Appendix A – Notation, Units, Constants and Useful Relations

A.1 Notation and units

Vectors are specified with ***bold italic*** face type. For example, the complex electric field vector is ***U***. Unit vectors are lower case bold italic and are identified with a carrot, like $\hat{\boldsymbol{s}}$.

The MKS system of units is used throughout this book. Unit vectors for Cartesian coordinates are $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$. $\boldsymbol{r}$ is a position vector with components along (*x*, *y*, *z*) axes. $\hat{\boldsymbol{k}}$ is the unit vector in the direction of propagation of a plane wave.

Direction cosines are used to indicate direction. The direction cosine for each of the Cartesian axes is the cosine of the angle from the propagation vector $\hat{\boldsymbol{k}}$ to the axis, as shown in Figs. A.1 and A.2.

Operators are specified with uppercase **bold** face type and usually include a subscript to indicate the output variable. For example, the Fourier transform operator operating on function *g*(*x*) with transform variable ξ is given by $\mathbf{F}_{\xi}[\, g(x)]$.

Transverse waves are periodic shapes that move in a specified direction. For example, $y(x,t) = A\cos(kx - \omega t)$ is a periodic wave where $k = 2\pi / \lambda$, and λ is the spatial period in m^{-1}. Associated notation is shown in Table A.1

Table A1. Notation

Variable	**Definition**	**Units**
temporal period	$\tau = \dfrac{\lambda}{v}$	S
temporal frequency	$\nu = \dfrac{1}{\tau}$	s^{-1}
wave number	$\kappa = \dfrac{1}{\lambda}$	m^{-1}
Phase	$\psi = kx - \omega t$	radians (rad)
angular frequency	$\omega = \left\lvert \left(\dfrac{\partial \psi}{\partial t} \right)_x \right\rvert = \dfrac{2\pi}{\tau}$	rad s^{-1}
propagation number	$k = \left\lvert \left(\dfrac{\partial \psi}{\partial x} \right)_t \right\rvert = \dfrac{2\pi}{\lambda}$	rad m^{-1}
initial phase φ	$y(x,t) = A\cos\left(kx - \omega t + \varphi\right)$	rad
phase velocity	$v = \left(\dfrac{\partial x}{\partial t} \right)_{\psi} = \dfrac{\omega}{k}$	m s^{-1}

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group velocity	$v_g = \frac{\partial \omega}{\partial k}$	m s^{-1}

Other variables and symbols used in his book include those listed in Table A.2. Handy conversions are listed in Table A.3. Values of constants are shown in Table A.4.

Table A.2. Other variables and symbols

Variable	**Definition**	**Units**
E (real-valued field) *u* or *U*(complex-valued field)	electric field amplitude	V m^{-1}
I	irradiance	W m^{-2}
H	magnetic intensity	A m^{-1}
m	Mass	kg
n	real part of the refractive index	--
κ	imaginary part of the refractive index	--
$N = n + \kappa j$	complex refractive index	
α	linear absorption coefficient	m^{-1}
ε_r	relative dielectric constant	--
t	time	S
x, *y*, *z*	spatial position	m
α, β, γ	direction cosines	--
ξ, η	spatial frequencies	m^{-1}
P	macroscopic electronic polarization	C m^{-2}
γ	molecular concentration	m^{-3}
ζ	molecular polarizability	C m^2 V^{-1}
χ	dielectric susceptability	--
V	fringe visibility	--
V_p	degree of polarization	--
N_f	Fresnel number	--
OPL	optical path length	m
OPD	optical path difference	m
NA	numerical aperture	--
j or *i*	$\sqrt{-1}$	--
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	m^{-2}

Table A.3. Handy Conversions

1 W	1 J s^{-1}
1 km	10^3 m
1 m	39.37 in
1 m	10^3 mm
1 m	10^6 μm
1 m	10^9 nm
1 in	25.4 mm
1 eV	1.602×10^{-19} J
1 J	10^7 ergs

