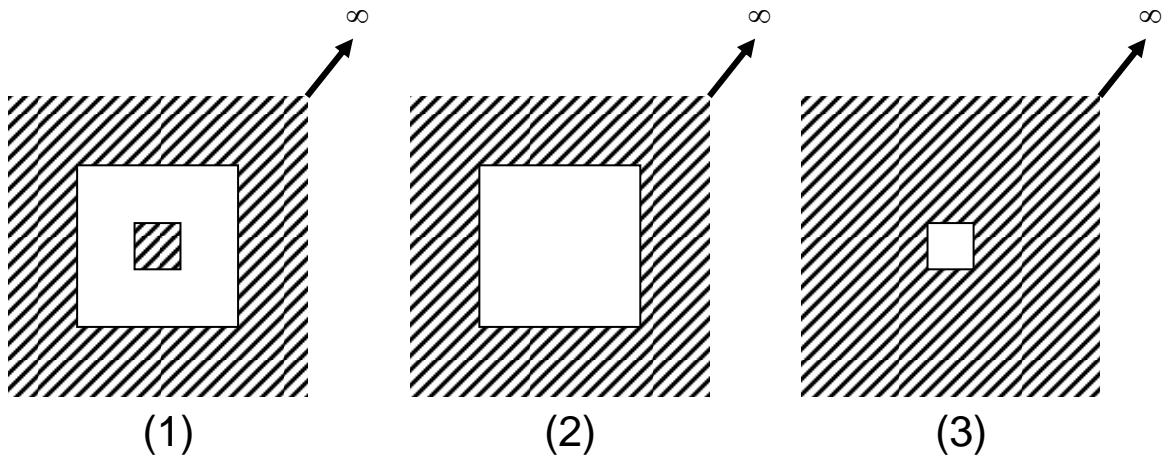
D-B5) Find an expression for the intensity distribution in the Fraunhofer diffraction pattern of the aperture shown below. Assume unit-amplitude, normally incident plane-wave illumination.



D-B6) A 1 mm diameter hole is illuminated with a normally incident plane wave at  $1 \text{ W/cm}^2$  and  $\lambda = 500 \text{ nm}$ .

- a.) What is the on-axis irradiance at a distance of 0.167 m behind the aperture?
- b.) What is the on-axis irradiance at a distance of 0.125 m behind the aperture?
- c.) Roughly sketch the irradiance profile for part (a). Indicate the boundary of the geometrical shadow.
- d.) Roughly sketch the irradiance profile for part (b). Indicate the boundary of the geometrical shadow.

D-B7) The diffracted amplitude and phase from each aperture shown below is measured at a distance  $z$  from the aperture. The field measurements are denoted  $f_i(x,y)$ , where  $i = 1, 2, \dots, 10$ . Find four combinations of apertures such that  $f_i(x,y) = f_j(x,y) + f_k(x,y)$ , where  $i \neq j \neq k$ . Assume identical measurement conditions for each aperture. (Shading indicates an opaque region.)



(9) transparent

(10) opaque

$$f_i(x,y) = f_j(x,y) + f_k(x,y)$$

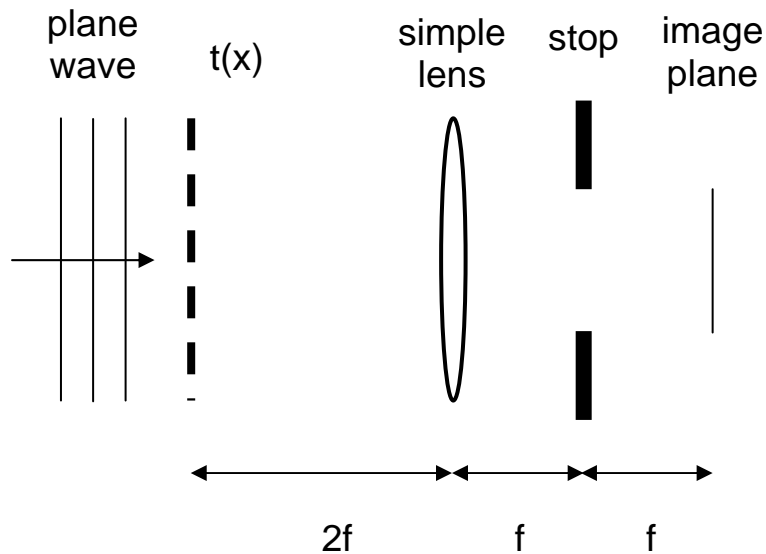
$i$	$j$	$k$

D-B8) The Fraunhofer diffraction pattern of an aperture has the condition that  $I(x,y) = I(-x,-y)$ . What do we know about the aperture?

D-B9) A Fraunhofer diffraction pattern is formed with a plane diffracting screen having an aperture of any shape. Let the incident beam be normal to the screen, and let the coordinates at any point  $P$  on the rim of the aperture be  $(x_p, y_p)$ . Show that if the aperture is deformed by changing  $x_p$  to  $hx_p$  for all points  $P$ , where  $h$  is a constant (so for example, a circle becomes an ellipse), then the irradiance at  $(x_o, y_o)$  after the deformation is  $h^2$  times that at  $(hx_o, y_o)$  before the deformation.

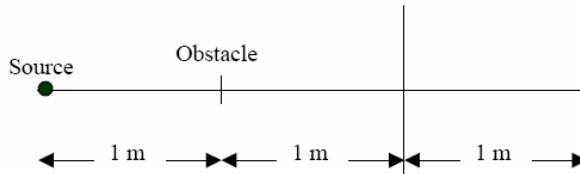
D-B10) A grating with amplitude transmittance  $t(x) = 1 + \cos(2\pi\alpha_0x)$  is illuminated by an on-axis monochromatic plane wave with  $\lambda = 0.5 \mu\text{m}$ . The grating is imaged with a simple lens as shown below, where  $f = 30 \text{ mm}$ . The diameter of the stop is 2 mm.

- Describe the angular spectrum of the light between the grating and the lens
- Describe the irradiance distribution in the region between the grating and the lens. Where are the planes of reconstruction of the grating pattern? Of the inverse pattern?
- Plot the contrast (visibility) of the grating image as a function of  $\alpha_0$ . What is the cutoff frequency  $\alpha_0/\lambda$ , where no contrast is observed in the image?



D-B11) A 1 mm diameter circular obstacle is illuminated with a spherical wave diverging from a point source 1 meter to the left of the circular obstacle. In the plane of the circular obstacle the irradiance of the illuminating beam is 1 watt/cm<sup>2</sup>. What is the on-axis irradiance.

- a.) (10 pts) 1 meter to the right of the circular obstacle?  
 b.) (5 pts) 2 meters to the right of the circular obstacle?



D-B12) A 100 μm pitch amplitude grating is illuminated by two collimated laser beams at normal incidence. The laser wavelengths are 400 nm and 800 nm. What observation is made at a distance of 12.5 mm from the grating? The transmittance of the grating is ( $m \ll 1$ ,  $D = 50\mu\text{m}$ ):

$$t(x_1) = 1 + m \frac{1}{D} \text{rect}\left(\frac{x_1}{D}\right) * \frac{1}{2D} \text{comb}\left(\frac{x_1}{2D}\right).$$