

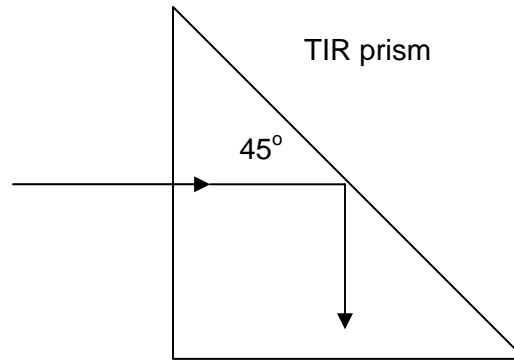
- 1.) A plane wave travels in air with direction cosines given by  $(\alpha, \beta) = (0.4, 0)$ . The maximum amplitude of the electric field is 1 V/m and the wavelength is  $0.6 \mu\text{m}$ .
- (4 pts.) Plot the electric field amplitude as a function of position along the  $z$  axis for several periods of the wave at  $t = 0$ .
  - (3 pts.) What is the velocity of the wave crest projection (peak of the electric field amplitude) that travels in this direction?
  - (3 pts.) Plot the time-averaged irradiance along the  $z$  axis.

2.) A left-hand circularly polarized laser beam illuminates the total-internal reflection prism as shown below. The angle of incidence on the turning surface is  $45^\circ$ . The Fresnel equations given below describe the reflection coefficient for  $s$  and  $p$  polarized light as a function of the angle of incidence, and the index of refraction of the glass is  $N_1 = 1.6$ . Assume that the prism is in air with  $N_2 = 1$  and ignore effects of the entrance and exit surfaces of the prism.

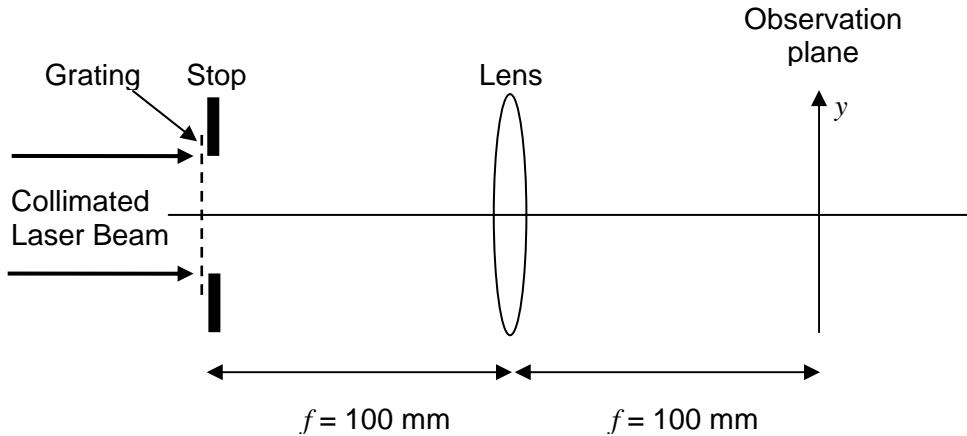
- (5 pts) What is the state of polarization after reflection from the turning surface?  
(Include a calculation of the ellipticity and the rotation angle  $\theta$  of the major axis.
- (5 pts) Sketch the locus plot (trace of the tip of the electric vector in a plane perpendicular to the direction of propagation) for the electric vector after reflection from the turning surface.

$$r_p = \frac{-N_2 \cos \theta_i + N_1 \cos \theta_t}{N_2 \cos \theta_i + N_1 \cos \theta_t}$$

$$r_s = \frac{N_1 \cos \theta_i - N_2 \cos \theta_t}{N_1 \cos \theta_i + N_2 \cos \theta_t}$$



3.) The optical system below is used to analyze a laser beam. The laser passes through a rectangular stop with dimension  $w = 10 \text{ mm}$  in the  $y$  direction. A grating is placed before the stop with period  $\Lambda = 10 \text{ }\mu\text{m}$ . The grating produces diffracted orders in the  $y$  direction. You may simplify the analysis by only calculating values in the  $y$  dimension.



a.) (2 pts) Assume that the grating is removed. If  $\lambda = 0.5 \text{ }\mu\text{m}$  and  $f = 100 \text{ mm}$ , what is the width of the focused spot irradiance in the observation plane? (Calculate the width of the central bright lobe, where width is defined as the distance between points of zero irradiance.)

