$\boxed{\text{KEY}}$

1.)

a.) (5 pts.) Determine the refractive index and thickness of a thin film to be deposited on a glass surface with $n = 1.5$ such that no normally incident light of wavelength 600 nm is reflected.

$$n_{film} = \sqrt{1.5} = \boxed{1.22}$$

$$t = \frac{\lambda}{4 n_{film}} = \frac{600}{4 \cdot 1.22} = \boxed{123\ nm}$$

b.) (5 pts.) If a white-light source is used to illuminate the surface, what color is observed in transmission after the film is deposited? Choose between red, green and blue. Justify your answer.

$\boxed{\text{Red}}$ portion of the spectrum is transmitted. Eye is not as sensitive to blue.

c.) (5 pts.) If the light illuminating the surface ~~id~~ s incident at 45 degrees, what is the wavelength at which ~~peak~~ minimum reflection occurs?

$$\sin\theta_2 = \frac{1}{1.225}\sin 45^\circ$$

$$\theta_2 = 35.26^\circ$$

$$2nt\cos 35.26^\circ = m\lambda_2 \qquad m = \frac{1}{2}$$

$$\lambda_2 = \cancel{4n}\frac{\lambda_1}{\cancel{4n}}\cos 35.26^\circ = 600 \cdot \cos 35.26^\circ$$

$$= \boxed{489.9\ nm}$$

(Part 1c has a typo in it. It should ask for the wavelength of minimum reflection. If students attempt to find the peak wavelength in a correct way, give them full credit.)

2.) Consider diffraction by a plane screen.

a.) (3 pts.) Write the Green's function G associated with each boundary condition named below.

i.) Kirchhoff

$$\frac{e^{jkr_{01}}}{r_{01}}$$

ii.) Neumann

$$\frac{e^{jkr_{01}}}{r_{01}} + \frac{e^{jkr_{01}'}}{r_{01}'}$$

iii.) Dirichlet

$$\frac{e^{jkr_{01}}}{r_{01}} - \frac{e^{jkr_{01}'}}{r_{01}'}$$

b.) (3 pts.) In order to find the diffraction at an observation point to the right of the screen, what conditions must be placed on U and $\frac{\partial U}{\partial n}$ in the open part of the aperture for each G in (a)?

i) u and $\frac{\partial u}{\partial n}$ same as w/o screen

ii) $\frac{\partial u}{\partial n}$ same as w/o screen

iii) u same as w/o screen

c.) (3 pts.) In order to find the diffraction at an observation point to the right of the screen, what conditions must be placed on U and $\frac{\partial U}{\partial n}$ in the shadow of the screen for each G in (a)?

i) $u=0$ & $\frac{\partial u}{\partial n}=0$

ii) $\frac{\partial u}{\partial n}=0$

iii) $u=0$

3.) A grating with amplitude transmittance $t(x) = 1 + \cos\left(\frac{2\pi}{\lambda}\alpha_o x\right)$ is illuminated from the left by an on-axis monochromatic plane wave with $\lambda = 0.5$ μm.

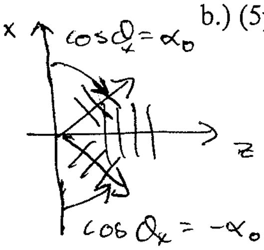
a.) (5 pts.) Find the angular spectrum $A_o(\alpha, \beta)$ of the electric field immediately behind the grating.

$$A_o(\alpha,\beta) = \lambda^2 \mathcal{F}\{U(u,v)\}_{\alpha,\beta} \qquad u = \frac{x}{\lambda} \quad v = \frac{y}{\lambda}$$

$$= \lambda^2 \mathcal{F}\{1 + \cos(\tfrac{2\pi}{\cancel{\lambda}} \alpha_o \cancel{\lambda} u)\}_{\alpha,\beta}$$

$$= \lambda^2 \left[\delta(\alpha,\beta) + \tfrac{1}{2}\{\delta(\alpha-\alpha_o) + \delta(\alpha+\alpha_o)\}\delta(\beta)\right]$$

b.) (5pts.) Give a physical interpretation of the result you found in (a).



Three plane waves, One on-axis, Two symmetrically propagating plane waves with direction cosines $\pm\alpha_o$.

c.) (5 pts.) If $\alpha_o = 0.01$, what is the electric field distribution a distance z = 5 mm from the grating?

$$H(\gamma; z) = e^{jk\gamma z} \approx e^{jkz} e^{-j\frac{kz}{2}\sigma^2}$$

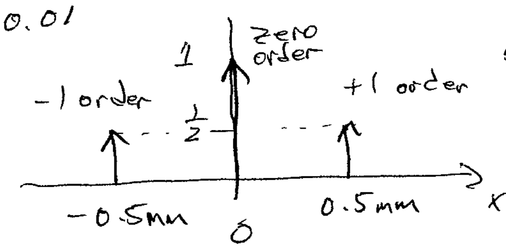
$$A(\alpha,\beta; z) = \lambda^2 e^{jkz}\left[\delta(\alpha,\beta) + \tfrac{1}{2} e^{-j\pi}\{\delta(\alpha+\alpha_o) + \delta(\alpha+\alpha_o)\}\delta(\beta)\right]$$

$$= \lambda^2 e^{jkz}\left[\delta(\alpha,\beta) - \tfrac{1}{2}\{\delta(\alpha-\alpha_o) + \delta(\alpha+\alpha_o)\}\delta(\beta)\right]$$

$$\Rightarrow U(x_o, y_o) = e^{jkz}\left[1 - \cos(\tfrac{2\pi}{\lambda}\alpha_o x)\right]$$

d.) (5 pts.) A simple lens is placed a distance 10 mm to the right of the grating. Assume that the lens is perfect and is infinitely large. The focal length of the lens is 50 mm. What is the relative distribution of the irradiance in the focal plane of the lens?

for $\alpha_o = 0.01$



$\Delta x = f\alpha_o$

$= 0.5$ mm

e.) (5 pts.) How does the irradiance pattern in part (d) change if the grating frequency $\frac{\alpha_o}{\lambda}$ increases?

