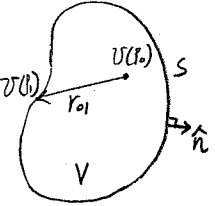


Chapter 10.

6.2.2 Integral Theorem of Helmholtz and Kirchhoff = (Page 4)

Divergence Theorem: $\underbrace{\iiint_V (G \nabla^2 U - U \nabla^2 G) dV}_{\text{integral over closed volume } V} = \underbrace{\iint_S (G \frac{\partial U}{\partial n} - U \frac{\partial G}{\partial n}) ds}_{\text{surface integral surrounding the volume over surface } S}$

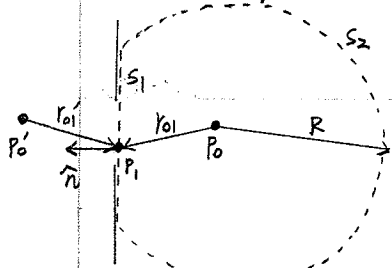
substituting $(\nabla^2 + k^2)U = 0$ & $(\nabla^2 + k^2)G = \delta(r_0)$, $r_0 = \sqrt{(x_0-x)^2 + (y_0-y)^2 + (z_0-z)^2}$



$$U(P_0) = \iint_S [U(P_i) \frac{\partial G(P_0; P_i)}{\partial n} - G(P_0; P_i) (\frac{\partial U(P_i)}{\partial n})] ds \quad (6.6)$$
 by choosing $G(P_0; P_i) = \frac{-e^{jk r_0}}{4\pi r_0}$

$$U(P_0) = \frac{1}{4\pi} \iint_S \left[\frac{e^{jk r_0}}{r_0} \frac{\partial U(P_i)}{\partial n} - U(P_i) \frac{\partial}{\partial n} \left(\frac{e^{jk r_0}}{r_0} \right) \right] ds \quad (6.8)$$

6.2.3 Diffraction by Plane Screen (Page 4)



- at surface 1, the Green function equals 0, $\frac{\partial G}{\partial n} U(P_i) \neq 0$

surface 2, the Green function equals 0, by Sommerfeld Radiation Condition.

as $R \rightarrow \infty$, $U(P_i) = 0$, so $\frac{\partial U}{\partial n} U(P_i) = 0$, nothing contributes to $U(P_0)$

- only $\frac{\partial G}{\partial n} U(P_i)$ at S_1 contributes to $U(P_0)$

$$U(P_0) = \iint_{S_1} \left(\frac{-j}{\lambda} + \frac{1}{2\pi r_0} \right) \gamma_z \frac{e^{jk r_0}}{r_0} U(P_i) ds \quad (6.19)$$
 $\gamma_z = \frac{z_0}{r_0} = \cos(\hat{n}, \hat{r}_0)$

when $r_0 \gg \lambda$,
$$U(P_0) = \frac{-j}{\lambda} \iint_{S_1} \gamma_z \frac{e^{jk r_0}}{r_0} U(P_i) ds \quad (6.20)$$
 Rayleigh-Sommerfeld diffraction formula

Huygen's Principle

$$U(P_0) = \iint_S W(P_0; P_i) U(P_i) \frac{e^{jk r_0}}{r_0} ds \quad (6.21)$$

the field at the observation point is due to a weighted summation of spherical wave multiplying by the incident field.