b.) (Assume that the grating is placed as shown in the figure.)

i.) (1 pt) What is the separation between the  $m = 0$  order and the  $m = 1$  order in the observation plane?

ii.) (1 pt) What is the separation between the  $m = 0$  order and the  $m = 2$  order in the observation plane?

c.) (2 pts) What is the smallest wavelength change that can be measured if the minimum detectable position change of the  $m = 1$  order in the observation plane is one-half the spot diameter? This is the resolution  $\Delta\lambda_{RES}$  of the system.

d.) (2 pts) If a second wavelength is added to the laser beam, how far can the wavelength be increased before the  $m = 1$  order of the second wavelength overlaps the  $m = 2$  order of the  $\lambda = 0.5 \text{ }\mu\text{m}$  component?

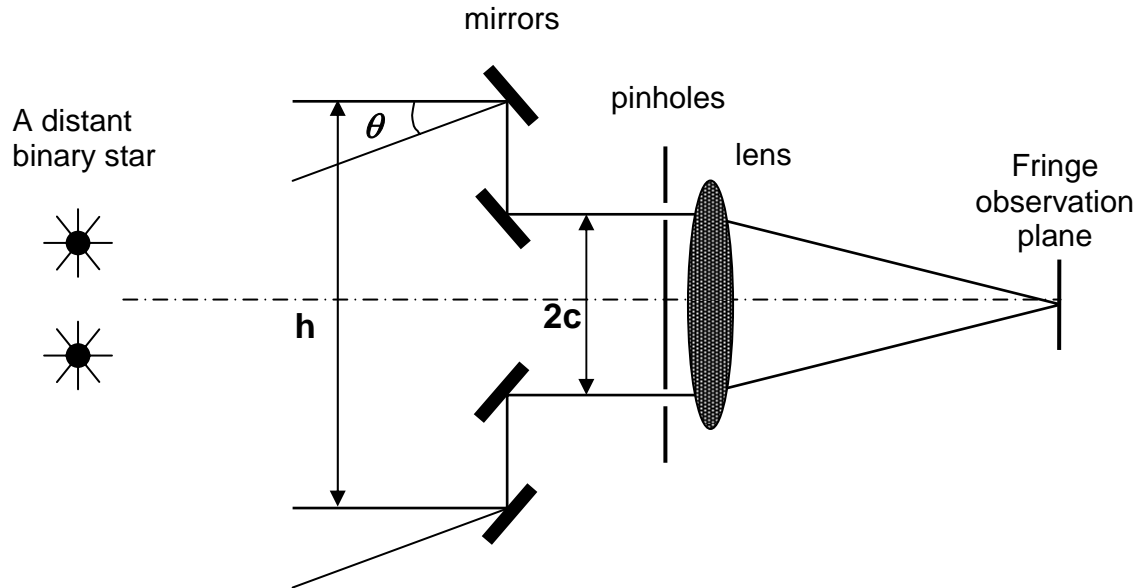
e.) (1 pt) The difference between the wavelength found in (4) and  $\lambda = 0.5 \text{ }\mu\text{m}$  is the free spectral range  $\Delta\lambda_{FSR}$ . What is the free spectral range for this grating?

f.) (1 pt.) What is the Finesse of the system, where Finesse =  $\frac{\Delta\lambda_{FSR}}{\Delta\lambda_{RES}}$ ?

4.) The Michelson Stellar interferometer shown below is used to measure the separation of a particular binary star. The radiance distribution is

$$a^2(\theta) = I_0 \{ \delta(\theta + \Delta\theta) + \delta(\theta - \Delta\theta) \},$$

where  $\pm \Delta\theta$  are the angles between the optical axis and the ray coming from each star, and  $I_0$  is the radiance of each star. Assume that a narrow-band optical filter is used to obtain quasimonochromatic illumination at  $\bar{\lambda} = 850$  nm. If the first zero in fringe visibility occurs at a mirror separation of  $h = 15$  meters, what is the angular separation of the stars?



5.) Two plane waves with the same wavelength, but different amplitudes, interfere in space.

a.) (4 pts) If  $\vec{k}_1 = k\hat{k}_1$  and  $\vec{k}_2 = k\hat{k}_2$ , where  $\hat{k}_1 = 0.1\hat{x} + 0.99499\hat{z}$  and  $\hat{k}_2 = \hat{z}$ , what is the orientation of fringe planes?

b.) (3 pts) What is the minimum spacing between fringe planes if  $\lambda_1 = \lambda_2 = 632.8 \text{ nm}$  ?