The two orders separate away from the zero order.

4.) A 1 mm diameter hole is illuminated with a λ = 600 nm normally incident plane wave.

a.) (10 pts.) Sketch the on-axis irradiance from z= 0.1 m to z = 0.25 m, indicating the positions of odd and even Fresnel numbers.

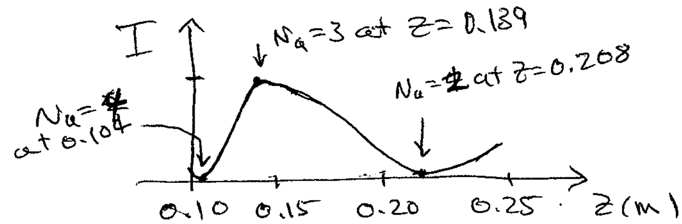
$$z = 0.1\ m \Rightarrow N_a = \frac{(0.5\times10^{-3})^2}{(0.6\times10^{-6})(0.1)} = \frac{0.25}{0.6}\cdot 10 = 4.16$$

$$z = 0.25\ m \Rightarrow N_a = \frac{(0.5\times10^{-3})^2}{(0.6\times10^{-6})(0.25)} = \frac{0.25}{0.6}\cdot 4 = 1.67$$

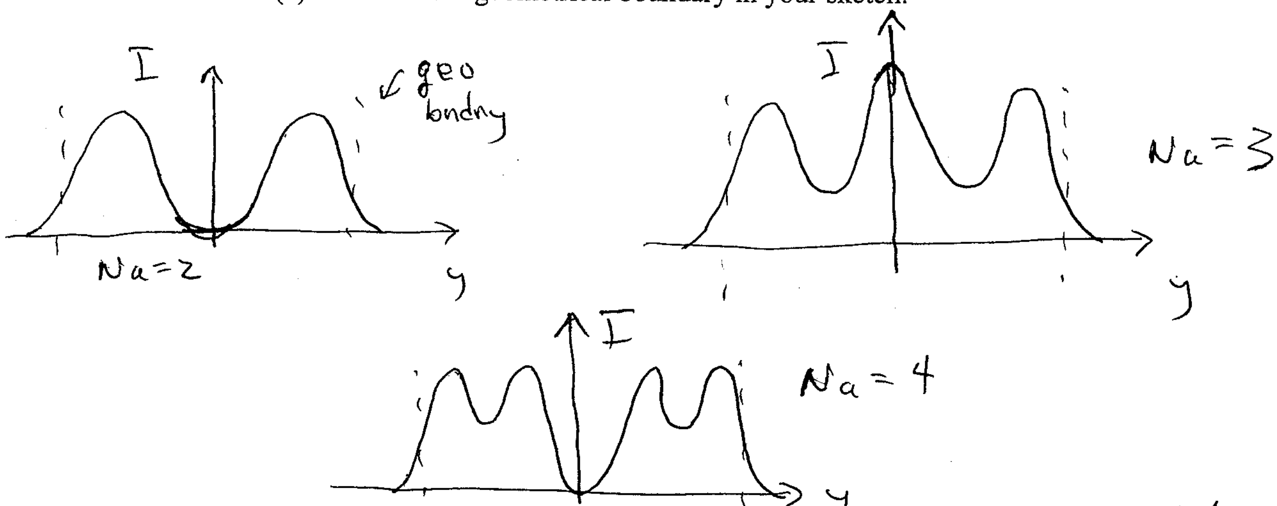
$$N_a = 2 \Rightarrow z = \frac{(0.5\times10^{-3})^2}{(0.6\times10^{-6})(2)} = \frac{0.25}{0.6}\cdot\frac{1}{2} = 0.208$$

$$N_a = 3 \Rightarrow z = \frac{0.25}{0.6}\cdot\frac{1}{3} = 0.139$$

$$N_a = 4 \Rightarrow z = \frac{0.25}{0.6}\cdot\frac{1}{4} = 0.104$$



b.) (5 pts.) Sketch the transverse irradiance profiles at each integer Fresnel number in part (a). Indicate the geometrical boundary in your sketch.