6.2.4 Huygen's Wavelet (Page 11)

$$U(P_0) = \iint_S U(P_i) h(P_0; P_i) ds \quad (6.24)$$

$U(P_i) \rightarrow$ total field at the aperture plane

$h(P_0; P_i) \rightarrow$ Point Spread Function

$$h(P_0; P_i) = c e^{j\phi} \gamma_z e^{jk r_0} = \frac{\sqrt{1 + (k r_0)^2}}{2\pi r_0^2} \gamma_z e^{j[k r_0 - \tan^{-1}(k r_0)]} \quad (6.28)$$
 phase $\phi = -\tan^{-1}(k r_0)$

amplitude ratio $\left(\frac{A_{sph}}{A_{Huy}} \right) \uparrow \uparrow$ with r_0 when $r_0 < \lambda$, $\left(\frac{A_{sph}}{A_{Huy}} \right) \approx 1$ with large r_0 , ϕ changes a lot when $r_0 < \lambda$

6.2.5 The Fresnel Approximation (Page 14)

$r_0 \gg \lambda$, $e^{-j\phi} \rightarrow -j$; $h(P_0; P_i) = \frac{-j}{\lambda} \gamma_z \frac{e^{jk r_0}}{r_0}$, $\gamma_z = \frac{z_0}{r_0}$

by approximation $r_0 \approx z_0 \left\{ 1 + \frac{1}{2} \left[\frac{(x_0-x)^2}{z_0^2} + \frac{(y_0-y)^2}{z_0^2} \right] \right\}$, $\frac{1}{r_0^2} \approx \frac{1}{z_0^2}$

Fresnel Diffraction Formula

$$U(P_0) = \frac{-j e^{jk z_0}}{\lambda z_0} \iint_S U(P_i) e^{\left\{ \frac{jk}{2z_0} [(x_0-x)^2 + (y_0-y)^2] \right\}} ds \quad (6.38)$$

Fresnel Wavelet

$$h(P_0; P_i) = \frac{-j e^{jk z_0}}{\lambda z_0} e^{\left\{ \frac{jk}{2z_0} [(x_0-x)^2 + (y_0-y)^2] \right\}} \quad (6.39)$$

→ by $V(P) = V_{z=0}(x_1, y_1) = V(x_1, y_1) \text{tap}(x_1, y_1)$

Amplitude transmittance of the Aperture
 $\text{tap}(x_1, y_1) = \begin{cases} 1, & \text{if } P \in \Sigma \\ 0, & \text{otherwise} \end{cases}$

Fresnel Diffraction Formula

$$V(x_0, y_0) = \frac{-j e^{jkz_0}}{\lambda z_0} e^{j \frac{k}{2z_0}(x_0^2 + y_0^2)} F_\eta F_\xi \left\{ V_{z=0}(x_1, y_1) e^{j \frac{k}{2z_0}(x_1^2 + y_1^2)} \right\}$$

$$\xi = \frac{x_0}{\lambda z_0}, \eta = \frac{y_0}{\lambda z_0}$$

condition: $z_0 \gg \sqrt{\frac{\pi}{\lambda} \left[\left(\frac{x_0 - x_1}{\lambda} \right)^2 + \left(\frac{y_0 - y_1}{\lambda} \right)^2 \right]_{\max}}$ (6.44)

Stationary Phase: $(x_1 - x_0)^2$ is large: cosine oscillates rapidly contribute less to the integrand.
 ⇒ % error in approximation ↑↑ when oscillatory frequency ↑
 errors decreases when aperture size increases.

6.2.6 The Fraunhofer Approximation (Page.20) Far field

→ Condition: $z_0 \gg \frac{k}{2}(x^2 + y^2)_{\max} \Rightarrow e^{j \frac{k}{2z_0}(x^2 + y^2)} \rightarrow 1$

Fraunhofer Diffraction Formula

$$V(x_0, y_0) = \frac{-j e^{jkz_0}}{\lambda z_0} e^{j \frac{k}{2z_0}(x_0^2 + y_0^2)} F_\eta F_\xi \left\{ V_{z=0}(x_1, y_1) \right\} \Big|_{\xi = \frac{x_0}{\lambda z_0}, \eta = \frac{y_0}{\lambda z_0}} \quad (6.49)$$

