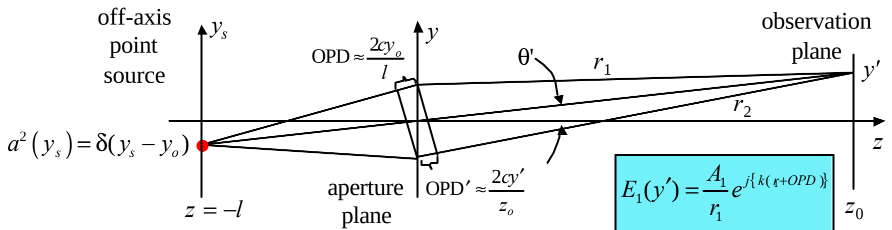
Coherence
(Chapter 5 – Part B)

Diffraction and Interferometry

YDPI – Off-Axis Source Point



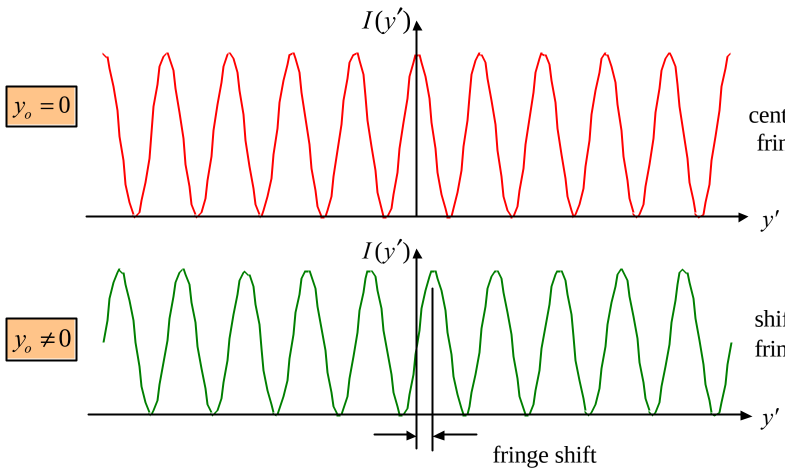
$$E_1(y') = \frac{A_1}{r_1} e^{j\{k(r_1 + OPD)\}}$$

$$E_2(y') = \frac{A_2}{r_2} e^{jkr_2}$$

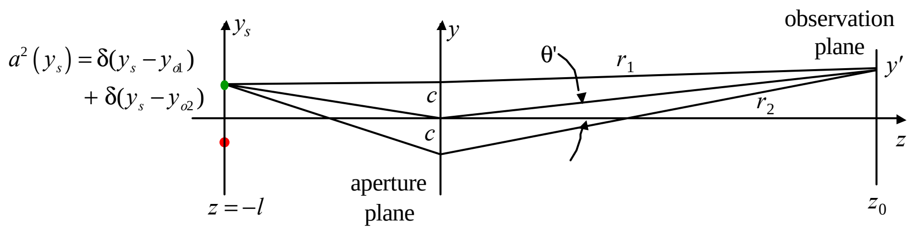
$$\begin{aligned}
I(y') &= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\{k(r_2 - r_1 - OPD)\} \\
&= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\{k(OPD' - OPD)\} \\
&= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left\{k\left(\frac{2cy'}{z_o} - OPD\right)\right\} \\
&= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left\{\frac{k2c}{z_o}\left(y' - \frac{z_o OPD}{2c}\right)\right\}
\end{aligned}$$

The net result of a source shift is a shift in the center of the fringe pattern, where shift $= \frac{z_o OPD}{2c} = \frac{z_o y_o}{l}$

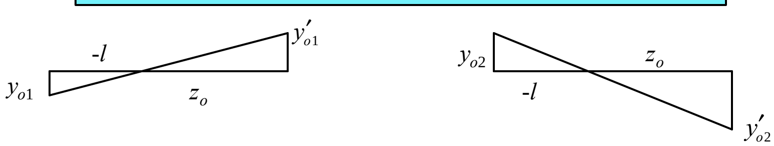
YDPI – Off-Axis Source Point



YDPI – Two Off-Axis Source Points



The second source point contributes its own fringe pattern in the observation plane that is shifted with respect to the fringe pattern from the first source point.



