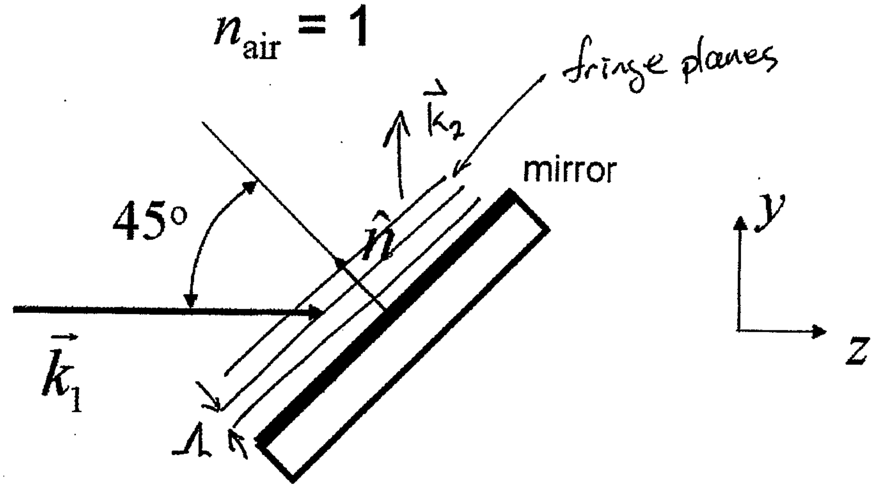
1.) A polarized plane wave illuminates a mirror, as shown below. The angle of incidence on the mirror is 45°. Parameters of the problem include:

Parameter	Value	Description	Units
$\vec{k}_1$	$k_0\hat{z}$	wave vector	m^{-1}
k_0	$2\pi/\lambda_0$	wave vector amplitude	m^{-1}
λ_0	550e-9	free-space wavelength	m
$\hat{n}$	$\frac{\hat{y}-\hat{z}}{\sqrt{2}}$	surface normal unit vector	-
n_{air}	1	refractive index of air	-
N_{mirror}	1+6.6j	complex refractive index of the mirror	-
$\hat{a}_1$	$\frac{\hat{x}+j\hat{y}}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\j\end{pmatrix}$	state of polarization for incident plane wave	-



a.) (2.5 pts) Determine the orientation of the wave reflected from the mirror.

$\uparrow \hat{k}_2 = \hat{y}$ (Snell's Law)

/2.5

b.) (5 pts) Determine the state of polarization of the reflected beam using the convention found from the "Chapter 3 - Polarization Convention for ps and TR Jones Matrices" link on the 505R class web site. Express your answer as a Jones vector with respect to the xy coordinates of the reflected beam.

$J_2 = M J_1$

(using computer program attached)

$r_s = -0.9487 - 0.2i$

$r_p = -0.8601 - 0.3794i$

$M = \begin{pmatrix} r_s & 0 \\ 0 & -r_p \end{pmatrix}$

$J_2 = \begin{pmatrix} -0.6709 - 0.1414i \\ -0.2683 + 0.6082i \end{pmatrix}$

$J_2 / |J_2| = \begin{pmatrix} -0.7025 - 0.1480i \\ -0.2809 + 0.6369i \end{pmatrix}$

$= \frac{1}{-0.7025 + 0.1480i} \begin{pmatrix} 1 \\ 0.9696\, e^{-j0.434\pi} \end{pmatrix}$

c.) (2.5pts) Write the answer for part (b) in terms of unit vector $\hat{a}_2$ of the reflected beam. That is, the y component of part (b) becomes the $-z$ component of the new vector, and the x component remains the same. Be sure to normalize the unit vector.

$\hat{a}_2 = (-0.7025 - 0.1480i)\hat{x} + 0\hat{y} - (-0.2809 + 0.6369i)\hat{z}$

d.) (5pts) Find the reduction in visibility due to polarization of the resulting fringe pattern by taking the absolute value of the dot product of $\hat{a}_1$ and $\hat{a}_2$. [Remember that the dot (or scalar) product for complex vectors is defined as $\hat{a}_1 \bullet \hat{a}_2 = a_{1x}^* a_{2x} + a_{1y}^* a_{2y} + a_{1z}^* a_{2z}$, where the asterisk (*) denotes complex conjugate. Note: This calculation is a little different than the way I used the dot product on slides 4A-10 and 4A-11. It is more mathematically correct here.]