6.2.7 Transfer Function of Free Space (Page.22)

$$V(x_0, y_0) = V_{z=0}(x_1, y_1) \ast \ast h(x_0 - x_1, y_0 - y_1; z_0), \quad h(x_0 - x_1, y_0 - y_1; z_0) = \frac{j}{2z_0} \frac{e^{jk|r_0|}}{2\pi |r_0|}$$

Weyl's Integral

$$\frac{e^{jkr}}{-jkr} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{jk(\alpha x + \beta y + \delta z)} d\omega, \quad d\omega = \frac{d\alpha d\beta}{r}$$

$$\frac{j}{2z_0} \left(\frac{e^{jkr}}{2\pi r} \right) \Downarrow = F_y^{-1} F_x^{-1} \left\{ e^{jk\delta z} \right\} \quad (6.64)$$

Transfer Function of Free Space $H(\gamma; z_0) = F_\eta F_\xi [h(P_0; P_1)] = e^{jk\delta z_0} \quad (6.66)$

$$F_\eta F_\xi [V(P_0)] = F_\eta F_\xi [V_{z=0}(P_1)] H(\gamma; z_0) = F_\eta F_\xi [V_{z=0}(P_1)] e^{jk\delta z_0} \quad (6.67)$$

Spatial Frequency Spectrum $A(\xi, \eta; z_0) = A_{z=0}(\xi, \eta) e^{jk\gamma z_0}, \quad V(x_0, y_0; z_0) = F_y^{-1} F_x^{-1} [A_{z=0}(\xi, \eta) e^{jk\delta z_0}]$

6.2.8 Angular Spectrum of Plane Wave (Page.26)

$$V_{z=0}(P_1) e^{-i\omega t} = \iint_{\infty} A_{z=0}(\xi, \eta) e^{j(k \cdot r - \omega t)} \Big|_{z=0} d\xi d\eta$$

$\begin{cases} K = k(\alpha \hat{x} + \beta \hat{y} + \delta \hat{z}) & \text{direction cosine} \\ A_{z=0}(\xi, \eta) & \text{can be treated as weighting} \end{cases}$

$$V_{z=0}(P_0) e^{-i\omega t} = \iint_{\infty} A_{z=0}(\xi, \eta) e^{j(k \cdot r - \omega t)} d\xi d\eta = \text{this field is composed of a summation of plane wave (weighting is the same as } z=0)$$

if $\sigma = \sqrt{\alpha^2 + \beta^2} \leq 1$, there is an angle between \hat{z} & \hat{k}

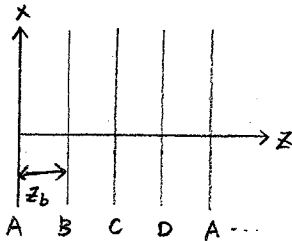
$\begin{cases} \text{small angle} : \text{lower frequency} \\ \text{large angle} : \text{large frequency} \end{cases}$

Procedure [calculating diffraction from field $U_{z=0}(P_1)$ to field $U(P_0)$ on a plane at z_0]

- ① Find $F_3 F_2 [U_{z=0}(x_1, y_1)] = A_{z=0}(\xi, \eta)$
- ② Multiplying $A_{z=0}(\xi, \eta)$ with $H(\gamma, z) = e^{jk\gamma z_0}$, $\gamma = \sqrt{1 - (\lambda\xi)^2 - (\lambda\eta)^2}$, then we find $A(\xi, \eta; z_0)$
- ③ Find $F_3^{-1} F_2^{-1} [A(\xi, \eta; z_0)]$ with respect to spatial variables x_0 and y_0 . Result = physical field on observation plane at distance z_0 .

From $H(\gamma, z_0) = e^{jk\gamma z_0} \Rightarrow$ its phase is $\psi = k\gamma z_0 = kz_0(1 - \sigma^2)^{\frac{1}{2}}$; $|\sigma| \leq 1$
 ψ is quadratic for small σ ; $\psi \uparrow$ as $z_0 \uparrow$ (small σ), when $\sigma=0$; ψ is max.

