WP-B1) I have two electric fields

$$\boldsymbol{U}_1(z,t) = E_1\hat{\boldsymbol{a}}_2 \exp(j\varphi_1)\exp\left[j(kz-\omega t)\right]$$

and

$$\boldsymbol{U}_2(z,t) = E_2\hat{\boldsymbol{a}}_2 \exp(j\varphi_2)\exp\left[j(kz-\omega t)\right],$$

where $\hat{\boldsymbol{a}}_1$ and $\hat{\boldsymbol{a}}_2$ are unit vectors in the direction of oscillation of the electric field. Assume that the fields are linearly polarized, where $\hat{\boldsymbol{a}}_1$ and $\hat{\boldsymbol{a}}_2$ do not contain any phase terms. By breaking the electric field into components in the x and y directions, show that the time average irradiance is proportional to

$$I \propto E_1^2 + E_2^2 + 2E_1E_2\left(\hat{\boldsymbol{a}}_1 \bullet \hat{\boldsymbol{a}}_2\right)\cos(\varphi_2 - \varphi_1).$$

WP-B2) Show that the tangential component of the electric field intensity $\boldsymbol{E}$ is continuous across an interface.

WP-B3) Show that the nodes and antinodes of the magnetic field component of an electromagnetic standing wave alternate with those of the electric field.

WP-B4) Show that the polarization angles for internal and external reflection at an interface are complementary.

WP-B5) Prove using Jones calculus that a retarder at 45^o between vertical and horizontal quarter-wave plates is equivalent to a rotator. (Orientation of sandwich not important.)

WP-B6) Use Jones calculus to show that a half-wave plate converts right handed circularly polarized light into left handed circularly polarized light and the phase of the light can be changed by rotating the half-wave plate.

WP-B7) Derive a Mueller matrix for a half-wave plate having a vertical fast axis. Utilize your result to convert a right-handed state into a left-handed state. Verify that the same wave plate will convert a left-handed state to a right-handed state. Advancing or retarding the relative phase by π should have the same effect. Check this by deriving the matrix for a horizontal fast axis half-wave plate.

WP-B8)

a) Assume I want to rotate by 90^o the polarization of a linearly polarized plane wave using only "perfect" (no loss, other than from the mathematical projection along the transmission axis) linear polarizers. What is the maximum power transmission I can obtain using only two polarizers?

What is the minimum number of perfect polarizers required to have a power transmission greater than 95%?

b) The indices of refraction for quartz for the sodium yellow line are n_o=1.544 and n_e=1.553. Calculate the thickness of a quarter-wave plate made from quartz.

WP-B9) Use Jones calculus to show that two half-wave plates at angle θ between them are equivalent to a rotator through angle 2θ.

WP-B10) Let

$$\begin{aligned} \boldsymbol{U}(z,t) &= U_x \hat{\boldsymbol{x}} + U_y \hat{\boldsymbol{y}} \\ &= \left[A_x \hat{\boldsymbol{x}} + A_y \exp(j\phi) \hat{\boldsymbol{y}} \right] \exp\left[j(kz - \omega t) \right] \end{aligned} \quad ,$$

where

$$\begin{aligned} E_x(z,t) &= \mathrm{Re}(U_x) \\ &= A_x \cos(kz - \omega t) \end{aligned}$$

and

$$\begin{aligned} E_y(z,t) &= \mathrm{Re}(U_y) \\ &= A_y \cos(kz - \omega t + \phi) \end{aligned} \quad .$$

Show that

$$\left[\frac{E_x(z,t)}{A_x} \right]^2 + \left[\frac{E_y(z,t)}{A_y} \right]^2 - 2\left[\frac{E_x(z,t)}{A_x} \right]\left[\frac{E_y(z,t)}{A_y} \right] \cos\phi = \sin^2\phi \quad .$$

WP-B11) Show that, in terms of the Stokes parameters, the degree of polarization can be written as

$$V_P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad .$$

WP-B12) A linearly polarized laser source is used with a Twyman-Green interferometer. A 50-50 non-polarization sensitive beam splitter is used in the interferometer. The flat mirror in the reference arm of the interferometer is tilted slightly to give straight, equally spaced fringes in the output. A quarter-wave plate is placed in the test arm of the interferometer. The fast axis of the quarter-wave plate makes an angle θ with respect to the direction of polarization of the light incident upon the quarter-wave plate. A perfect polarizer is placed in the output of the interferometer. The transmission axis of the polarizer is in the direction of polarization of the light coming from the reference arm.

a) Calculate the irradiance of the interference pattern as a function of θ. What is the fringe visibility as a function of θ?

b) Assume the polarizer is non-perfect in that it transmits 95% of the light having a polarization along the transmission axis of the polarizer and 5% of the light having the orthogonal polarization. What is the observed fringe visibility as a function of θ?

WP-B13)

a) What are the four filters we associate with the four Stokes parameters?

b) Suppose that an ideal polarizer is rotated at a rate ω between a similar pair of stationary crossed polarizers. Give the ratio of the modulation frequency of the emergent power density (irradiance) to the rotation frequency of the polarizer.

c) I am working on a problem involving partially polarized light. Which should I use, Jones or Mueller matrices? Explain (briefly).

d) I am working on a problem using completely polarized light where I want to keep track of the phase of the beam. Should I use Jones or Mueller matrices? Explain (briefly).