$\hat{a}_1 = \frac{1}{\sqrt{2}}\hat{x} + \frac{j}{\sqrt{2}}\hat{y} + 0\hat{z}$

$|\hat{a}_1 \cdot \hat{a}_2| = \left| \left(\frac{1}{\sqrt{2}}\right)(-0.7025 - 0.1480i) + 0 + 0 \right|$

$= 0.5077$

e.) (2.5pts) Determine the orientation of the fringe pattern with respect to the mirror surface. Draw the fringes on the figure above in their correct orientation.

|| to mirror surface

f.) (2.5pts) What is the minimum spacing of the fringes?

$\Lambda_{min} = \frac{\pi}{|\vec{K}_m|} = \frac{\pi}{\frac{2\pi}{\lambda}\left|\frac{1}{2}(\hat{k}_1 - \hat{k}_2)\right|} = \frac{\lambda}{|\hat{z} - \hat{y}|}$

$= \lambda/\sqrt{2} = 389 \times 10^{-9}\ \mathrm{m}$

17.5

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thetai=45*pi/180;
Nt=1+6.6j;
Ni=1;
J1=1/sqrt(2)*[1;j];

thetat=asin(Ni/Nt*sin(thetai));

rs = (Ni*cos(thetai)-Nt*cos(thetat))/(Ni*cos(thetai)+Nt*cos(thetat))
rp = (-Nt*cos(thetai)+Ni*cos(thetat))/(Nt*cos(thetai)+Ni*cos(thetat))
disp(['rs = ' num2str(rs) ' Rs = ' num2str(abs(rs).^2)])
disp(['rp = ' num2str(rp) ' Rp = ' num2str(abs(rp).^2)])

M=[rs 0;0 -rp];

J2=M*J1

J2_norm= J2./sqrt(sum(abs(J2).^2));

disp(['magnitude of J2(2): ' num2str(abs(J2_norm(2))) ' angle of J2(2)/pi: ' num2str(angle
(J2_norm(2))/pi) ])
disp(['Reflected power = ' num2str(sum(abs(J2).^2))])

J2_3D           = [J2_norm(1) 0 -J2_norm(2)];
J1_3D           = [J1(1) J1(2) 0];