YDPI – Two Off-Axis Source Points

• We know that the beat-frequency term between the two sources is not observable if the frequency difference between the sources is greater than the detection bandwidth. In terms of wavelength,

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta\nu$$

• For example, a 1GHz difference frequency yields a wavelength difference of

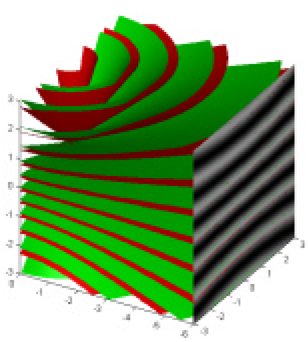
$$\Delta\lambda = \frac{\left(0.5\times10^{-6}\right)^2}{3\times10^8}(10^9) = 1.7\times10^{-12}\text{m} = 0.017\ \text{Å}$$

•Effects, like fringe period, for this wavelength difference may be very difficult to measure in the observation plane.

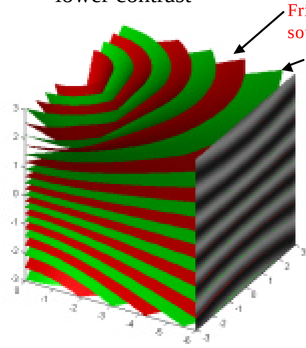
• Therefore, a very narrow bandwidth distributed source can be considered as a collection of independent radiators for which the resulting fringe fields from each source point add in irradiance.

YDPI – Two Off-Axis Source Points

Small source separation yields closely spaced fringe fields and high contrast

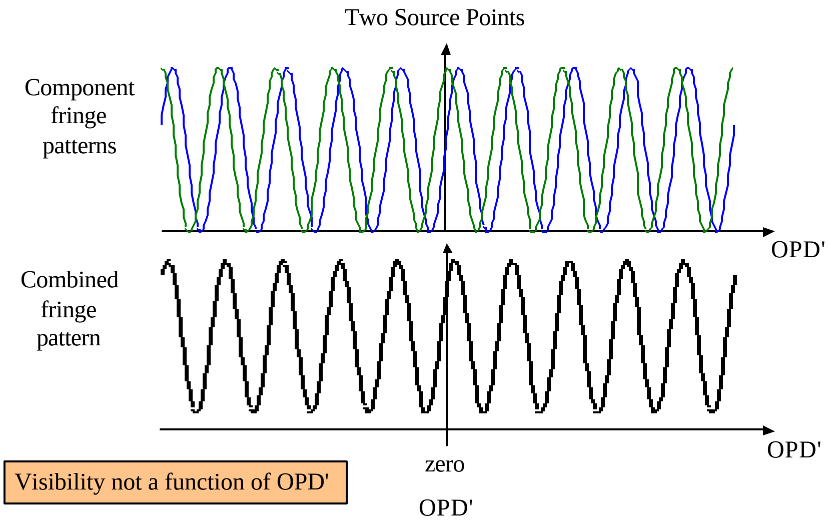


Larger source separation yields more widely spaced fringe fields and lower contrast



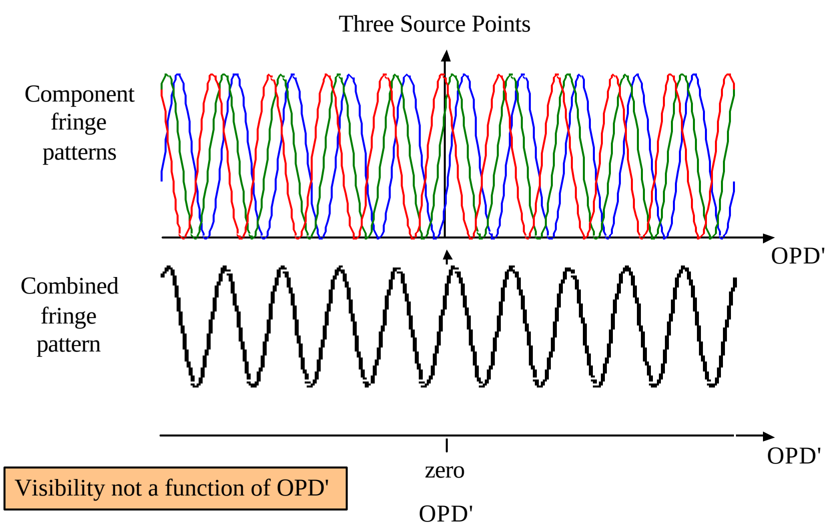
The effect of varying the source separation is to change the contrast.

