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OPTI 505
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good

CHAPTER 9: DIRECT PHASE MEASUREMENT

- Advantages: Rapid, high accuracy measurements, results independent of intensity variation across the pupil, fixed grid points, good results with low contrast fringes
- 90 degrees phase difference between measurements
- Three unknowns therefore a minimum of three measurements to solve $I_{dc} + I_{ac} \cos[\phi(x,y)]$
- Phase can be shifted by PZT, diffraction grating, bragg cell, or holographic element
- Requires adjacent pixels have less than pi difference. If greater than pi, N2pi needs to be added/subtracted.
- Vibration is major error source, so strong need for simultaneous data acquisition.
- Vertical Scanning Interferometer (VSI) uses a longer wavelengths, or shorter coherence length sources such as white light, to avoid step ambiguities.

CHAPTER 10: DIFFRACTION-fsPSF

- Diffraction theory is the study of light propagation effects that are not predicted by geometrical optics.
- Solving the problem requires: 1) Properties of illumination field U(r); 2) Properties of the aperture (transmission amplitude and phase; 3) Location of the observation.
- Can be solved by 1) Free-Space Point Spread Function (fsPSF) or by 2) Transfer Function

fsPSF Method

- General Solution: The general solution for the electric field at an observation point P0 can be found as Equation 1. At this point a clever substitution is needed for the *Greens Function* (G) to simplify the problem. Table 1 provides multiple options with boundary conditions necessary for their use.
- Using the *Dirichlet* conditions we can simplify Equation 1 to Equation 2 immediately following the aperture. This is a form of the *Rayleigh-Sommerfield Diffraction Formula.*
- Depending on the distance to the observation plane differing approximations of Equation 2 can be used: Huygens Region, Fresnel Region. Fraunhoffer Region.

fsPSF Method: Huygens Region

- Rewriting Equation 2 into the form of Equation 3 yields a form where we can separate out the PSF h(P0;P1). This function can be written as Equation 4 and Equation 5, which is referred to as the *Huygens Wavelet.*
- Huygens Region valid for any observation distance and the wavelet arriving at the observation plane is a complete Huygens wavelet.

fsPSF Method: Fresnel Region

- The Fresnel approximation occurs when r01, of the *Huygens Wavelet*, is >> than the wavelength. This allows the *Fresnel Wavelet* to be written as Equation 6.
- The *Fresnel Wavelet* allows Equation 2 to be rewritten as Equation 7, which is valid as long as the observation distance satisfies Equation 8 and the arriving wavelet is parabolic.

fsPSF Method: Fraunhofer Region

- The approximation allows the rewriting of Equation 4 as Equation 10. This approximation gives us the electric field as Equation 11, which is a handy Fourier transform of the original electric field which prefixed with constant values.
- Valid when the observation distance is satisfied by Equation 9 and the arriving wavelet is planar.

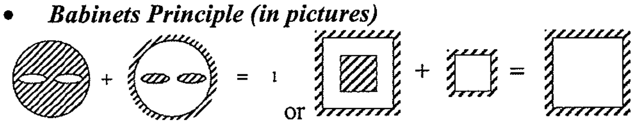
CHAPTER 11: DIFFRACTION-TRANSFER FUNCTION

- Related to the fsPSF by the *Weyl's integral*, and is expressed mathematically in Equation 12 which is gained through the following procedure:
  - Find the Fourier transform of $U_{z=0}(x_1,y_1)$ with respect to the frequency variables ξ and η. This result is the angular spectrum $A_{z=0}(\xi,\eta)$.
  - Multiply $A_{z=0}(\xi,\eta)$ by the transfer function of free space $H(\gamma;z) = \exp(jk\,\gamma\, z_0)$, where $\gamma = [1 - (\lambda\xi)^2 - (\lambda\eta)^2]^{1/2}$. This result is the angular spectrum $A(\xi,\eta;z_0)$. Simplify, if possible.
  - Find the inverse Fourier transform of $A(\xi,\eta;z_0)$ with respect to spatial variables x_0 and y_0. This result is the physical field on the observation plane at distance z_0.
- The A term in the above procedure can be called the *spatial frequency spectrum, plane wave spectrum*, or the *angular spectrum of plane waves.*
- With respect to the *angular spectrum of plane waves*; The $\hat{k}$'s of the planewave components are directed with an angle equal to or less than 90° with respect to the z axis if $\sigma = (\alpha^2 + \beta^2)^{1/2} <= 1$. The lower spatial frequnices have a lower diffraction angle while the higher spatial frequencies have higher diffraction angles. If $\sigma>1$ waves are *non-propagating* or *evanescent* components.