1 lb	4.448 N
1 V	1 J C^{-1}
1 N	1 kg m s^{-1}
1 cal	4.184 J
1 rad	57.30^{o}
1 ft s^{-1}	0.3048 m s^{-1}
1 rpm	0.1047 rad s^{-1}
1 mile	1.609 km

Table A.4. Values of Constants

Constant	**Value**	**Units**
c, speed of light in vacuum	$3\text{x}10^{8}$	m s^{-1}
ε_0 , permittivity of free space	$8.85\text{x}10^{-12}$	C^2 N^{-1} m^{-2}
μ_0 , permeability of free space	$4\pi\text{x}10^{-7}$	N m^2 C^{-2}
e^-, electronic charge	$1.602\text{x}10^{-19}$	C
h, Plank's constant	$6.626\text{x}10^{-34}$	J s
k, Boltzman's constant	$1.381\text{x}10^{-23}$	J K^{-1}
N_A, Avogadro's number	$6.022\text{x}10^{23}$	particles mole^{-1}
m_e, electron mass	$9.1095\text{x}10^{-31}$	kg
m_p, proton mass	$1.673\text{x}10^{-27}$	kg
m_n, neutron mass	$1.675\text{x}10^{-27}$	kg

A.2 Complex notation

For much of this book, complex variables are used to mathematically describe wave optics phenomena. A complex number has the form $z = a + bj$, where $j = \sqrt{-1}$. The complex number is often written as

$$z = A\exp\left(j\phi\right) \quad ,$$

where

$$A = \left|z\right| = \sqrt{a^2 + b^2}$$

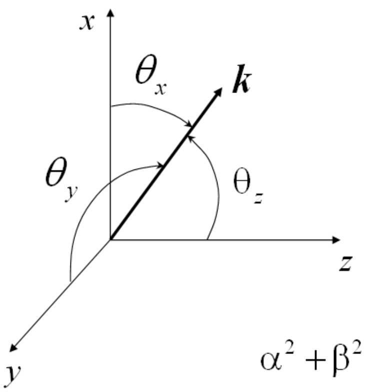
is the magnitude and

$$\phi = \angle z = \tan^{-1}\frac{b}{a}$$

is the phase. E is the real value of the electric field, and U is the complex electric field. Otherwise, no special indication is given to indicate whether a parameter is a real or complex quantity, unless Re{} or Im{} is used in specific cases.

The complex exponential can be expanded using Euler's identity

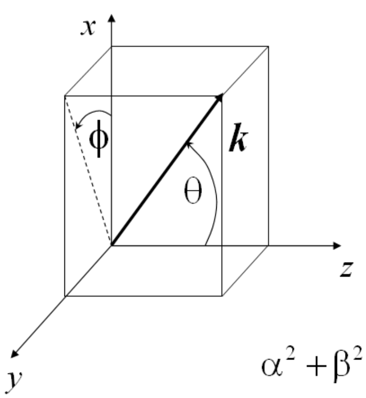
$$\exp\left(j\phi\right) = \cos\phi + j\sin\phi \quad .$$



$$\alpha = \cos\theta_x$$
$$\beta = \cos\theta_y$$
$$\gamma = \cos\theta_z$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$
$$\hat{k} = \alpha\hat{x} + \beta\hat{y} + \gamma\hat{z}$$
$$\boldsymbol{k} = |\boldsymbol{k}|\hat{k} = k\,\hat{k}$$

Figure A.1 Direction cosines using axis angles



$$\alpha = \cos\phi\sin\theta$$
$$\beta = \sin\phi\sin\theta$$
$$\gamma = \cos\theta$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$
$$\hat{k} = \alpha\hat{x} + \beta\hat{y} + \gamma\hat{z}$$
$$\boldsymbol{k} = |\boldsymbol{k}|\hat{k} = k\,\hat{k}$$

Figure A.2 Direction cosines using spherical angles

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