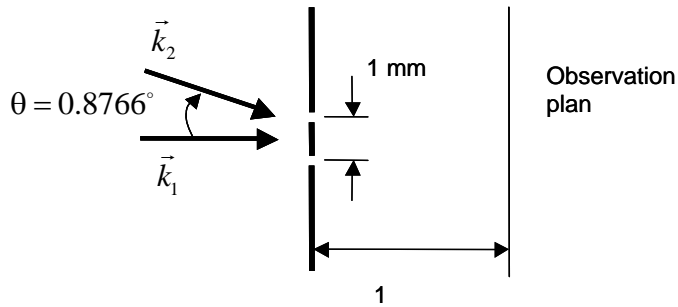
c.) (3 pts) Does visibility depend on orientation of the observation screen?

6.) I am looking through a window screen made up of a square wire mesh of wires at a point source 10 meters away.

a.) (5 pts) If the mesh has wires 0.25 mm apart and the wavelength is 530 nm, what is the spacing of the square arrangement of bright spots that I see located about the point source?

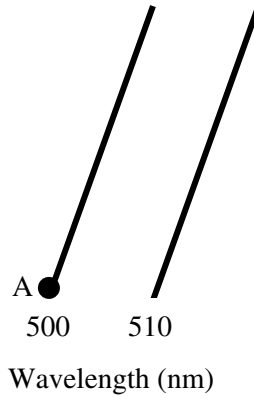
b.) (5 pts) What determines the relative intensities of the bright spots?

7.) (10 pts) Two plane waves illuminate a Young's double pinhole interferometer as shown.  $\lambda_1 = 532 \text{ nm}$  and  $\lambda_2 = 541.4 \text{ nm}$ . The angle between the plane waves is  $0.8766$  degrees.  $\vec{k}_1$  is normally incident onto the aperture plane. The pinhole spacing is  $1 \text{ mm}$ , and the distance from aperture plane to observation plane is  $1 \text{ m}$ . Ignore finite size of the pinholes, and assume that the observation region is limited to a small area around the axis. Sketch the observation-plane irradiance pattern, showing any important features. It may not be possible to show both fine and macroscopic features in the same sketch, so show fine features in one sketch and macroscopic features (features  $> 10$  times larger than the smallest feature) in a second sketch.



8.) The following interferogram is obtained using a FECO interferometer to test a 5 cm x 5 cm square flat mirror.

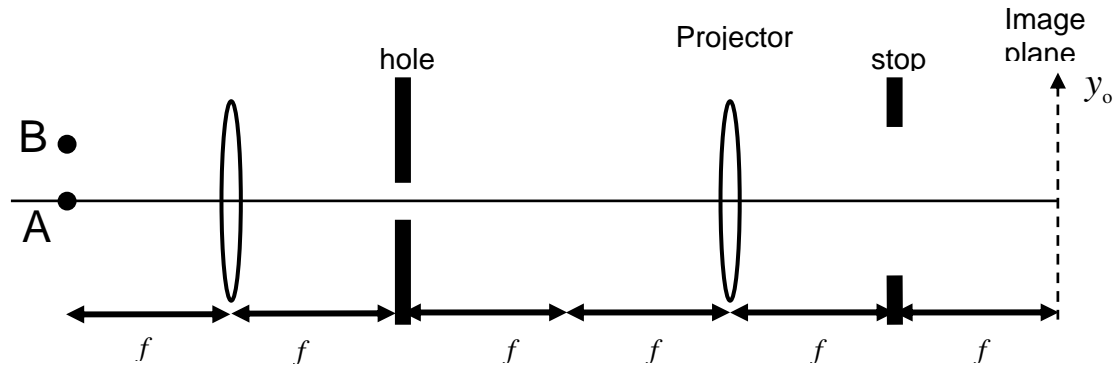
- (3 pts) Sketch the interferometer.
- (4 pts) For point A, what is the separation between the sample being measured and the reference surface?
- (3 pts) What is the wedge angle between the sample being measured and the reference surface?



State any assumptions you are making.