YDPI – Multiple Source Points



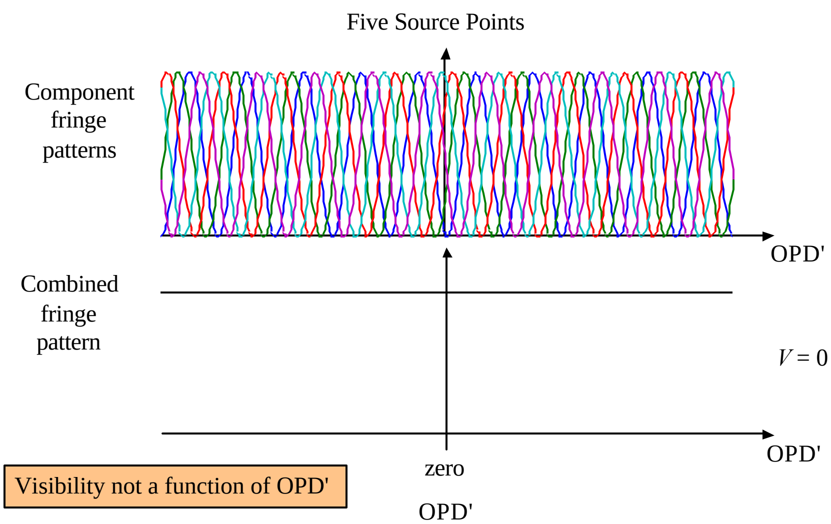
Visibility not a function of OPD'

YDPI – Multiple Source Points



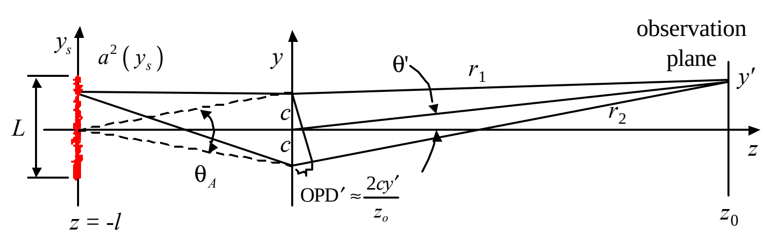
Visibility not a function of OPD'

YDPI – Multiple Source Points



Visibility not a function of OPD'

YDPI Extended Source



$$I\left(\mathrm{OPD}',y_s\right)=K_1a^2(y_s)+K_2a^2(y_s)+2\sqrt{K_1K_2}\,a^2(y_s)\cos\left\{k\left(\mathrm{OPD}'-\frac{2cy_s}{l}\right)\right\}$$

$$I\left(\mathrm{OPD}'\right)=\int_{source} I\left(\mathrm{OPD}',y_s\right)dy_s$$

K's are diffraction constants

YDPI Extended Source

$$I(\text{OPD}') = \int_{source} I(\text{OPD}', y_s)\, dy_s$$

$$= \int_{source} \left[K_1 a^2(y_s) + K_2 a^2(y_s) + 2\sqrt{K_1 K_2}\, a^2(y_s) \cos\left\{ k\left(\text{OPD}' - \frac{2cy_s}{l} \right) \right\} \right] dy_s$$

$$= (K_1 + K_2) \int_{source} a^2(y_s)\, dy_s \;+ 2\sqrt{K_1 K_2} \int_{source} a^2(y_s) \cos\left\{ k\left(\text{OPD}' - \frac{2cy_s}{l} \right) \right\} dy_s$$

$$= (K_1 + K_2) I_L \;+ 2\sqrt{K_1 K_2}\, \text{Re}\left\{ e^{jk\text{OPD}'} \int_{source} a^2(y_s) e^{-jk\theta_A y_s}\, dy_s \right\}$$