6.2.9 Talbot Effect (Page. 34)



- A. Phase
- B. Amplitude
- C. Conjugate Phase
- D. Reversed Contrast Amplitude

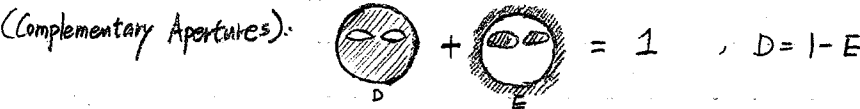
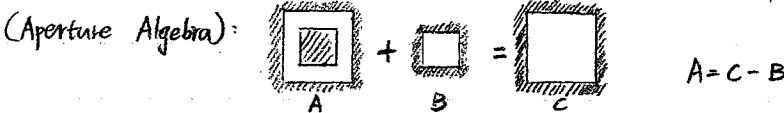
$$z_{\text{base}} = \frac{\lambda}{2\alpha_0^2} = \frac{1}{2\lambda\xi_0^2} = \frac{T^2}{2\lambda}$$

$$\xi_0 = \frac{1}{T}$$

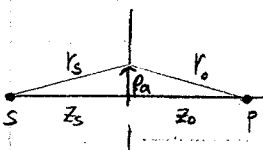
$$\alpha_0 = \lambda\xi_0, \beta_0 = \lambda\eta_0$$

6.2.10 Babinet's Principle (Page. 38)

- scalar diffraction is a linear and shift invariant process. $U_{\text{total}} = U_1 + U_2 + \dots + U_n$



6.3 Fresnel Diffraction (Page. 41)



$$U(P) = \frac{-jA}{\lambda z_s z_0} \iint_{\Sigma} e^{ik(r_s+r_0)} dx dy = \frac{-jU_{\infty}(P)}{\lambda} \int_0^a e^{jkOPD(\rho/a)} d\rho$$

Effective Propagation Distance $L = \frac{z_0 z_s}{z_s + z_0}$

$$OPD = r_s + r_0 - z_s - z_0$$

- \Rightarrow by approximation ($a \ll r_s, a \ll r_0$); $OPD \approx \frac{a^2}{2L}$
- \rightarrow Fresnel # $N_f = \frac{a^2}{\lambda L}$; $\rho_m = \sqrt{m\lambda L}$; $A_f = \pi\lambda L = \text{constant}$
- \rightarrow $U(P) = -U_{\infty}(P) [e^{j\pi N_f} - 1]$ (6.109) only depends on N_f
 $I(P) = 4I_{\infty}(P) \sin^2\left(\frac{\pi N_f}{2}\right)$ (6.110) ($I(P) = \text{max}$ at $N_f = \text{odd}$, $I(P) = 0$ at $N_f = \text{even}$)

Procedure to [determine the irradiance at the observation point.] (Page. 49)

- ① Divide the Aperture into Fresnel zones based on the OPD between the source pt and observation point.
- ② (A_{odd} - A_{even}) / (Total Area)
- ③ Multiply the square of this fraction by $4I_{\infty}(P)$. This is the irradiance at observation pt.

Procedure [Calculating the near-field irradiance behind the aperture for any type of illuminating laser beam.]

- ① Calculate the effective propagation distance L .
- ② Calculate the field observed a distance L behind an aperture with collimated illumination.
- ③ Apply scaling factor $\frac{z_0 + z_s}{z_s}$
- ④ Scale the total energy in the pattern to the total energy at the aperture plane.

6.3.3 Poisson's Spot (Page. 58)

- ↳ When a plane wave illuminates a circular disk, a bright spot is observed on-axis.
- ↳ the axial irradiance with the disk has the same value as the irradiance without the disk.
- ↳ the axial irradiance behind the disk is not a function of z .