- 9.) Newton's interference fringes are observed with a plano-convex lens resting on a plane glass surface. The radius of curvature of the convex surface is 20 meters. Assume the lens is slowly moved away from the plane glass surface until the separation of the plane glass surface and the convex surface is 0.1 mm.
- a) (5 pts) Do the radii of the fringes increase or decrease?
  - b) (5 pts) How many times does the irradiance at the center of the interference fringes go through a maximum if the incident light is monochromatic and has a wavelength of 500 nm?

10.) A round hole of diameter 1 mm is illuminated by collimating  $\lambda = 0.193 \mu\text{m}$  point sources A and B, as shown below. The focal length of the ideal thin lenses is 50 mm. Various grating-like objects are placed in the hole. The diameter of the stop is 50 mm.

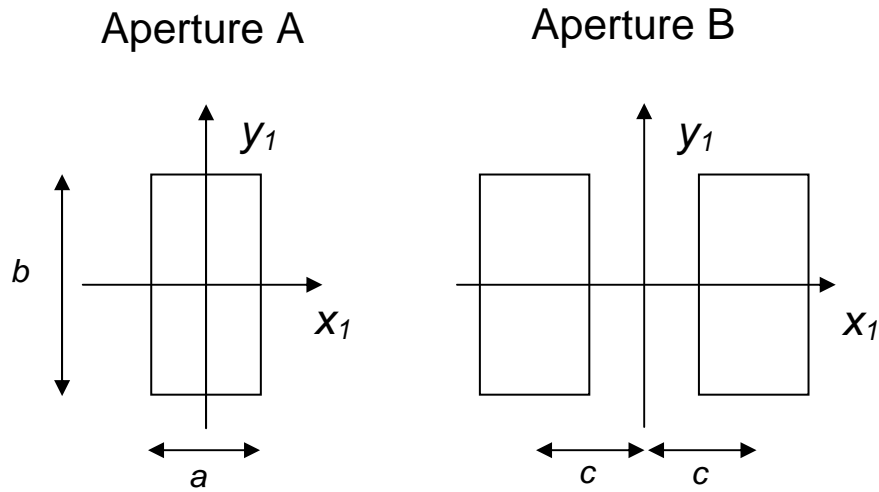


(The drawing is not to scale)

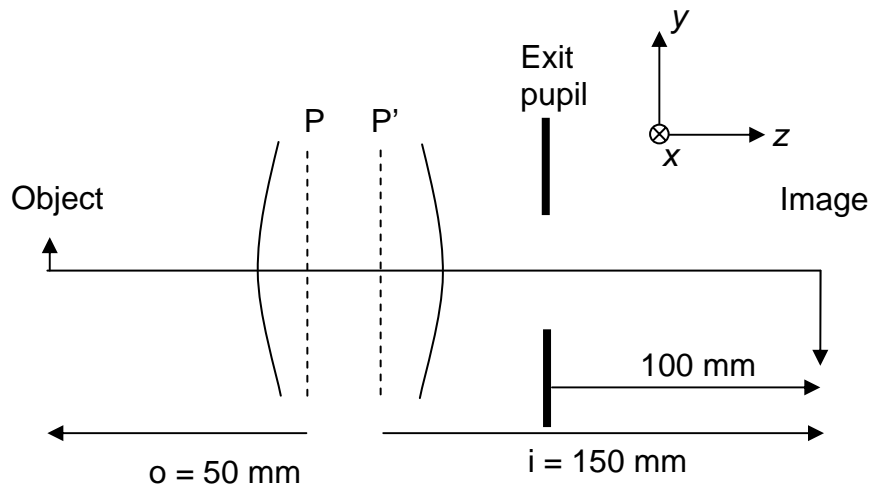
a.) (5 pts) Sketch the CTF in the  $y_0$  direction as a function as spatial frequency if only on-axis point A is used in the source plane.

b.) Point B is off axis by 25 mm. Sketch the CTF in the  $y_0$  direction as a function of spatial frequency if only point B is used in the source plane.

- 1.) The apertures below are illuminated with an on-axis plane wave with wavelength  $\lambda$ . The areas outside the rectangles are opaque.
- a.) (5 pts.) Determine a mathematical expression for the Fraunhofer diffraction pattern (electric field amplitude) from Aperture A at a distance  $z_0$ .
- b.) (5 pts.) Determine a mathematical expression for the Fraunhofer diffraction pattern (electric field amplitude) of Aperture B at distance  $z_0$  using Babinet's Principle, the shift theorem, and the Fraunhofer diffraction pattern from Aperture A.