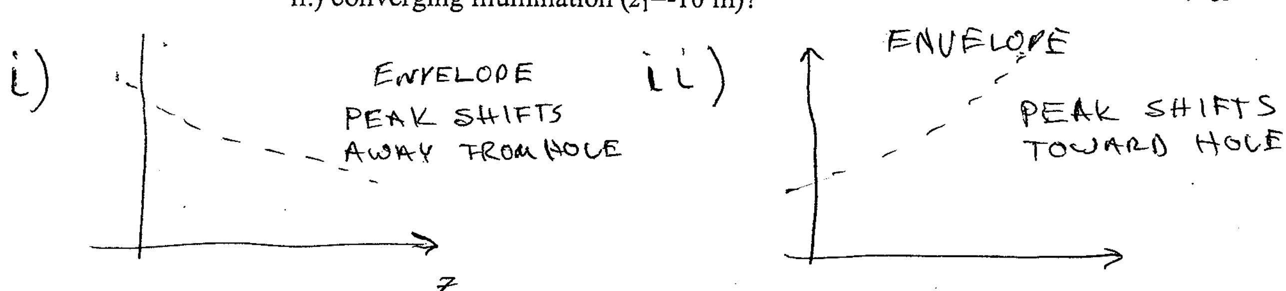
c.) (6 pts.) How would the the on-axis irradiance of part (a) change for the following cases. Draw a rough sketch indicating the envelope of the modulation. Also indicate whether the peak locations shift toward the hole or away from it.

i.) diverging illumination (z_1 = 10 m)?

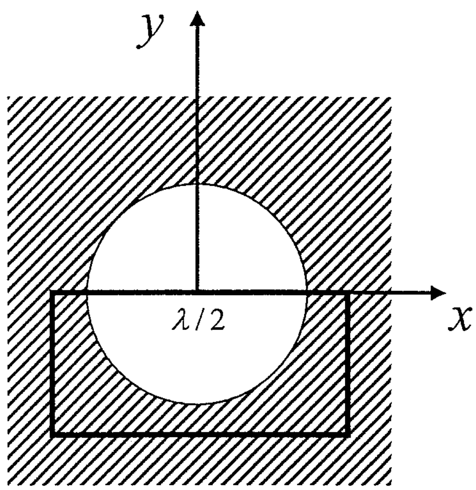
ii.) converging illumination (z_1=-10 m)?

$$L \simeq z_2\left(1 - \frac{1}{z_1}\right)$$

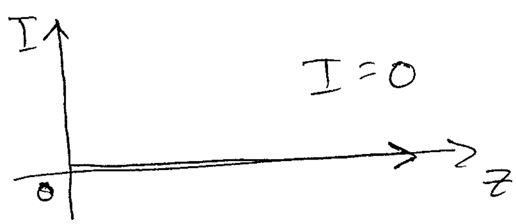
$$z_2 \simeq \frac{a^2}{N_a\lambda}\left(1 + \frac{1}{z_1}\right)$$



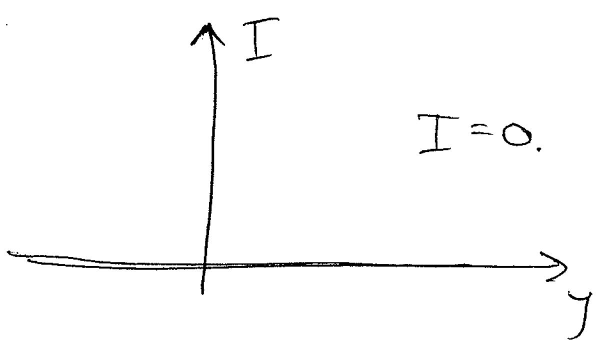
5.) Consider the 1 mm diameter hole of Problem (4), where a $\lambda/2$ plate is placed so that it covers exactly half of the open area, as shown below.



a.) (10 pts.) Sketch the on-axis irradiance from $z = 0.1$ m to $z = 0.25$ m.



b.) (5 pts.) Draw a rough sketch of the transverse irradiance profile along $y = 0$ at $z = 0.1$ m.



6.) A collimated 1 W/cm^2 laser beam of wavelength 632.8 nm illuminates a Fresnel zone plate.

a.) (5 pts.) The primary focus occurs at z = 100 mm. What is the radius of the first open area in the zone plate?

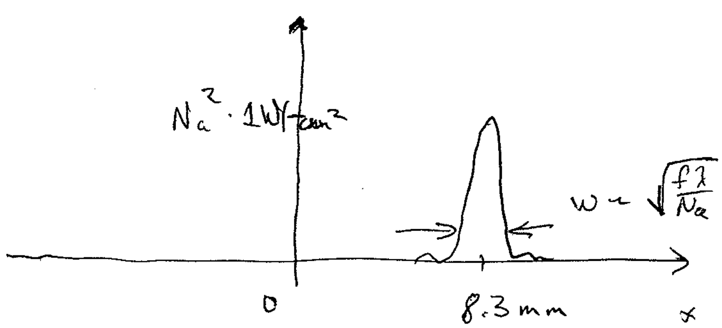
$$\rho_1 = \sqrt{\lambda f_1} = \boxed{0.252 \text{ mm}}$$

b.) (5 pts.) If the zone plate is heated, it uniformly expands so that the open and closed areas of the plate increase their diameter. If the plate is heated so that the diameter of the first area increases by 1% , where is the new position of the primary focus?

