3.6.3.2 Stokes Parameters

Ref: Optics by Hecht and Polarized Light by Shurcliff

Let us have 4 filters, each of which transmits 50% of natural radiation

#1 isotropic filter, passing all states equally, transmits I_o

#2 linear polarizer, transmission axes are horizontal, transmits I_1

#3 linear polarizer, at 45°, transmits I_2

#4 circular polarizer, opaque to L-state, I_3

Define Stokes parameters

$S_o = 2\, I_o$

$S_1 = 2\, I_1 - 2\, I_o$

$S_2 = 2\, I_2 - 2\, I_o$

$S_3 = 2\, I_3 - 2\, I_o$

S_o is the incident irradiance.

The other 3 Stokes parameters specify the state of polarization

- ***S_1***

If $S_1 > 0$, resembles horizontal polarization

If $S_1 < 0$, resembles vertical polarization

If $S_1 = 0$, beam shows no preferential orientation
(i.e. elliptical at ± 45 °, circular, unpolarized)

- ***S*$_2$**

  If $S_2 > 0$, resembles polarization at $+45°$

  If $S_2 < 0$, resembles polarization at $-45°$

  If $S_2 = 0$, neither

- ***S*$_3$**

  If $S_3 > 0$, tendency toward right handed

  If $S_3 < 0$, tendency toward left handed

Calculating Stokes parameters

- **Electric field**

  $\vec{E}_x = A_x \operatorname{Cos}[k z - \omega t] \hat{\imath}$

  $\vec{E}_y = A_y \operatorname{Cos}[k z - \omega t + \phi] \hat{\jmath}$

  $\vec{E}_z = 0$

- **Stokes Parameters**

  $S_o = \langle A_x^2 \rangle + \langle A_y^2 \rangle$

  $S_1 = \langle 2 A_x^2 \rangle - \langle A_x^2 + A_y^2 \rangle = \langle A_x^2 \rangle - \langle A_y^2 \rangle$

  $S_2 = \left\langle 2 \left| \frac{1}{\sqrt{2}} (A_x + A_y e^{i \phi}) \right|^2 \right\rangle - \langle A_x^2 + A_y^2 \rangle = \langle 2 A_x A_y \operatorname{Cos}[\phi] \rangle$

  S_3 = We must calculate using Jones Calculus

  Multiplying the incident radiation times a right circular polarizer yields

  $$\frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} . \begin{pmatrix} \langle A_x \rangle \\ \langle A_y e^{i \phi} \rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{2} (\langle A_x \rangle + i \langle e^{i \phi} A_y \rangle) \\ \frac{1}{2} (-i \langle A_x \rangle + \langle e^{i \phi} A_y \rangle) \end{pmatrix}$$

  $$\text{intensity} = \left\langle \frac{1}{4} 2 \left(A_x^2 + A_y^2 + 2 A_x A_y \operatorname{Cos}\left[\phi + \frac{\pi}{2}\right] \right) \right\rangle = \left\langle \frac{1}{2} (A_x^2 + A_y^2 - 2 A_x A_y \operatorname{Sin}[\phi]) \right\rangle$$

  $S_3 = \langle -2 A_x A_y \operatorname{Sin}[\phi] \rangle$

  Note : If $\phi = -\frac{\pi}{2}$ we have right handed circular.

  Often normalize Stokes parameters by dividing by S_o.

Degree of Polarization

If beam is unpolarized $S_o = \langle A_x^2 \rangle + \langle A_y^2 \rangle$ and $S_1 = S_2 = S_3 = 0$

For completely polarized light $S_o^2 = S_1^2 + S_2^2 + S_3^2$

It can be shown that the degree of polarization, V, is given by

$$V = \frac{I_p}{I_p + I_u} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_o}$$

Addition of Stokes parameters

For incoherent beams the Stokes parameters add

$$S_o''' = S_o' + S_o''$$
$$S_1''' = S_1' + S_1''$$
$$S_2''' = S_2' + S_2''$$
$$S_3''' = S_3' + S_3''$$

Stokes Vectors and Mueller Matrices

Stokes Vectors

Unpolarized Light

$$\texttt{upStokes} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

Linear Horizontal

$$\texttt{lhStokes} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix};$$

Linear Vertical

$$\texttt{lvStokes} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix};$$

Linear at +45 degrees

$$\texttt{lp45Stokes} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix};$$

Linear at -45 degrees

$$\texttt{lm45Stokes} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix};$$

Right Circular

$$\text{rcStokes} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

Left Circular

$$\text{lcStokes} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix};$$

Mueller Matrices

Horizontal linear polarizer

$$\text{hlpMueller} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

Vertical linear polarizer

$$\text{vlpMueller} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

Linear polarizer at + 45 degrees

$$\text{lpp45Mueller} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

Linear polarizer at - 45 degrees

$$\text{lpm45Mueller} = \frac{1}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

Quarter-wave plate with fast axis vertical

$$\textbf{qfavMueller} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix};$$

Quarter-wave plate with fast axis horizontal

$$\textbf{qfahMueller} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

Rotation Matrix

$$\textbf{rotMueller}[\theta\_] := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Cos}[2\theta] & \text{Sin}[2\theta] & 0 \\ 0 & -\text{Sin}[2\theta] & \text{Cos}[2\theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

RotatedMatrix = rotMueller[-θ] *R*[0 °] rotMueller[θ];

OutputPolarization = rotMueller[-θ] *R*[0 °] rotMueller[θ] InputPolarization;