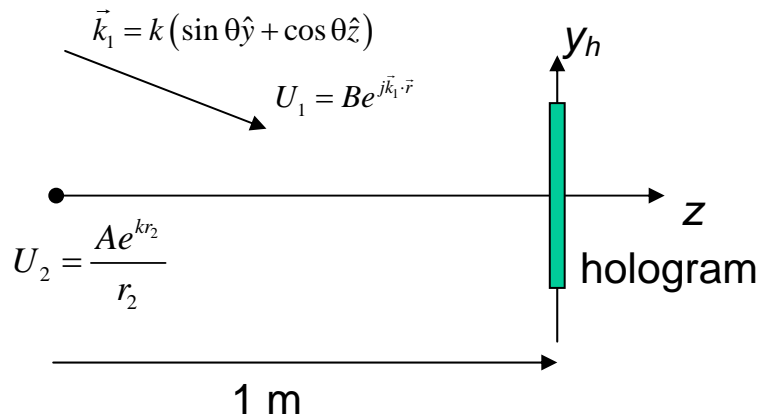


- 2.) a.) (5 pts.) Plot the PSF irradiance along  $(x_o, 0)$  and  $(0, y_o)$  of the system pictured below, which uses Aperture B of Problem (1) in the exit pupil. You may normalize the PSF to its peak value. Parameters are:  $a = 1.0$  mm,  $b = 4.0$  mm,  $c = 1.0$  mm and  $\lambda = 0.5$   $\mu\text{m}$ .
- b.) (5 pts.) Plot the coherent transfer function (CTF) in image space, where plot profiles are  $H(\xi, 0)$ ,  $H(0, \eta)$  and  $H(20\text{lp/mm}, \eta)$ . Label all significant spatial frequencies.
- c.) (5 pts.) Plot the optical transfer function (OTF) assuming a luminous object for profiles  $\text{OTF}(\xi, 0)$  and  $\text{OTF}(0, \eta)$ .
- d.) (5 pts.) A half-wave plate, which is described by the Jones matrix  $M_{\frac{\lambda}{2}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ , is placed over the right-hand-side rectangle in Aperture B. Consider two states of input polarization,  $E_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $E_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ . Plot  $\text{OTF}(\xi, 0)$  and  $\text{OTF}(0, \eta)$  for each state of input polarization.

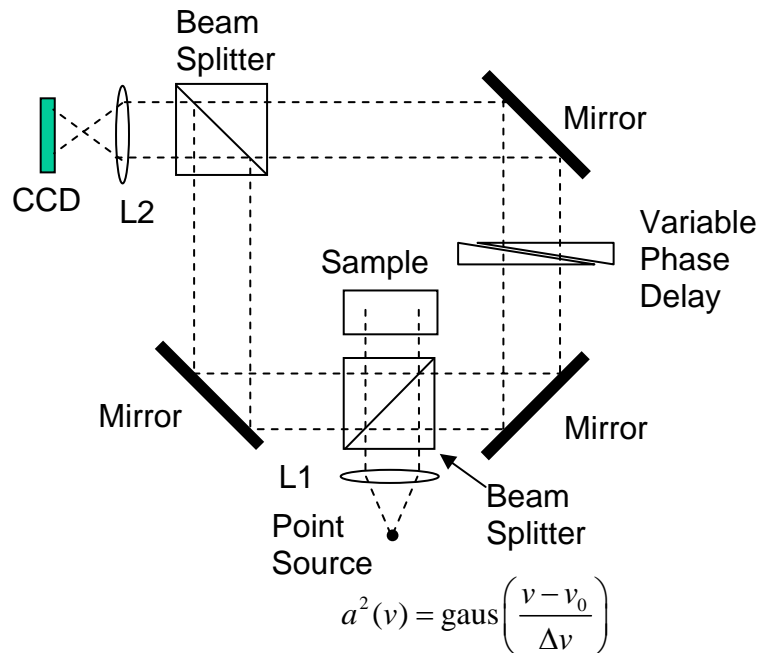


3.) Consider the holographic recording geometry shown below, where a point source ( $U_2$ ) is combined with a plane-wave reference ( $U_1$ ) to expose the hologram.  $\lambda = 0.5 \mu\text{m}$ .