6.3.4 Fresnel Zone Plate (P. 60)

- ↳ The zone plate exhibit lens-like properties.
- ↳ if first zone contributes $4 I(P)$ to on-axis irradiance, an aperture and mask with $N^2 z$ open Fresnel zones. Produces $I(P) = 4(N^2 z) I_{\infty}(P)$ (6.114) total on-axis irradiance
- ↳ The axial position and irradiance values of the maxima and zeros created by the first open area of the Fresnel zone plate are identical to the maxima and zeros observed from an aperture with radius $a_{z=1}$.
- ↳ As we move the observation plane closer to the aperture, total irradiance decreases.
- ↳ The concentration of energy from the Fresnel zone plate is similar to a refractive positive lens

(6.120) $\frac{1}{L} = \frac{1}{z_s} + \frac{1}{z_0}$, $L = a_{z=1}^2 / \lambda = f$ (image position of first axial maximum)

Image-position $f_n = a_{z=1}^2 / n\lambda$, $n = \pm 1, \pm 3, \pm 5 \dots$ (- : left side) (6.122)

Chapter 9.

Advantage of phase-shifting Interferometry

- rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil.
- Phase obtained at fixed grid of points.
- Easy to use with large solid-state detector arrays.
- High measurement accuracy.

$$I(x,y) = \underbrace{I_{dc}}_{\text{test mirror}} + \underbrace{I_{ac}}_{\text{reference mirror}} \cos \left[\underbrace{\phi(x,y)}_{\text{measured object phase}} + \underbrace{\phi(t)}_{\text{phase shift}} \right]$$

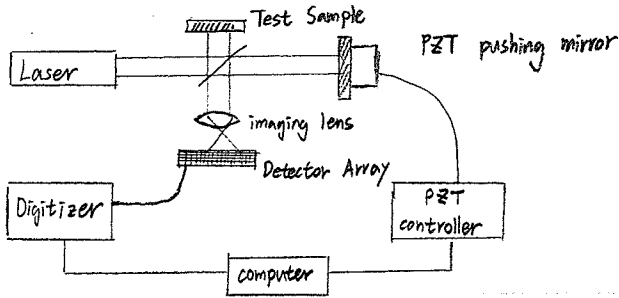
$$\begin{cases} I_1(x,y) = I_{dc} + I_{ac} \cos [\phi(x,y)] & \phi(t) = 0 \\ I_2(x,y) = I_{dc} - I_{ac} \sin [\phi(x,y)] & \frac{\pi}{2} \\ I_3(x,y) = I_{dc} - I_{ac} \cos [\phi(x,y)] & \pi \\ I_4(x,y) = I_{dc} + I_{ac} \sin [\phi(x,y)] & \frac{3\pi}{2} \end{cases}$$

3 measurement

$$\phi = \tan^{-1} \left[\frac{I_3 - I_2}{I_1 - I_2} \right]$$

4 measurement

$$\phi = \tan^{-1} \left[\frac{I_4 - I_2}{I_1 - I_3} \right]$$



Phase-Shifting

1. Diffraction Grating
2. Bragg Cell
3. Rotating Half-wave plate
4. Rotating Polarizer Phase shifter
5. Moving Mirror

Disadvantages: ① Phase Ambiguities. ② Vibration ③ Doesn't work with multiple λ (white light)

Remove

① Phase Ambiguities

- Require adjacent pixel less than π difference. ($\frac{1}{2}$ wave OPD)
- Trace path
- When phase jumps by $>\pi$, add or subtract $N2\pi$, adjust so $<\pi$
- Arctan Mod 2π (Mod 1 wave)

② Vibration

- Reduce Vibration
- Take data fast
- Take all frames at once.
- Measure vibration and introduce vibration 180° out of phase to cancel it out.