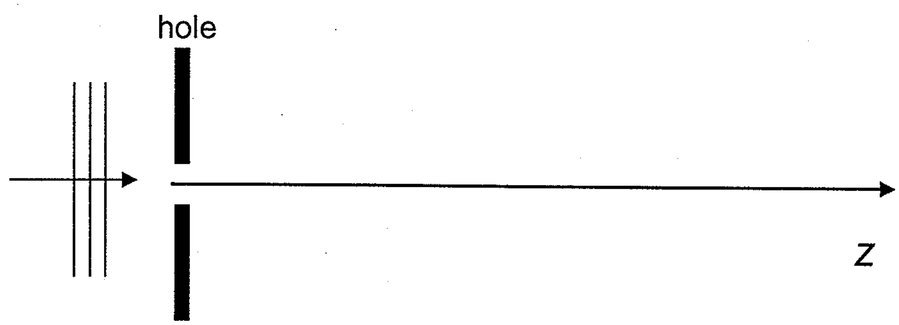
$$f_1' = \frac{a^2 (1.01)^2}{\lambda} = f_1 (1.01)^2 = \boxed{102 \text{ mm}}$$

c.) (5 pts.) If the x-angle of the illuminating plane wave is changed to 5 degrees with respect to the z axis in part (a), sketch the transverse irradiance at z = 100 mm along the x axis.

$$\Delta x \approx \frac{5}{60} \cdot 100 = 8.3 \text{ mm}$$



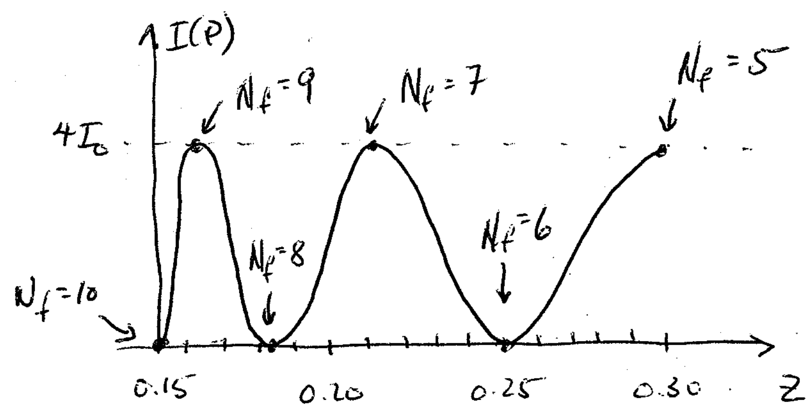
1.) A round hole of diameter 2 mm is illuminated with an on-axis $\lambda = 0.667$ µm plane wave as shown below.



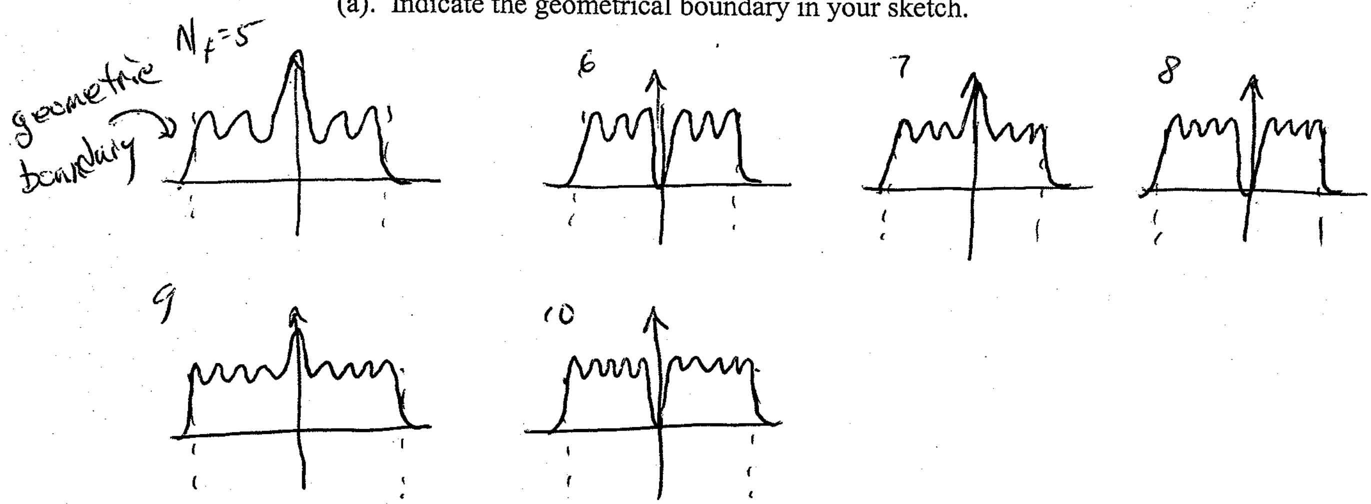
$z_1 \to \infty$

a.) (10 pts.) Sketch the on-axis irradiance from $z = 0.15$ m to $z = 0.3$ m, indicating the positions of odd and even Fresnel numbers.

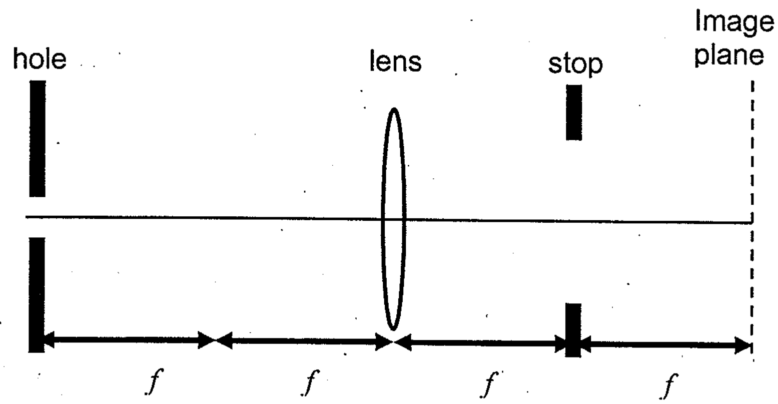
N_f	z_2
5	0.2999
6	0.2499
7	0.2142
8	0.1824
9	0.1666
10	0.1499



b.) (5 pts.) Sketch the transverse irradiance profiles at each integer Fresnel number in part (a). Indicate the geometrical boundary in your sketch.



