5.3.5 Edge Diffraction

There are several methods to solve the problem of diffraction from a straight edge. In fact, this problem is one of the most studied in diffraction theory. Section 5.3.5 describes two methods, one of which is an application of the geometrical procedure listed in Section 5.3.2.2 for calculating the irradiance behind an aperture. The second method uses Fresnel's Equations to evaluate the Fresnel diffraction integral of Eq. (5.38).

5.3.5.1 Geometrical procedure for calculating edge diffraction based on OPD

Application of the procedure listed in Section 5.3.2.2 involves dividing an aperture into Fresnel zones as a function of OPD between the source point and the observation point. With respect to a single, infinite straight edge, the zone map also becomes infinite in extent. In practice, a limited range is sufficient to demonstrate the concept and obtain reasonably accurate results.

Figure 5.53 shows progression of the geometrical-zone concept for a straight edge at four *y* positions of the observation point. Estimates of observation-point irradiance are calculated by normalizing the difference in odd and even zone areas and then squaring the result. Insignificant oscillation is observed in the boundary of the shadow, where odd and even zone areas are approximately equal, as shown in Figure 5.53(a). Like with the circular aperture, the largest oscillation in the diffracted field is observed when the central zone just passes the edge, as shown in Figure 5.53(c). Subsequent zone crossings also produce peaks in the oscillations, but at reduced magnitude.

A graph of the observation-point irradiance calculated from the model shown in Figure 5.53 is shown in Figure 5.54. Model parameters include: illumination with a collimated plane wave, 29.6 mm axial distance from the edge and λ = 670 nm. For different observations distances and laser wavelengths, the magnitude and width of the oscillations change, but the general characteristic of the diffraction pattern remains the same. Similar patterns are observed near edges in more complicated apertures at high Fresnel number, as shown in Section 5.3.6.

5.3.5.2 Fresnel Integrals

Consider a rectangular aperture in an otherwise opaque screen illuminated with a unit amplitude plane wave. Width of the aperture in the *x* direction is *a*, and width in the *y* direction is *b*. Assume that the observation distance z_0 is far enough to be in the Fresnel region. Application of Eq. (5.38) gives:

$$\begin{aligned} U_0(x_0, y_0) &= -\frac{je^{jkz_0}}{\lambda z_0} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \exp\left\{ \frac{jk}{2z_0} \left[(x_0 - x_S)^2 + (y_0 - y_S)^2 \right] \right\} dx_S dy_S \\ &= -\frac{je^{jkz_0}}{\lambda z_0} \int_{-a/2}^{a/2} \exp\left[\frac{jk}{2z_0} (x_0 - x_S)^2 \right] dx_S \int_{-b/2}^{b/2} \exp\left[\frac{jk}{2z_0} (y_0 - y_S)^2 \right] dy_S \quad . \end{aligned} \tag{5.123}$$

Let

$$Ae^{j\psi} = \sqrt{\frac{\lambda z_0}{2}} \int_{u_1}^{u_2} e^{j\frac{\pi}{2}t^2}\, dt \qquad , \tag{5.124}$$

where

$$t = \sqrt{\frac{2}{\lambda z_0}}\left(x_S - x_0\right) \qquad , \tag{5.125}$$

$$u_1 = \sqrt{\frac{2}{\lambda z_0}}\left(-\frac{a}{2} - x_0\right) \qquad , \tag{5.126}$$

and

$$u_2 = \sqrt{\frac{2}{\lambda z_0}}\left(\frac{a}{2} - x_0\right) \qquad , \tag{5.127}$$

so that

$$U_0\left(x_0, y_0\right) = -\frac{j}{2} e^{jkz_0} Ae^{j\psi} Be^{j\chi} \qquad . \tag{5.128}$$

Now define the *Fresnel Integrals* $C(\alpha)$ and $S(\alpha)$ such that

$$\begin{aligned} \int_0^{\alpha} e^{j\frac{\pi}{2}t^2}\, dt &= \int_0^{\alpha} \cos\frac{\pi}{2}t^2 dt + j\int_0^{\alpha} \sin\frac{\pi}{2}t^2 dt \\ &= C\left(\alpha\right) + jS\left(\alpha\right) \qquad . \end{aligned} \tag{5.129}$$

Now, Eq. (5.123) can be rewritten as

$$U_0\left(x_0, y_0\right) = -\frac{j}{2} e^{jkz_0} \left[C\left(u_2\right) + jS\left(u_2\right) - C\left(u_1\right) - jS\left(u_1\right)\right]\left[C\left(v_2\right) + jS\left(v_2\right) - C\left(v_1\right) - jS\left(v_1\right)\right] \ , \tag{5.130}$$

where v parameters are associated with y limits of the aperture. Equation (%.130) can be evaluated easily if values of the Fresnel integrals are known. Although there is some difficulty in evaluating the Fresnel integrals, many good public-domain programs are available for this purpose.

Historically, Fresnel integrals were calculated and graphed in the form of *Cornu's Spiral*, as shown in Fig. 5.55. The axis of the plot are the values of the integrals $C(\alpha)$ and

$S(\alpha)$. Once the parameter α is determined from the aperture geometry, location of the appropriate point along the curve is fixed. Values of $C(\alpha)$ and $S(\alpha)$ are found by simply reading the axial values associated with the point along the curve.