abs_dot_product = abs(dot(J2_3D,J1_3D))
```

2.) The fringe pattern of problem (1) is observed as the mirror is moved along the $+z$ direction.

a.) (5 pts) Do the fringes move relative to the global coordinates (xyz)? yes.

b.) (5pts) Do the fringes move relative to the mirror surface? no

The fringe pattern position relative to the mirror depends on the phase difference of the two beams, no the absolute phase. The phase difference is controlled by the mirror reflectivity, not the absolute position of the mirror.

/10

3.) Define the following terms mathematically and describe how the terms are used to determine properties of a light field.

a.) (2.5 pts) PSF of free space $h(P_0;P_1) = \left(-\frac{j}{\lambda} + \frac{1}{2\pi r_{01}}\right)\gamma_z \frac{e^{jkr_{01}}}{r_{01}}$

$h(P_0;P_1)$ is used with coherent light fields in a supperposition integral to relate the field distribution between two planes.

b.) (2.5 pts) Coherent PSF $cPSF = h(x_0, y_0) = F_{\frac{y_0}{\lambda z_2}} F_{\frac{x_0}{\lambda z_2}} \{H(\xi,\eta)\} = F_{\frac{y_0}{\lambda z_2}} F_{\frac{x_0}{\lambda z_2}} \left[P(-\lambda z_2 \xi, -\lambda z_2 \eta)\right]$

where $P(x_s, y_s)$ is the exit pupil and $H(\xi,\eta)$ is the coherent transfer function.

For a LSI imaging system* and monochromatic point-source illumination, cPSF is the image field distribution.

cPSF can be used to find the image field for on axis, pw illumination by convolution with the geometric (ideal) image field.

c.) (2.5 pts) Incoherent PSF

$iPSF = |h(x_0, y_0)|^2$ (square magnitude of cPSF)

Object must be luminous or illuminated with a wide spectrum of quasimonochromatic plane waves.

For an LSI imaging system*, a pinhole object produces the iPSF. iPSF can be used to find the image irradiance by a convolution integral with the geometrical (ideal) image irradiance.

d.) (2.5 pts) Transfer function of free space

FSTF = Fourier transform of (a) $= e^{jk\gamma z}$

Used with coherent light fields to relate the fourier transforms of the fields on two planes that are separated by distance z.

e.) (2.5 pts) Coherent transfer function $CTF = H(\xi,\eta) = P(-\lambda z_2 \xi, -\lambda z_2 \eta)$

For a LSI imaging system*, the CTF relates the spatial frequencies of the image and object fields when the object is illuminated with an on-axis plane wave.

f.) (2.5 pts) Incoherent transfer function $ITF = OTF = \mathcal{H}(\xi,\eta) = H(\xi,\eta) \star\star H^*(\xi,\eta)$ (can be normalized)

For a LSI imaging system* the ITF relates the spatial frequencies of the image and object irradiance when the object is luminous or illuminated with a wide spectrum of quasimonochromatic plane waves.

* Entrance pupil must be in the Fraunhofer region of the object.

/15

4.) A 1 mm diameter hole is illuminated with a unit amplitude $\lambda = 500$ nm normally incident plane wave.

a.) (2.5 pts) Graph the on-axis irradiance of the Fresnel diffraction pattern as a function of Fresnel number for observation distances from 0.5 m to 50 m. Use a log-log scale for the axes, and the horizontal axis should show lower Fresnel numbers (farther observation distances) on the right side. Use separate paper for your graph. (see attached program and chart)

b.) (2.5 pts) Graph the on-axis irradiance of the diffraction pattern assuming Fraunhofer diffraction (even though the calculation may not be valid) as a function of Fresnel number for observation distances from 0.5 m to 50 m. Use a log-log scale for the axes, and the horizontal axis should show lower Fresnel numbers (farther observation distances) on the right side. Plot the graph overlaid on top of the graph for part (a). (see attached program and chart)

c.) (2.5 pts) Graph the percentage difference between part (a) and part (b) as a function of Fresnel number for observation distances from 0.5 m to 50 m. Use a log-log scale for the axes, and the horizontal axis should show lower Fresnel numbers (farther observation distances) on the right side. Percentage difference is defined as $\left|\frac{I_{Fresnel} - I_{Fraunhofer}}{I_{Fresnel}}\right| \times 100\%$. Use separate paper for your graph.
(see attached program and chart)

d.) (2.5 pts) Based on the graph in part (c), approximately what Fresnel number is required before the error in on-axis irradiance reduces to 1%?

about 0.1 (see chart)

e.) (5 pts) Your graphs for parts (a) and (b) should decrease with decreasing Fresnel number. If this trend continues for increasing observation distances, the limit will approach zero. Describe physically how this result is consistent with conservation of power.

Although the peak values decrease, the energy becomes more distributed over a wider area.

/15

```
r          = 0.5e-3;
z2min      = 0.5;
z2max      = 50;
z1         = 1e10;
lambda     = 500e-9;
Nsamples   = 2000;