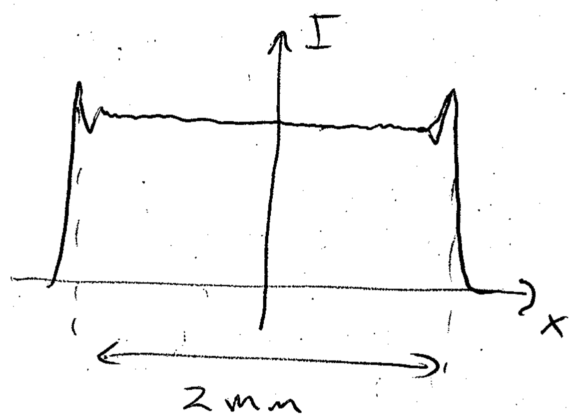
2.) A lens is positioned in back of the hole of Problem (1) as shown below. The focal length of the ideal thin lens is 15 mm. Other parameters are the same as Problem (1).



(The drawing is not to scale)

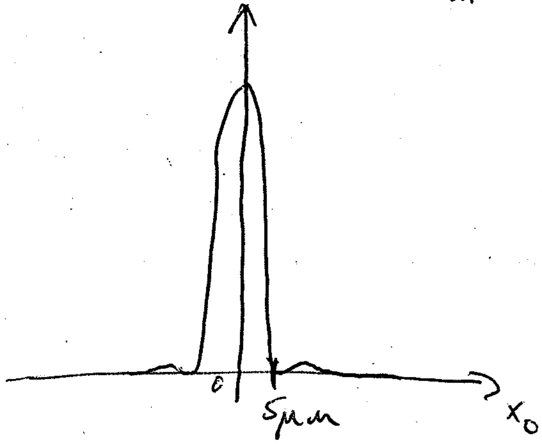
a.) (5 pts) Sketch the irradiance profile at the lens.

$$N_f = \frac{a^2}{\lambda z} = 50$$



b.) (5 pts) Sketch the irradiance profile in the back focal plane at distance f behind the lens in the x direction if the lens diameter is much greater than the irradiance boundary in part (a). Label the width of any primary features and their positions. (Hint: Treat this problem like a converging wave incident on an aperture, where the width of the irradiance in part (a) defines the aperture.)

$$I \sim \left| \mathcal{F}\left\{ \text{rect}\left(\frac{x}{d}\right)\right\}_{\frac{x_0}{\lambda f}} \right|^2 \sim \text{sinc}^2\left(\frac{d\, x_0}{\lambda f}\right)$$



distance to first zero

$$x_0 = \frac{\lambda f}{d} = \frac{0.667\times10^{-6} \cdot 15\times10^{-3}}{2\times10^{-3}}$$

$$= 5\times10^{-6}$$

3.) The hole of problem (2) is filled with a simple cosine grating that has an amplitude transmission function $t(x) = 1 + \cos\left(2\pi \frac{\alpha_0}{\lambda} x\right)$. $\frac{\alpha_0}{\lambda} = \nu_{x0} = \frac{1}{5} \times 10^6 m^{-1}$. That is, the grating period is 5 μm. Other parameters are identical to those for problem (2).

a.) (5 pts) Find an expression for the angular spectrum in the *x* direction of the transmitted light immediately behind the grating. You simplify the problem by assuming a one-dimensional geometry with the transmission of the aperture represented by a rect function.

$$\nu_{x0} = \frac{\alpha_0}{\lambda}$$

$$A_{z=0}(\nu_x) = \mathcal{F}\left\{ rect\left(\frac{x}{d}\right)\left[1 + \cos\left(2\pi \frac{\alpha_0}{\lambda} x\right)\right]\right\}_{\nu_x}$$

$$= d\,\text{sinc}(d\nu_x) * \left[\delta(\nu_x) + \frac{1}{2}\delta(\nu_x - \nu_{x0}) + \frac{1}{2}\delta(\nu_x + \nu_{x0})\right]$$

b.) (5 pts) Find an expression for the angular spectrum in the x direction of the light incident onto the lens.

$$A(\nu_x, z) = A_{z=0}(\nu_x)\, e^{jk\gamma z}$$

$$= d\, e^{jk\gamma z}\left[\text{sinc}(d\nu_x) * \left[\delta(\nu_x) + \frac{1}{2}\delta(\nu_x - \nu_{x0}) + \frac{1}{2}\delta(\nu_x + \nu_{x0})\right]\right]$$

c.) (5 pts) Accurately graph the amplitude and phase of (b) in the regions of non-zero amplitude. There should be three such regions (-1 order, zero order and +1 order). Limit the range of ν_x to +/- 4000 m^{-1} around each maximum amplitude. Notice that the phases around the +/- 1 orders are well represented by straight lines.