Talbot Effect

- The Talbot effect is a curious diffraction phenomenon that occurs for periodic objects illuminated with laser light. If the period of the object is large compared to λ, the maximum extent of the object's angular spectrum is limited to small angles where $\sigma \ll 1$. Various reconstructions of a phase grating can be observed at regular distances away from the aperture. The various observation planes are spaced by Equation 15. The pattern is A) Phase B) Amplitude C) Conjugate Phase D) Conjugate Amplitude. The key Transfer Function simplification is found in Equation 13.

- ***Babinets Principle (in pictures)***



CHAPTER 11: DIFFRACTION-FRESNEL DIFFRACTION

- By looking at the aperture as being a collection of spherical radiators a geometrical interpretation of diffraction can be garnered. $U(P) = \frac{-jU_\infty(P)}{\lambda}\int_0^a e^{jkOPD(\rho_a)}\frac{2\pi\rho_a}{L}d\rho_a$ Equation 14 is the electric field on axis at a point P, with the constant values described in $L = \frac{z_s z_0}{z_s + z_0}$; $\rho_a = \sqrt{x_a^2 + y_a^2}$; $OPD(\rho_a) \approx \frac{\rho_a^2}{2L}$
- Equation 16 and a being the aperture size.

Fresnel Zones

- The interference generated by these radiating point sources brings us to a nature discussion of *Fresnel Zones*. Which are described by zones across the aperture where the radiator causes the OPD = mλ, causing constructive interference, and OPD = (m + 1/2)λ, causing destructive interference.
- The number of *Fresnel Zones* across a circular aperture of radius a is called the *Fresnel Number*, Equation 17, the radius of the mth zone,
- Equation 18, and the area of each zone, Equation 19.

Fresnel Diffraction

- Incorporating the discussion on *Fresnel Zones* into $U(P) = \frac{-jU_\infty(P)}{\lambda}\int_0^a e^{jkOPD(\rho_a)}\frac{2\pi\rho_a}{L}d\rho_a$ Equation 14 yields an electric field calculation, Equation 20, with one variable being the *Fresnel Number*.
- This result written in terms of the axial irradiance becomes Equation 21, where for ***odd*** *Fresnel Numbers* the irradiance is ***maximum*** and for ***even*** *Fresnel Numbers* the irradiances is a ***minimum***.
- This leads to the physical interpretation that equal areas for odd and even zones produce zero field amplitude at the observation point P. Essentially equal areas ***cancel*** each other, which is why there is no irradiance when there is a Nf of 1 but zero irradiance at a Nf of 2.
- To find the approximate irradiance from an irregular aperture one needs to sum the odd and even areas, subtract them, divide by the area of one zone, and by squaring the result and multiplying by 4I(P) we have the approximate irradiance at the observation point.
- As the observation point goes off axis there will be a series of maximums and minimums with the total number of maximums equaling the *Fresnel Number*.

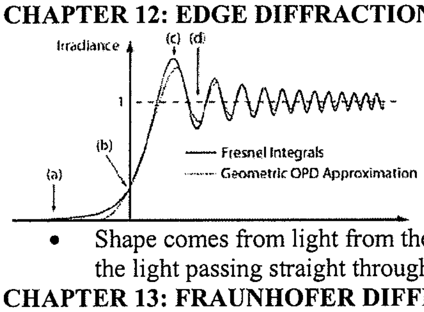
Poisson's Spot

- Basically it is black magic, given an opaque aperture the on axis irradiance is the same as if no aperture was present.

Fresnel Zone Plate

- By blocking either the odd or even zones the on-axis irradiances increases, Equation 22, due to only constructive interference.
- The axial irradiance observed due to diffraction from a Fresnel zone plate exhibits alternating maxima and zeros, where the locations of the maxima and minima are identical to those observed from an open aperture equal in radius to the radius of the first area of the zone plate. The off axis spot size is approximately Equation 22
- Zone Plate can be seen as a lens is described by $\frac{1}{f} = \frac{1}{z_s} + \frac{1}{z_0}$; $f_n = \frac{a_{ZP1}^2}{n\lambda}$
- Equation 24.

CHAPTER 12: EDGE DIFFRACTION (in Pictures)



- Shape comes from light from the edge to light passing straight through to the angle of interfence increasing between the light passing straight through and from the edge.

CHAPTER 13: FRAUNHOFER DIFFRACTION

- Expansion of earlier discussion, several aperture types are discussed with their characteristic electric field/irradiance
  - Plane wave on circular aperture Equation 25 and Equation 26

- Lens system with circular apertures Equation 27, with the Rayleigh limit, Equation 28, and the Sparrow limit, Equation 29.
- Plane wave on a rectangular aperture Equation 30 and Equation 31, with the width between the zeros of the central spot given by Equation 32.