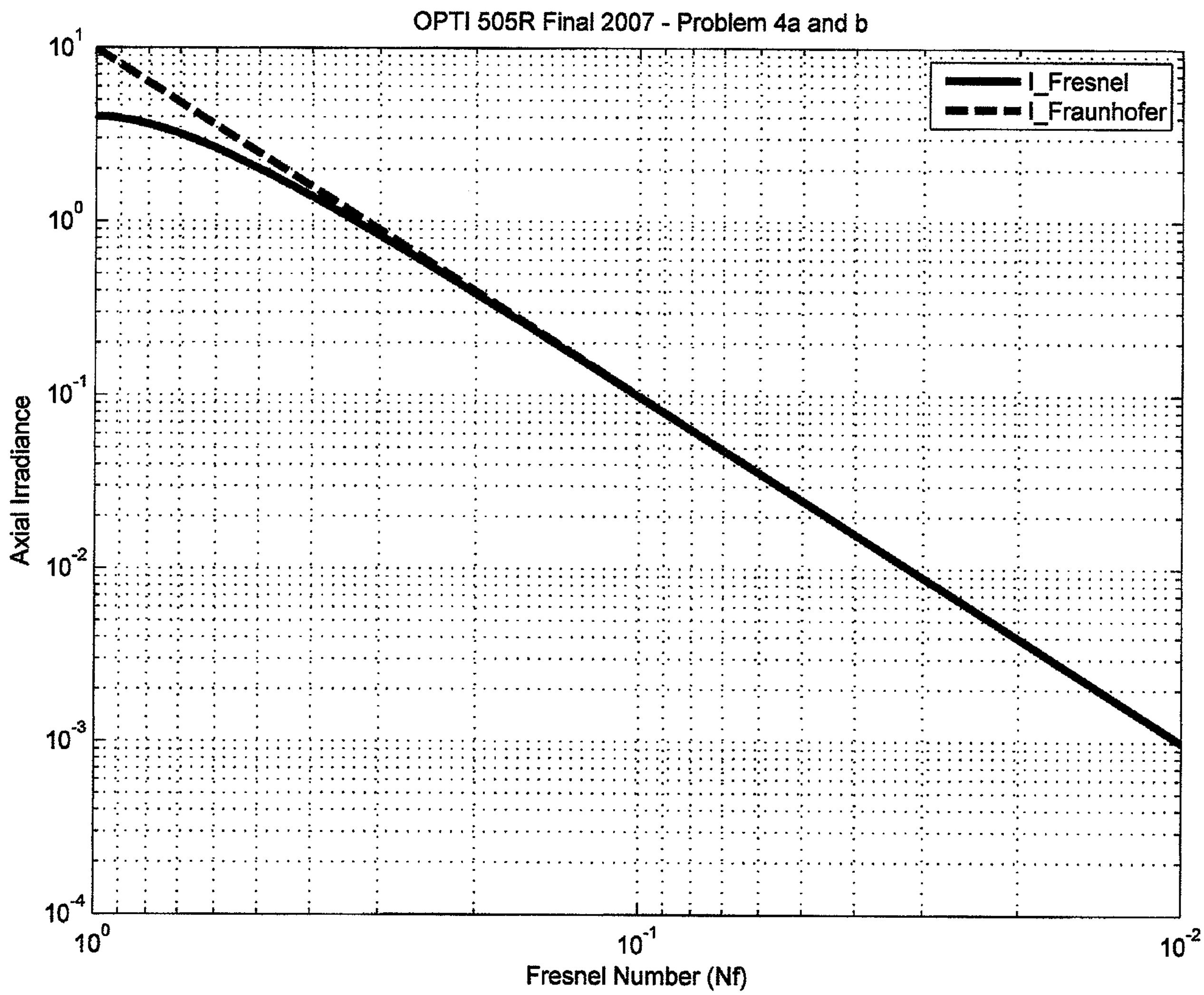
z2_vec     = linspace(z2min,z2max,Nsamples);
L_vec      = z1*z2_vec./(z1+z2_vec);