(Use an additional page)

d.) (5 pts) Find the coefficients for the approximation of the phase of $H(\gamma; z) = e^{jkz\sqrt{1-(\lambda\nu_x)^2}}$ as expressed below around $\nu_x = \nu_{x0} = 2\text{x}10^5\ m^{-1}$ with $z = 30$ mm and $\lambda = 0.667$ μm.(Hint: Remember that the Taylor series approximation of function *f*(*x*) around $x = a$ is given by

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots)$$

$$kz\sqrt{1-(\lambda\nu_x)^2} \approx 2\pi z\left[c_0 \mp c_1(\nu_x \pm \nu_{x0}) + c_2(\nu_x \pm \nu_{x0})^2\right]$$

$(\lambda \nu_{x0}) = 0.1334$

for c_0: $c_0 = \frac{1}{\lambda}\sqrt{1-(\lambda\nu_{x0})^2} = 1.4859 \times 10^{-6}$

for c_1: $c_1 = \frac{\partial}{\partial \nu_x}\left[\frac{1}{\lambda}\sqrt{1-(\lambda\nu_{x0})^2}\right] = \frac{-(\lambda\nu_x)}{\sqrt{1-(\lambda\nu_x)^2}} = -0.1346$

for c_2: $c_2 = \frac{\partial^2}{\partial \nu_x^2}\left[\frac{1}{\lambda}\sqrt{1-(\lambda\nu_x)^2}\right] = \frac{\lambda}{\left[1-(\lambda\nu_x)^2\right]^{3/2}} = 6.8521 \times 10^{-7}$

e.) (5 pts) Plot the difference between the true phase $kz\sqrt{1-(\lambda\nu_x)^2}$ and the linear approximation $2\pi z\left[c_0 + c_1(\nu_x \pm \nu_{x0})\right]$ in regions of the +/- 1st orders, and compare the residual phase with the phase of the zero order found in part (c). Notice that this phase distribution causes the diffraction effects observed in Problem 2(a).

(Use an additional page)

f.) (5 pts) Show through mathematical development or reasoning that the field incident onto the lens can be approximated by
$M(x) + \frac{1}{2}M(x+c_1 z)e^{j\phi}e^{-j2\pi\nu_{x0}x} + \frac{1}{2}M(x-c_1 z)e^{j\phi}e^{j2\pi\nu_{x0}x}$, where $M(x)$ is the field that produces the irradiance pattern of Problem 2(a).

$\phi = 2\pi z c_0$

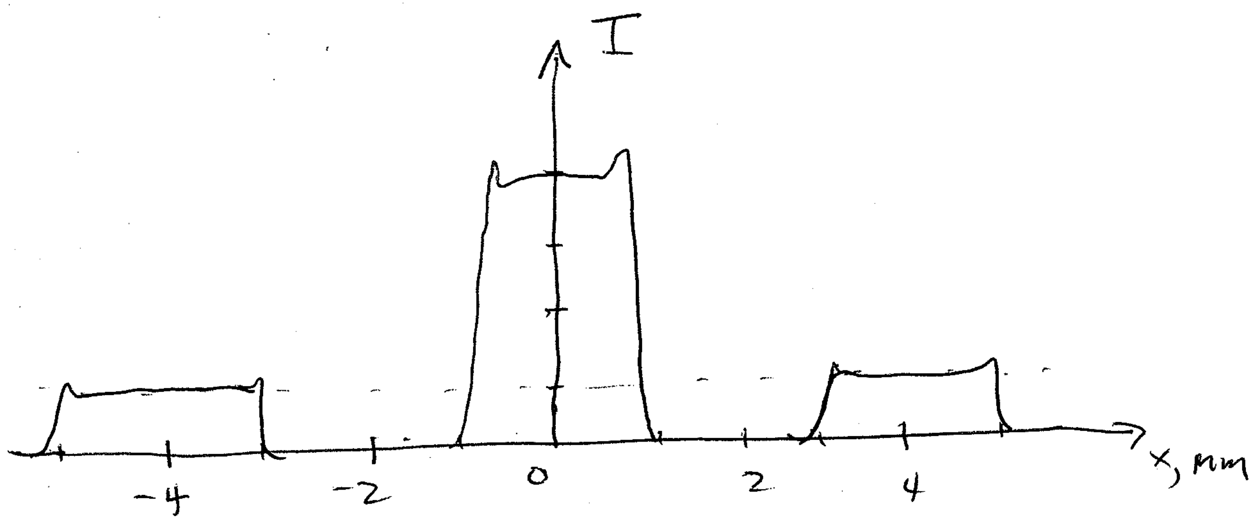
$$A(\nu_x; z) \approx d\, e^{j\gamma z}\,\mathrm{sinc}(d\nu_x) + \frac{d}{2}e^{j2\pi z c_0}\, e^{j2\pi z c_2(\nu_x - \nu_{x0})^2}\,\mathrm{sinc}(d[\nu_x - \nu_{x0}])\, e^{j2\pi z c_1(\nu_x - \nu_{x0})} + \frac{d}{2}e^{j2\pi z c_0}\, e^{j2\pi z c_2(\nu_x + \nu_{x0})^2}\,\mathrm{sinc}(d[\nu_x + \nu_{x0}])\, e^{-j2\pi z c_1(\nu_x + \nu_{x0})}$$