Example 5.8: Diffraction from a straight edge using Cornu's Spiral

Consider a straight edge in the xy half plane blocked below $y = 0$ that is illuminated by an on-axis plane wave. Assume that the observation distance z_o is far enough to be in the Fresnel region. An infinite edge in this half plane implies $a \to \infty$, $u_1 \to -\infty$ and $u_2 \to \infty$ in the expressions derived for the rectangular aperture. In the y direction, $b \to \infty$ in the $+y$ direction, so $v_2 \to \infty$. The only remaining variable is associated with v_1. By reading values from the Cornu Spiral, Eq. (5.130) is reduced to

$$U_0(x_0, y_0) = -\frac{j}{2} e^{jkz_0} [1 + j] \left[\frac{1}{2}(1 + j) - C(v_1) - jS(v_1) \right] . \tag{5.131}$$

Notice that, as y_0 increases inside the geometrical shadow (v_1 goes more negative), the value of Eq. (5.131) oscillates from the spiral. The oscillation agrees reasonably well with the geometrical diffraction effects predicted in Fig. 5.54.

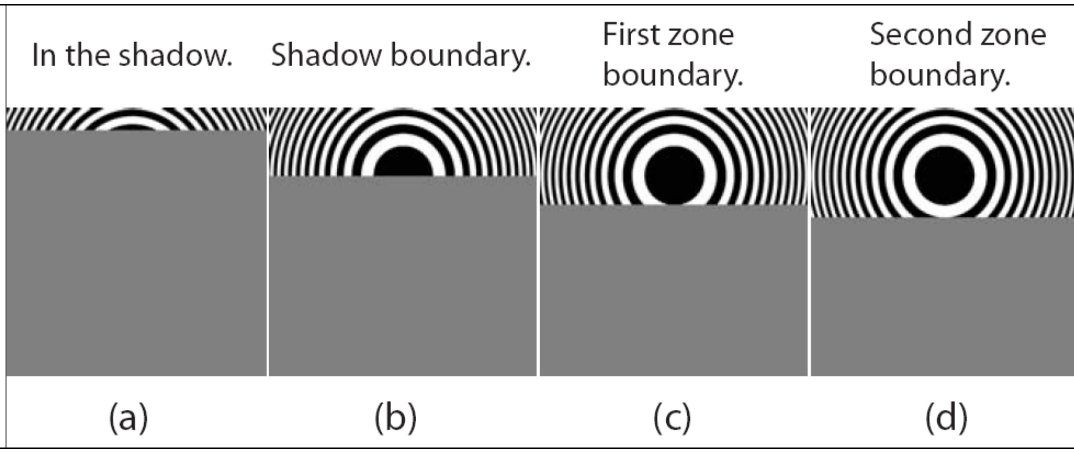

Figure 5.53. Geometrical OPD construction model of diffraction from a straight edge.

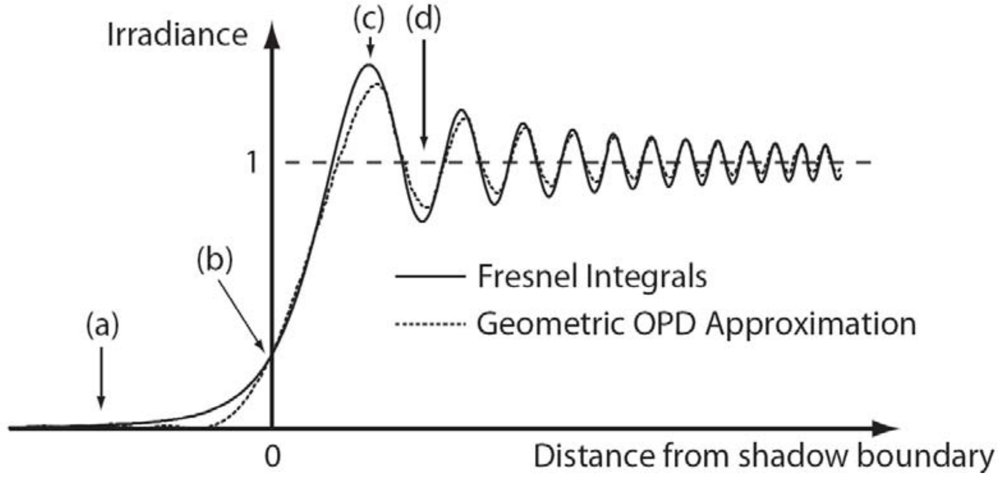

Figure 5.54. Edge diffraction calculation by two different calculations. (a) through (d) refer to the Geometrical OPD Model shown in Figure 5.53.

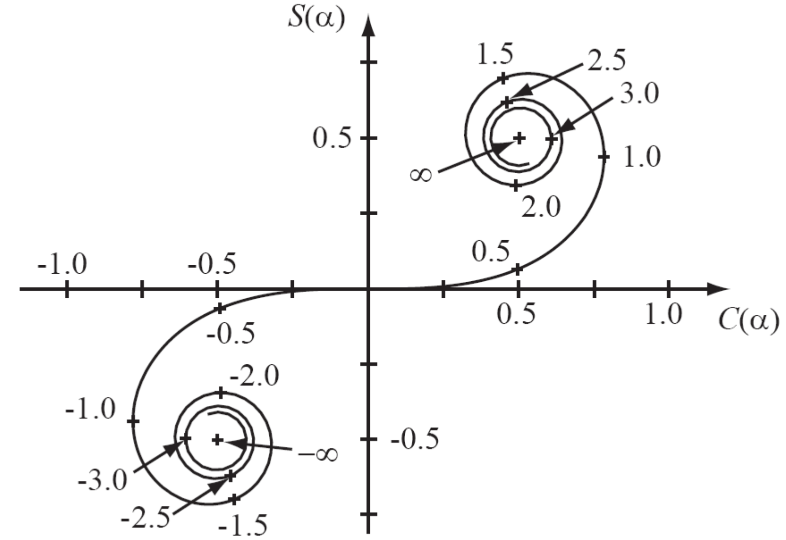

Figure 5.55. Cornu's Spiral, which graphically displays solutions to Fresnel's equations.