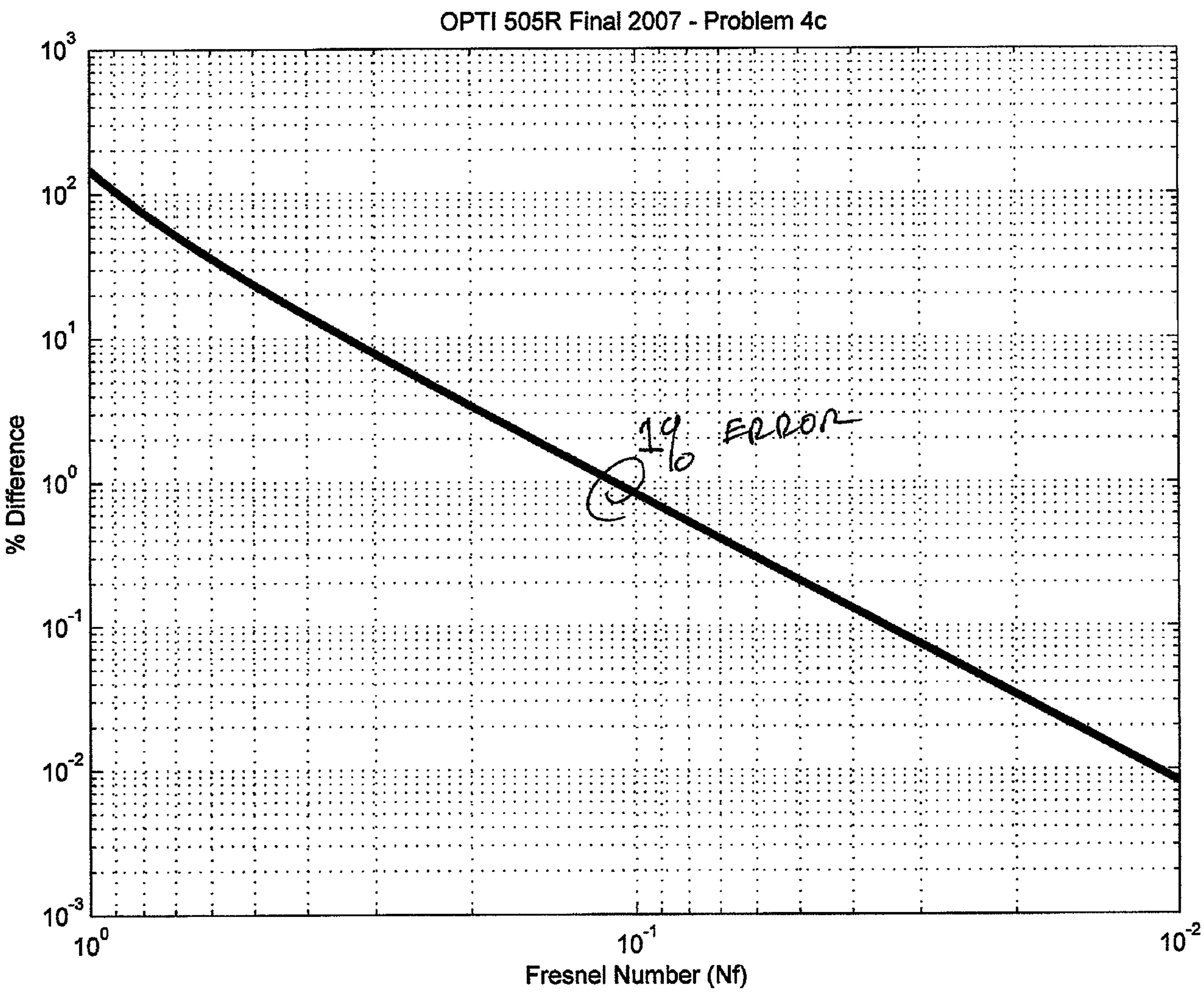
Nf_vec     = r^2/lambda./L_vec;

I_FN       = 4*sin(pi*Nf_vec/2).^2;
I_Fraun    = (pi*r^2 /lambda./z2_vec).^2;

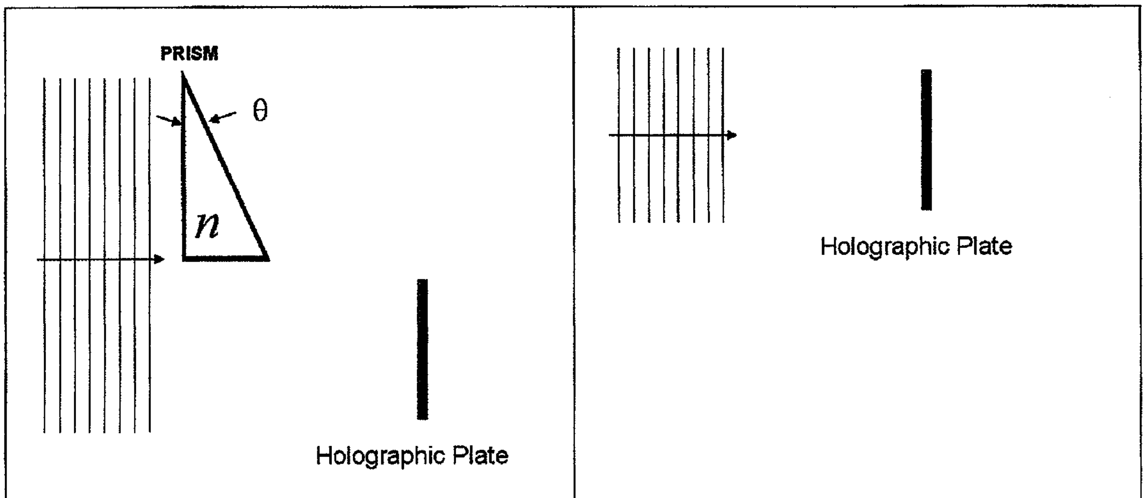
figure(10);loglog(Nf_vec,I_FN,Nf_vec,I_Fraun);grid
legend('I\_Fresnel','I\_Fraunhofer')
xlabel('Fresnel Number (Nf)')
ylabel('Axial Irradiance')
set(gca,'Xdir','reverse')
title('OPTI 505R Final 2007 - Problem 4a and b')