$$\mathcal{F}_1^{-1}\{A(\nu_x;z)\} = \mathcal{F}_1^{-1}\{d\,e^{jk\gamma z}\,\mathrm{sinc}(d\nu_x)\} + \frac{1}{2}\left[\mathcal{F}_1^{-1}\{d\,e^{j2\pi z c_2 \nu_x^2}\,\mathrm{sinc}(d\nu_x)\} * \delta(x + c_1 z)\right]e^{-j2\pi\nu_{x0}x}e^{j\phi} + \frac{1}{2}\left[\mathcal{F}_1^{-1}\{d\,e^{j2\pi z c_2 \nu_x^2}\,\mathrm{sinc}(d\nu_x)\} * \delta(x - c_1 z)\right]e^{j2\pi\nu_{x0}x}e^{j\phi}$$

$$= M(x) + \frac{1}{2}M(x - c_1 z)e^{j2\pi\nu_{x0}x}e^{j\phi} + \frac{1}{2}M(x + c_1 z)e^{-j2\pi\nu_{x0}x}e^{j\phi}$$

where $M(x) = \mathcal{F}_1^{-1}\{d\,e^{jk\gamma z}\,\mathrm{sinc}(d\nu_x)\}$

g.) (5 pts) Sketch the irradiance profile at the lens in the *x* direction.

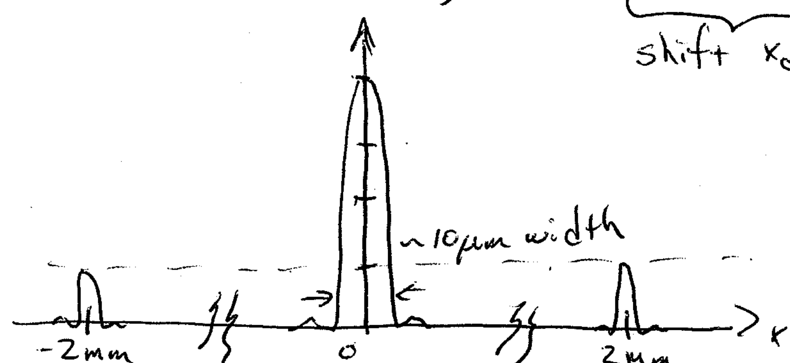


h.) (5 pts) Sketch the irradiance profile in the back focal plane at distance *f* behind the lens in the *x* direction if the lens diameter is much greater than the irradiance in part (g). Label the width of any primary features and their positions.

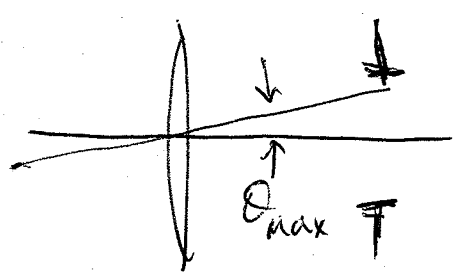
$$U(x_0) \sim \mathcal{F}\{U_{lens}(x_1)\}_{x_0/\lambda f} = A(\nu_x; z) \qquad \nu_y = \frac{x_0}{\lambda f}$$

$$\therefore I(x_0) \propto \text{sinc}^2\left(\frac{dx}{\lambda f}\right) + \frac{1}{4}\underbrace{\text{sinc}^2\left[d\left(\frac{x_0}{\lambda f} - \nu_{x0}\right)\right]}_{\text{shift } x_0 - \frac{\lambda f}{T}} + \underbrace{\frac{1}{4}\text{sinc}^2\left[d\left(\frac{x_0}{\lambda f} + \nu_{x0}\right)\right]}_{\text{shift } x_0 + \frac{\lambda f}{T}}$$

$$\frac{\lambda f}{T} = \frac{0.667\times10^{-6}\times 15\times10^{-3}}{5\times10^{-6}} = 2\text{ mm}$$



i.) (5 pts) If a round hole of diameter 5 mm is used as a stop in the back focal plane, what is the minimum grating period that can be imaged by the lens?