③ Multiple Wavelengths (White light)

- use vertical scanning interferometer

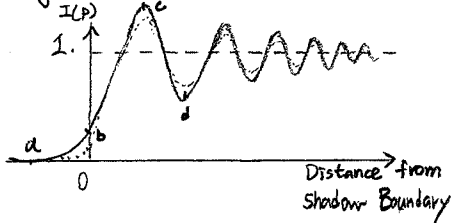
Comparison [VSI v.s PSI]

Advantages. ① can operate over large step height without ambiguity. ② uses 'White light' ③ less sensitive to vibration

Disadvantages ① slower ② Not as accurate ③ requires high-contrast fringes. ④ sensitive to irradiance fluctuation.

Chapter 11.

Edge Diffraction: ①



- a - in the shadow (odd & even zone areas are approximately equal)
- b - Shadow boundary
- c - first zone boundary (where central zone just pass the edge - max I)
- d - 2nd zone boundary

Chapter 12.

Fraunhofer Diffraction.

$$U(P_0) = \frac{-j e^{jkz_0}}{\lambda z_0} e^{[j \frac{k P_0^2}{2z_0}]} B_{\sigma_0 = \frac{P_0}{z_0}} \left[\text{circ} \left(\frac{P_1}{2a} \right) U_{z=0}(P_1) \right] \quad (6.134)$$

obs plane.

When an observation plane is far from the aperture illuminated by a laser, a FT determines the distribution in

6.4.1

Circular Aperture

$$U(P_0) = \frac{-j e^{jkz_0}}{\lambda z_0} e^{[j \frac{k P_0^2}{2z_0}]} (\pi a^2) \text{somb} \left(\frac{2a P_0}{\lambda z_0} \right) \quad (6.137)$$

$$I(P_0) = \left(\frac{1}{\lambda z_0} \right)^2 (\pi a^2)^2 \text{somb}^2 \left(\frac{2a P_0}{\lambda z_0} \right) \quad (6.138)$$

$a \rightarrow$ radius of the aperture, $\pi a^2 \rightarrow$ aperture area.

6.4.2

Exit pupil of an imaging system

of the exit pupil.

The field at the image plane of a perfect imaging system resulting from a pt-source is the FT

$$\text{Rayleigh's criterion: } P_{RES} = 1.22 \frac{\lambda z_0}{2a} = 0.61 \frac{\lambda}{NA} = 1.22 \lambda (f/\#)$$

$$\text{Sparrow criterion: } P_{RES} = 0.94 \frac{\lambda z_0}{2a} = 0.47 \frac{\lambda}{NA} = 0.94 \lambda (f/\#)$$

6.4.3 Rectangular Aperture

Field at the rectangular aperture $U_{z=0}(x_0, y_0) = A \text{rect}\left(\frac{x_1}{2a}\right) \text{rect}\left(\frac{y_1}{2b}\right)$ (6.143)

$$U(x_0, y_0) = \frac{-jAe^{jkz_0}}{\lambda z_0} e^{\left[\frac{k}{2z_0}(x_0^2 + y_0^2)\right]} (4ab) \text{sinc}\left(\frac{4ax_0}{\lambda z_0}\right) \text{sinc}\left(\frac{4by_0}{\lambda z_0}\right) \quad (6.144)$$

$$I(x_0, y_0) = \left(\frac{A}{\lambda z_0}\right)^2 (4ab)^2 \text{sinc}^2\left(\frac{2ax_0}{\lambda z_0}\right) \text{sinc}^2\left(\frac{2by_0}{\lambda z_0}\right) \quad (6.145)$$

6.4.4 Diffraction from Slits (long but narrow rectangle)

2-slits. $U_{z=0}(x_0, y_0) = A \text{rect}\left(\frac{x_1}{2a}\right) \text{rect}\left(\frac{y_1}{2b}\right) * \left[\delta\left(y_1 - \frac{d}{2}\right) + \delta\left(y_1 + \frac{d}{2}\right)\right]$ (6.147), $d \rightarrow$ slit separation

$$U(0, y_0) = \frac{-jAe^{jkz_0}}{\lambda z_0} e^{\left(\frac{ky_0^2}{2z_0}\right)} (4ab) \text{sinc}\left(\frac{2by_0}{\lambda z_0}\right) 2 \cos\left(\frac{2\pi d y_0}{2\lambda z_0}\right) \quad (6.148)$$

$$I(0, y_0) = 4 \left(\frac{A}{\lambda z_0}\right)^2 (4ab)^2 \text{sinc}^2\left(\frac{2by_0}{\lambda z_0}\right) \left[\frac{1}{2} + \frac{1}{2} \cos\left(\frac{2\pi d y_0}{\lambda z_0}\right)\right] \quad (6.149)$$