CHAPTER 14: GRATINGS

- The Grating equation is Equation 33.
- Table 2 provides information on diffraction types and efficiencies.

$$U(P_0) = \iint_S \left[U(P_1)\frac{\partial G(P_0;P_1)}{\partial n} - G(P_0;P_1)\left(\frac{\partial U(P_1)}{\partial n}\right)\right]ds$$

Equation 1

$$U(P_0) = \iint_\Sigma \left(-\frac{j}{\lambda} + \frac{1}{2\pi r_{01}}\right)\gamma_z \frac{e^{jkr_{01}}}{r_{01}} U(P_1)\,ds$$

Equation 2

$$U(P_0) = \iint_\Sigma U(P_1)h(P_0;P_1)\,ds$$

Equation 3

$$h(P_0;P_1) = \frac{\partial}{\partial z_1}\frac{e^{jkr_{01}}}{2\pi r_{01}} = \left(-\frac{j}{\lambda} + \frac{1}{2\pi r_{01}}\right)\gamma_z \frac{e^{jkr_{01}}}{r_{01}}$$

Equation 4

$$h(P_0;P_1) = \frac{\partial}{\partial z_1}\frac{e^{jkr_{01}}}{2\pi r_{01}} = \frac{\sqrt{1+(kr_{01})^2}}{2\pi r_{01}^2}\gamma_z e^{j[kr_{01} - \tan^{-1}(kr_{01})]}$$

Equation 5

$$h(P_0;P_1) = \frac{-j\gamma_z \exp(jkr_{01})}{\lambda\, r_{01}} = \frac{-je^{jkz_0}}{\lambda z_0}\exp\left\{\frac{jk}{2z_0}\left[(x_0-x_1)^2 + (y_0-y_1)^2\right]\right\}$$

Equation 6

$$-\frac{je^{jkz_0}}{\lambda z_0}\exp\left[j\frac{k}{2z_0}(x_0^2+y_0^2)\right]\mathbf{F}_\eta \mathbf{F}_\xi\left\{U_{z=0}(x_1,y_1)\exp\left[\frac{jk}{2z_0}(x_1^2+y_1^2)\right]\right\}$$

Equation 7

$$z_0 \gg \sqrt[3]{\frac{\pi}{4}}\lambda\left[\left(\frac{x_0-x_1}{\lambda}\right)^2 + \left(\frac{y_0-y_1}{\lambda}\right)^2\right]^{2/3}_{max}$$

Equation 8

$$z_0 \gg \frac{k}{2}(x_1^2+y_1^2)_{max}$$

Equation 9

$$h(P_0;P_1) = -\frac{je^{jkz_0}}{\lambda z_0}e^{j\frac{k}{2z_0}(x_0^2+y_0^2)}\exp\left[\frac{2\pi j}{\lambda z_0}(x_0x_1+y_0y_1)\right]$$

Equation 10

$$-\frac{je^{jkz_0}}{\lambda z_0}e^{j\frac{k}{2z_0}(x_0^2+y_0^2)}\mathbf{F}_{\eta=\frac{y_0}{\lambda z_0}}\mathbf{F}_{\xi=\frac{x_0}{\lambda z_0}}[U_{z=0}(x_1,y_1)]$$

Equation 11

$$U(x_0,y_0;z_0)e^{-j\omega t} = \iint_\infty A_{z=0}(\xi,\eta)e^{j2\pi\left(\frac{\gamma}{\lambda}z_0\right)}e^{j2\pi\left(\frac{\alpha}{\lambda}x_0+\frac{\beta}{\lambda}y_0\right)}e^{-j\omega t}\,d\xi\, d\eta$$

Equation 12

$$e^{jkz_0\gamma} \approx e^{jkz_0}e^{-j\frac{kz_0}{2}\alpha^2} = e^{jkz_0}e^{-j\frac{kz_0}{2}(\xi\lambda)^2}$$

Equation 13

$$U(P) = \frac{-jU_\infty(P)}{\lambda}\int_0^a e^{jkOPD(\rho_a)}\frac{2\pi\rho_a}{L}d\rho_a$$

Equation 14

$$z_{base} = \frac{1}{2\lambda\xi_0^2} = \frac{\lambda}{2\alpha_0^2}$$

Equation 15

$$L = \frac{z_s z_0}{z_s+z_0};\quad \rho_a = \sqrt{x_a^2+y_a^2};\quad OPD(\rho_a) \approx \frac{\rho_a^2}{2L}$$