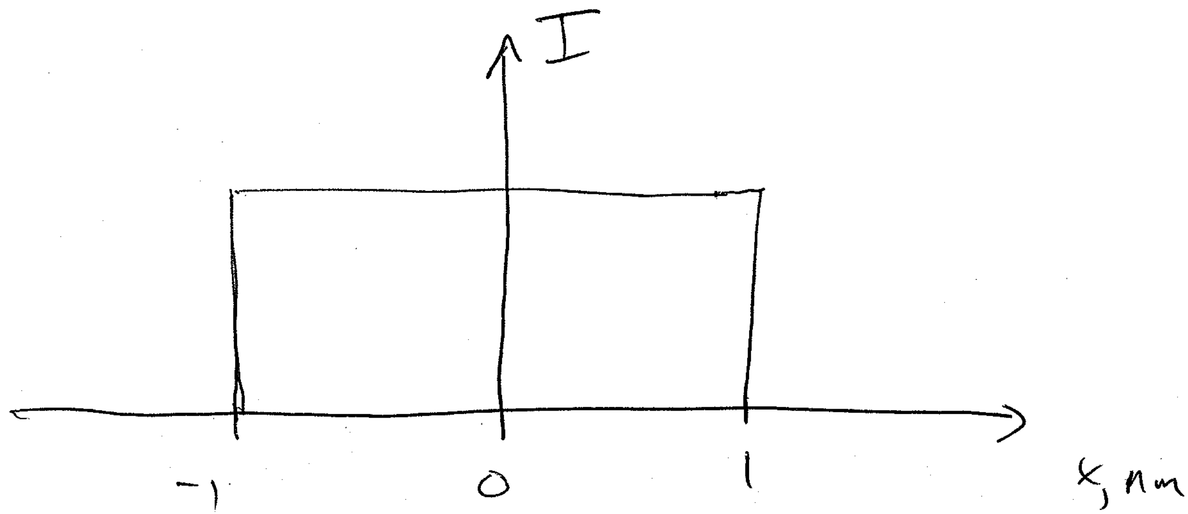


$$\theta_{max} = \tan^{-1}\left(\frac{2.5}{15}\right) = 9.462^\circ$$

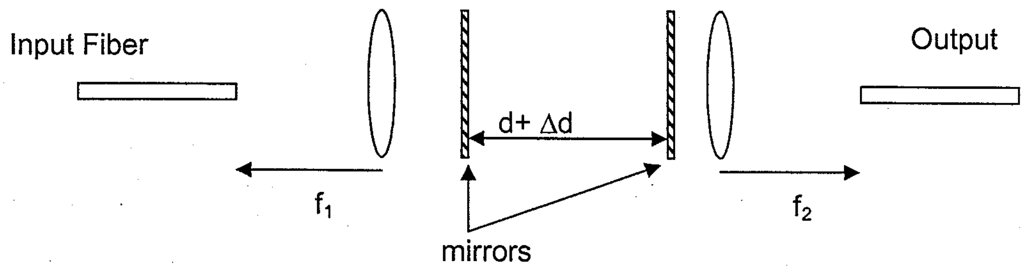
$$\Rightarrow (\alpha_0)_{max} = \sin\theta_{max} = 0.1644$$

$$\Rightarrow T_{min} = \frac{0.667\,\mu m}{0.1644} = 4.06\,\mu m$$

j.) (5 pts) For grating periods less than the minimum period found in part (i), describe the irradiance in the image plane by making a simple sketch of the irradiance in the *x* direction.



4.) Design a wavelength filter for a fiber communications network as shown below. Light from the input fiber is collimated and passed through a Fabret-Perot resonator. The transmitted light is collected by the output lens and coupled into the output fiber. By changing the distance between the mirrors, one wavelength band is selected at a time. The bands are centered around a wavelength of 1550 nm, they each have a bandwidth of 10 GHz, they are separated by 100 GHz, and there are 15 bands.



(a) (5 pts) What should be the nominal distance *d* and the distance Δ*d* in order to change from one band to the next?

$$\Delta\nu_{FSR} = 1.4\,THz = 1.4\times10^{12}\,Hz = \frac{c}{2nd}$$

$$d_{min} = \frac{c}{2\Delta\nu_{FSR}} = \frac{3\times10^{8}\,m/s}{2\cdot 1.4\times10^{12}\,1/s} = 1.07\times10^{-4} = \boxed{0.107\,mm}$$

$$\frac{\Delta d}{d} = \frac{\Delta\nu}{\nu} \Rightarrow \Delta d = \frac{\Delta\nu d}{c/\lambda} = \frac{\Delta\nu d\lambda}{c} = \frac{100\times10^{9}\times1.07\times10^{-4}\times1.55\times10^{-6}}{3\times10^{8}}$$

$$= \boxed{553\times10^{-9}\,m}$$

(b) (5 pts) What is the proper value for the reflectance of the mirrors?

$$\mathcal{F} = 15 \doteq \frac{\pi\sqrt{R}}{1-R} \Rightarrow R \sim 0.81$$

5.
a) (5 pts) Determine the refractive index and thickness of a thin film to be deposited on a glass surface (n_g=1.54.). Such that no normally incident light of wavelength 633nm is reflected.

$$n_1 = \sqrt{1.54} = 1.241$$

$$d = \lambda/4/n_1 = 127.52\ nm$$

b) (5 pts) Plot reflectance vs wavelength from 400nm to 1000nm at zero degrees and 45 degrees angle of incidence. For the 45 degrees angle of incidence, plot both *s* and *p* polarization responses. Assume that the indices of refraction for the film and the glass are not a function of wavelength.

$$r = \frac{-m_{21} + m_{11}\gamma_0 - m_{22}\gamma_s + m_{12}\gamma_0\gamma_s}{m_{21} + m_{11}\gamma_0 + m_{22}\gamma_s + m_{12}\gamma_0\gamma_s}$$

$m_{11} = \cos k_0 h$ $\qquad k_0 h = \frac{2\pi}{\lambda} n_1 d \cos\theta_1$

$m_{12} = -i \sin k_0 h/\gamma_1$ $\qquad n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_s \sin\theta_s$

$m_{21} = -\gamma_1 i \sin k_0 h$ $\qquad n_0 = 1 \quad n_1 = 1.241$

$m_{22} = \cos k_0 h = m_{11}$ $\qquad n_s = 1.54 \quad d = 127.52 \times 10^{-9}$

$\gamma_{0s} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_0 \cos\theta_0$ $\qquad \gamma_{0p} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_0 / \cos\theta_0$

$\gamma_{ss} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_s \cos\theta_s$ $\qquad \gamma_{sp} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_s / \cos\theta_s$

$\gamma_{1s} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_1 \cos\theta_1$ $\qquad \gamma_{1p} = \sqrt{\frac{\epsilon_0}{\mu_0}}\, n_1 / \cos\theta_1$

(see attached plots and matlab mfile for results).

TEST 2 PROB 3C