figure(11);loglog(Nf_vec,abs(I_FN-I_Fraun)./I_FN*100);grid;
xlabel('Fresnel Number (Nf)')
ylabel('% Difference')
set(gca,'Xdir','reverse')
title('OPTI 505R Final 2007 - Problem 4c')
```


OPTI 505R Final 2007 - Problem 4a and b
I_Fresnel
I_Fraunhofer
Axial Irradiance
Fresnel Number (Nf)
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{0}
10^{-1}
10^{-2}


OPTI 505R Final 2007 - Problem 4c
% Difference
Fresnel Number (Nf)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^0
10^{-1}
10^{-2}
1% ERROR

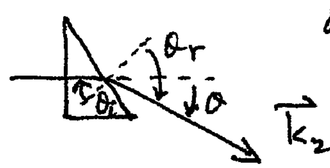
5.) (10 pts) A $\lambda = 488$ nm collimated plane wave illuminates a prism as shown in A below. The apex angle of the prism $\theta = 30$ degrees. Part of the beam is refracted by the prism, which has refractive index $n = 1.532$. An amplitude-type holographic plate is placed in the overlap region of the two beams. A hologram is recorded, and a $\lambda = 632$ nm plane-wave reconstruction beam is used to illuminate the processed hologram, where the reconstruction geometry is shown in B below. Describe the angular orientation of the beams that result from transmission through the hologram. State any assumptions that you make.



A. Exposure	B. Reconstruction

RECORDING:

Through prism:

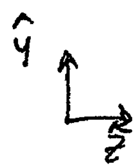


$\theta_i = 30^\circ$

$\theta_R = \sin^{-1}(1.532 \sin 30^\circ)$

$= 50^\circ$

$\therefore\ \theta = \theta_R - 30^\circ = 20^\circ$



$\hat{k}_2 = -\sin 20^\circ \hat{y} + \cos 20^\circ \hat{z}$

$\hat{k}_1 = \hat{z}$

$$\Lambda = \frac{\pi}{|\vec{K}_m|} = \frac{\cancel{\pi}}{\frac{\cancel{2\pi}}{\lambda} \cdot \frac{1}{\cancel{2}} |\hat{k}_1 - \hat{k}_2|} = \frac{\lambda}{|\sin 20^\circ \hat{y} + (1 - \cos 20^\circ)\hat{z}|}$$

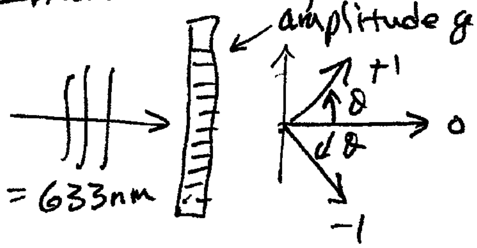
$$= \frac{\lambda}{0.3473} = 1.405 \times 10^{-6}\ \text{m}$$

$$\Lambda' = \frac{\Lambda}{\cos 10^\circ} = 1.4267 \times 10^{-6}\ \text{m}$$

Reconstruction:

$t(y) = 1 + \cos(2\pi y / \Lambda')$