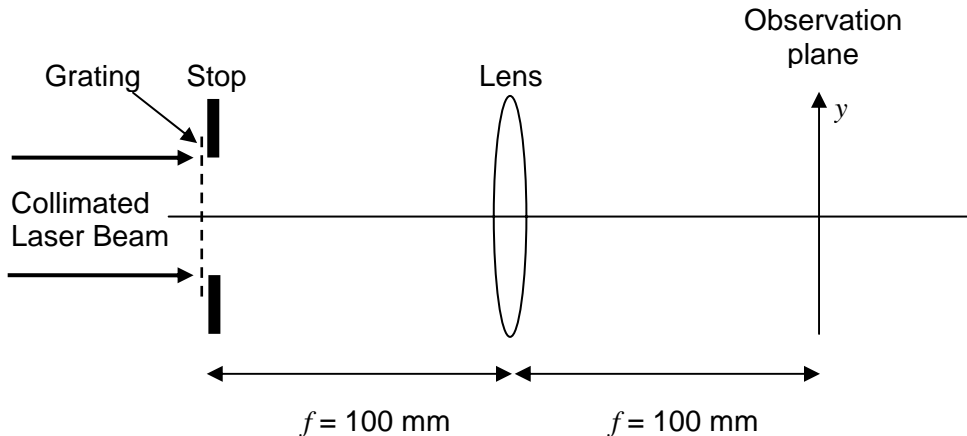
- a.) (5 pts.) Write an expression  $I(x_h, y_h)$  that is proportional to the irradiance that exposes the hologram.
- b.) (5 pts.) Assume that  $A/r_2 = B$  at the hologram. Plot the fringe pattern  $I(x_h, y_h)$  along  $I(x_h, 0)$  and  $I(0, y_h)$  for  $\theta = 0$ . Limit the extent of your graph to  $-3 \text{ mm} \leq y_h \leq 3 \text{ mm}$  and  $-3 \text{ mm} \leq x_h \leq 3 \text{ mm}$ .
- c.) Assume that the hologram is recorded with  $\theta = 0$  so that, after development, illumination by the same reference wave produces  $U^+ = K_2 U_1 - K_3 U_2^* U_1^2 - K_3 I_1 U_2$  in transmission.
  - i.) (5 pts.) Where does the  $K_3 U_2^* U_1^2$  term come to focus? Determine the position and draw a sketch. Show  $U_1$  and the transmitted  $K_3 U_2^* U_1^2$  term in your sketch.
  - ii.) (5 pts.) Where does the  $K_3 I_1 U_2$  term appear to originate from? Determine the position and draw a sketch. Show  $U_1$  and the transmitted  $K_3 I_1 U_2$  term in your sketch.
  - iii.) (5 pts.) Describe the similarities and differences of this hologram compared to a binary amplitude zone plate with  $a_{ZP1} = \frac{1}{\sqrt{2}}$  mm.
- d.) (5 pts.) Repeat part (b) for  $\theta = 5^\circ$ . Limit the extent of your graph to  $-0.1 \text{ mm} \leq y_h \leq 0.1 \text{ mm}$  and  $-3 \text{ mm} \leq x_h \leq 3 \text{ mm}$ .
- e.)
  - i.) (5 pts.) Repeat (c)(i) for  $\theta = 5^\circ$ .
  - ii.) (5 pts.) Repeat (c)(ii) for  $\theta = 5^\circ$ .



- 4.) (10 pts.) The system pictured below is used to obtain information about the distribution of defects throughout the depth of the glass sample. Lens L1 collimates the point source, which illuminates the sample. Lens L2 images reflected light from the sample onto the CCD camera. An interference pattern is generated at the CCD from combination of the two interferometer beams. Because the defects produce only weak reflections, the interference can improve contrast in the image. However, if a narrow-bandwidth laser source is used, it is difficult to separate information about depth of the defects, due to the fact that good contrast is obtained throughout the entire depth of the sample. To improve differentiation of defect distributions at different depths, a wider-bandwidth point source is used with  $a^2(\nu) = \text{gaus}\left(\frac{\nu - \nu_0}{\Delta\nu}\right)$ , where  $\bar{\lambda} = 530 \text{ nm}$  and the refractive index of the sample is 1.5. A variable phase delay is added to the reference arm in order to equalize path lengths. When the path lengths are equal for a certain depth in the sample, good contrast is achieved. Reflections from defects at other depths do not have high contrast, due to a reduction of temporal coherence. What is  $\Delta\lambda$  of the new source that is required to differentiate between defects spaced  $5 \mu\text{m}$  apart in depth?