Equation 16

$$N_f = \frac{a^2}{\lambda L}$$

Equation 17

$$\rho_m = \sqrt{m\lambda L}$$

Equation 18

$$A_f = \pi\rho_{m+1}^2 - \pi\rho_m^2 = \pi\lambda L$$

Equation 19

$$-U_\infty(P)(e^{j\pi N_f} - 1)$$

Equation 20

$$I(P) = U(P)U^*(P) = 4I_\infty(P)\sin^2\left(\frac{\pi N_f}{2}\right)$$

Equation 21

$$I(P) = 4N_{FZ}^2 I_\infty(P)$$

Equation 22

$$w = \frac{\lambda z_0}{a}$$

Equation 23

$$\frac{1}{f} = \frac{1}{z_s} + \frac{1}{z_0};\quad f_n = \frac{a_{ZP1}^2}{n\lambda}$$

Equation 24

$$U(\rho_0) = -\frac{je^{jkz_0}}{\lambda z_0}\exp\left(j\frac{k\rho_0^2}{2z_0}\right)\pi a^2 \mathrm{somb}\left(\frac{2a\rho_0}{\lambda z_0}\right)$$

Equation 25

$$I(\rho_0) = \left(\frac{1}{\lambda z_0}\right)^2(\pi a^2)^2\mathrm{somb}^2\left(\frac{2a\rho_0}{\lambda z_0}\right)$$

Equation 26

$$\frac{je^{jkz_0}}{\lambda z_0}\exp\left(j\frac{k\rho_0^2}{2z_0}\right)\pi a^2 \mathrm{somb}\left(\frac{2a\rho_0}{\lambda z_0}\right)$$

Equation 27

$$\rho_{RES} = 1.22\frac{\lambda z_0}{2a} = 0.61\frac{\lambda}{NA} = 1.22\lambda f/\#$$

Equation 28

$$\rho_{RES-SPARROW} = 0.94\frac{\lambda z_0}{2a} = 0.47\frac{\lambda}{NA} = 0.94\lambda f/\#$$

Equation 29

$$-\frac{jAe^{jkz_0}}{\lambda z_0}\exp\left[\frac{k}{2z_0}(x_0^2+y_0^2)\right]4ab\,\mathrm{sinc}\left(\frac{4ax_0}{\lambda z_0}\right)\mathrm{sinc}\left(\frac{4by_0}{\lambda z_0}\right)$$

Equation 30

$$I(x_0,y_0) = \left(\frac{A}{\lambda z_0}\right)^2(4ab)^2\mathrm{sinc}^2\left(\frac{2ax_0}{\lambda z_0}\right)\mathrm{sinc}^2\left(\frac{2by_0}{\lambda z_0}\right)$$

Equation 31

$$s_x = \frac{\lambda z_0}{a} = \frac{\lambda}{NA_x}$$

Equation 32

$$n_S\sin\theta_S + n_0\sin\theta_m = \frac{m\lambda}{d}$$

Equation 33

Table 1 Possible Green's Functions

Table 6.1 Summary of boundary conditions, obliquity factors and diffraction integrals.

Boundary Conditions	$G(P_0,P_1)$	Across Σ	In Shadow	Obliquity Factor	Diffraction Integral	Comment
Kirchhoff[3] (Cauchy)	$-\frac{e^{jkr_{01}}}{4\pi r_{01}}$	U and $\partial U/dn$ same as without screen	$U=0$, $\frac{\partial U}{\partial n}=0$	$\frac{\cos(\hat{n},\hat{r}_{21})-\cos(\hat{n},\hat{r}_{01})}{2}$	$U(P_0)=\frac{jA}{\lambda}\iint_\Sigma \frac{e^{jk(r_{21}+r_{01})}}{r_{21}r_{01}}\left[\frac{\cos(\hat{n},\hat{r}_{01})-\cos(\hat{n},\hat{r}_{21})}{2}\right]ds$ (6.22)	Fresnel-Kirchhoff Diffraction Formula
Neumann	$-\frac{e^{jkr_{01}}}{4\pi r_{01}}-\frac{e^{jk\tilde{r}_{01}}}{4\pi \tilde{r}_{01}}$	$\partial U/dn$ same as without screen	$\frac{\partial U}{\partial n}=0$	contained in $\partial U/dn$	$U(P_0)=\frac{1}{4\pi}\iint_\Sigma \frac{\partial U(P_1)}{\partial n}\frac{2e^{jkr_{01}}}{r_{01}}ds$ (6.23)	
Dirichlet	$-\frac{e^{jkr_{01}}}{4\pi r_{01}}+\frac{e^{jk\tilde{r}_{01}}}{4\pi \tilde{r}_{01}}$	U same as without screen	$U=0$	$\gamma_z=\cos(\hat{n},\hat{r}_{01})$	$U(P_0)=\iint_\Sigma\left(\frac{-j}{\lambda}+\frac{1}{2\pi r_{01}}\right)\gamma_z\frac{e^{jkr_{01}}}{r_{01}}U(P_1)ds$	Rayleigh-Sommerfeld Diffraction Formula