N-slits. $U(0, y_0) = \frac{-jAe^{jkz_0}}{\lambda z_0} e^{\left(\frac{ky_0^2}{2z_0}\right)} (4ab) \text{sinc}\left(\frac{2by_0}{\lambda z_0}\right) \frac{\sin\left(2\pi N \frac{dy_0}{2\lambda z_0}\right)}{\sin\left(2\pi \frac{dy_0}{2\lambda z_0}\right)}$ (6.150)

6.5 Theory of Grating

Gratings are periodic structures that, when illuminated by a coherent light beam, diffract light into diffraction orders that are copies of the illuminating wave separated in angle.

Square-wave transmission [Phase Grating]

$$f_d(y) = \begin{cases} e^{j\alpha\phi} & 0 < y < s \\ 1 & s < y < d \end{cases}$$

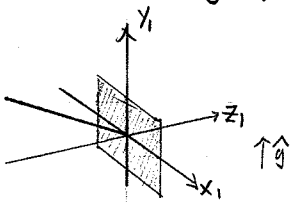
[Amplitude Grating]

$$f_d(y) = \begin{cases} 0 & 0 < y < s \\ 1 & s < y < d \end{cases}$$

[reflective Grating]

$$f_d(y) = \begin{cases} e^{j\alpha\phi} & 0 < y < s \\ 1 & s < y < d \end{cases}$$

$$\text{Grating Equation: } n_s \sin \theta_s + n_o \sin \theta_m = \frac{m\lambda}{d}$$



$$\hat{g} = \hat{y}_1 \quad \text{OPD} = n_s d \hat{k}_s \cdot \hat{g} + n_o d \hat{k}_d \cdot \hat{g} = m\lambda$$

$$n_s \beta_s + n_o \beta_m = \frac{m\lambda}{d}$$

$$\vec{k}_m = \frac{2\pi n_o}{\lambda} \left[ds \hat{x} + \left(\frac{m\lambda}{n_o d} - \frac{n_s \beta_s}{n_o}\right) \hat{y} + \sqrt{1 - ds^2 - \left(\frac{m\lambda}{n_o d} - \frac{n_s \beta_s}{n_o}\right)^2} \hat{z} \right]$$

$$U(x_0, y_0) = \frac{-\pi a^2 A_i}{\lambda z_0 d} e^{jkz_0} e^{\frac{jkx_0^2}{2z_0}} \sum_{m=-\infty}^{\infty} F_m \left[f_d(y_1) \right] \text{somb} \left(\frac{2a}{\lambda z_0} \sqrt{x_0^2 + \left(y_0 + \frac{m\lambda z_0}{d} - \beta_s z_0\right)^2} \right)$$

$$I(x_0, y_0) \approx \left(\frac{\pi a^2 A_i}{\lambda z_0 d}\right)^2 \sum_{m=-\infty}^{\infty} \left| F_m \left[f_d(y_1) \right] \right|^2 \text{somb}^2 \left[\frac{2a}{\lambda z_0} \sqrt{x_0^2 + \left(y_0 + \frac{m\lambda z_0}{d} - \beta_s z_0\right)^2} \right]$$

Absolute Diffraction Efficiency $\equiv \frac{\text{Power in order } m}{\text{incident power}}$

$$E_m = \left| \frac{1}{d} F_m \left[f_d(y) \right] \right|^2$$

Conical diffraction