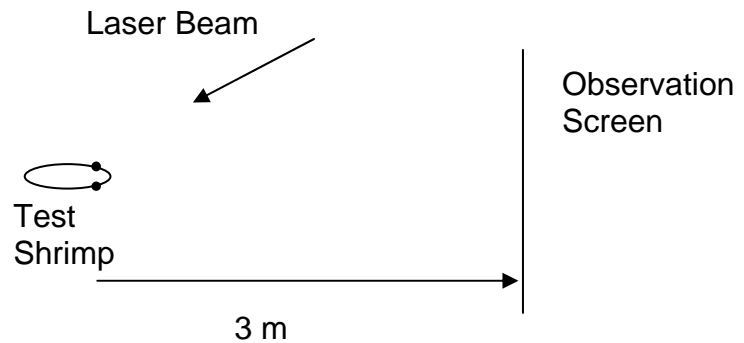


5.) (10 pts.) The optical system below is used to analyze a laser beam. The laser passes through a rectangular stop with dimension  $w = 5 \text{ mm}$  in the  $y$  direction. A grating is placed before the stop with period  $\Lambda = 10 \text{ }\mu\text{m}$ . The grating produces diffracted orders in the  $y$  direction. You may simplify the analysis by only calculating values in the  $y$  dimension.



- a.) (2 pts) Assume that the grating is removed. If  $\lambda = 1 \text{ }\mu\text{m}$  and  $f = 100 \text{ mm}$ , what is the width of the focused spot in the observation plane? (Calculate the width of the central bright lobe, where width is defined as the distance between points of zero irradiance.)
- b.) (Assume that the grating is placed as shown in the figure.)
  - i.) (1 pt) What is the separation between the  $m = 0$  order and the  $m = 1$  order in the observation plane?
  - ii.) (1 pt) What is the separation between the  $m = 0$  order and the  $m = 2$  order in the observation plane?
- c.) (2 pts) What is the smallest wavelength change that can be measured if the minimum detectable position change of the  $m = 1$  order in the observation plane is one-half the spot diameter? This is the resolution  $\Delta\lambda_{RES}$  of the system.
- d.) (2 pts) If a second wavelength is added to the laser beam, how far can the wavelength be increased before the  $m = 1$  order of the second wavelength overlaps the  $m = 2$  order of the  $\lambda = 1 \text{ }\mu\text{m}$  component?
- e.) (1 pt) The difference between the wavelength found in (d) and  $\lambda = 1 \text{ }\mu\text{m}$  is the free spectral range  $\Delta\lambda_{FSR}$ . What is the free spectral range for this grating?
- f.) (1 pt.) What is the Finesse of the system, where 
$$\text{Finesse} = \frac{\Delta\lambda_{FSR}}{\Delta\lambda_{RES}} ?$$

- 6.) (10 pts.) The two eyes of a shrimp act like very good point sources when illuminated with a laser beam. (See the picture below of a happy shrimp.) The separation between their eyes increases as they grow. It is desirable for shrimp farmers to know how large shrimp are by some method. You are hired by a large shrimp company as a consultant. You remember from your 505R class that the diffraction pattern from two point sources produces a specific fringe pattern that is a function of the separation of the sources. In the simplified geometry below, you set up an experiment to measure the separation of a shrimp's eyes by measuring the separation of the fringes in the center of an observation screen 3 m away from the test shrimp. You measure a fringe separation of 1.9 mm when a HeNe laser beam ( $\lambda = 632.8 \text{ nm}$ ) illuminates the shrimp. What is the separation between the eyes of the test shrimp?



Compare and contrast the optical system transfer function (both CTF and OTF) with the transfer function of free space. Provide examples, other than what are in the notes or in the homework problem listings. Original thought and interpretation will be scored highly, rather than simply restating the notes. Limit your text to no more than 2500 words, excluding figures, tables, and equations. The paper does not need to be typed, but it is appreciated. Use of outside reference material, other than the notes and lectures, is not required, but, if you use outside reference material, provide a bibliography. The format of the paper should follow that used for submission to a journal publication, with abstract, introduction, body, results and conclusions. If you type your paper, please use double-spaced lines.