Illuminated by $\lambda = 633$ nm beam $\Rightarrow$



$90 - \theta = 90 - \cos^{-1}(\lambda / \Lambda')$

$= 90 - 63.661^\circ$

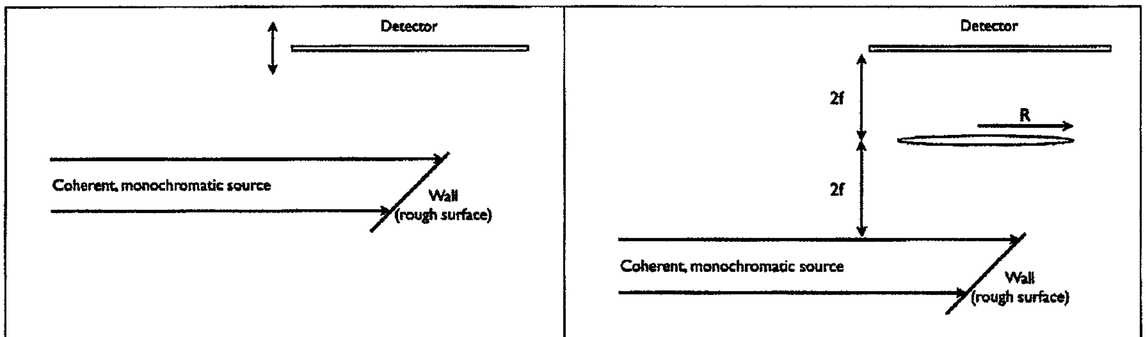
$= 26.339^\circ$

/10

6.) Imagine a monochromatic, coherent beam reflected off of a wall (i.e., a rough surface) and illuminating onto a detector (see A below). A speckle pattern will emerge on this detector. You may assume that the detector is in the far field (Fraunhofer region) of the spot illuminating the wall.

(a) (5 pts) You take many images with the detector, each at a different distance away from the wall. How does the speckle pattern change? Why?

(b) (5 pts) Now consider putting a lens with focal length f and radius R in between the wall and the detector, as shown in B below. How does this lens affect the speckle pattern? Compare the sizes of the speckles for two lenses of focal length f, one with radius R, and one with radius 0.5R. You may assume that the lens is in the Fraunhofer region of the spot illuminating the wall.



A. First experimental setup. B. Second experimental setup

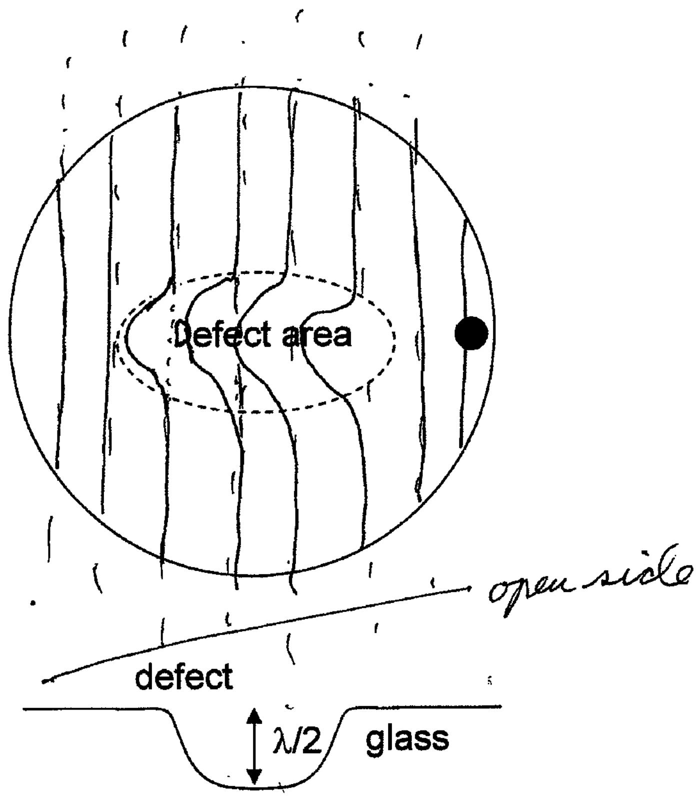
(a) As the detector moves farther from the wall, the speckle size increases, since the speckle size is proportional to the Fourier transform of the size of the spot illuminating the wall. The transform variable is $\frac{x_0}{\lambda z}$, where z is the distance to the wall from the detector.

(b) The PSF of the imaging system now determines the size of the speckle. The system with lens radius R will exhibit speckle half the diameter of the system with lens radius 0.5R.

/10

7.) (20pts) An almost flat surface has a hole defect as shown in the figure. A perfectly flat glass plate is placed on top of the surface, and Fizeau fringes are observed in reflection. Draw the fringes observed on the circular top view if the maximum wedge is 4λ across the part and the number of fringes decreases when light pressure is applied at the black dot.

Top view



/20