$$= (K_1 + K_2) I_L \;+ 2\sqrt{K_1 K_2}\, I_L\, \text{Re}\left\{ e^{jk\text{OPD}'} \frac{\mathscr{F}\left[a^2(y_s) \right]_{\frac{\theta_A}{\lambda}}}{I_L} \right\}$$

$$= (K_1 + K_2) I_L \;+ 2\sqrt{K_1 K_2}\, I_L \mu_{12} \cos\left(k\,\text{OPD}' + \beta_{12} \right)$$

YDPI Extended Source

$$I(\text{OPD}') = (K_1 + K_2) I_L \;+ 2\sqrt{K_1 K_2}\, I_L \mu_{12} \cos\left(k\,\text{OPD}' + \beta_{12} \right)$$

$$\mu_{12}(\text{OPD}') = \left| \frac{\mathscr{F}\left[a^2(y_s) \right]_{\frac{\theta_A}{\lambda}}}{I_L} \right| \neq f(\text{OPD}')$$

$$\beta_{12} = phase\left\{ \mathscr{F}\left[a^2(y_s) \right]_{\frac{\theta_A}{\lambda}} \right\} \neq f(\text{OPD}')$$

van Cittert – Zernike Theorem

$$I_{\max} = (K_1 + K_2) I_L \;+ 2\sqrt{K_1 K_2}\, I_L \mu_{12}$$
$$I_{\min} = (K_1 + K_2) I_L \;- 2\sqrt{K_1 K_2}\, I_L \mu_{12}$$

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{K_1 K_2}}{K_1 + K_2} \mu_{12}$$

Spatial Coherence and the Diffractometer

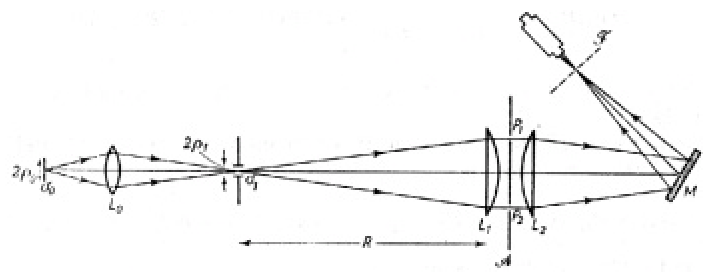
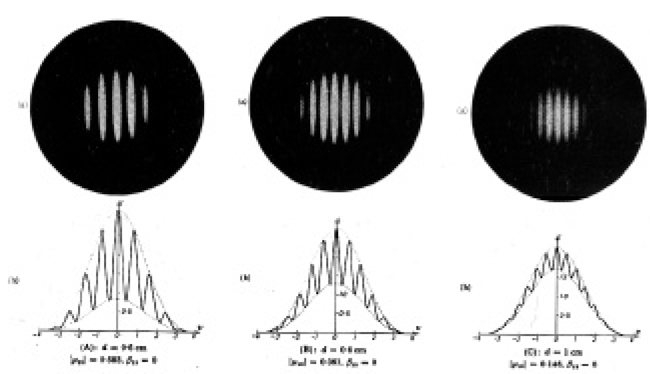


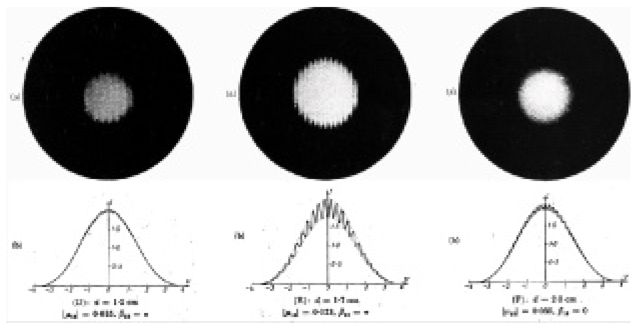
Fig. 10.4. The Diffractometer.

- Spatial coherence is controlled by changing iris radius ρ_1.
- Visibility is measured at *F* by varying pinhole separation *d*.
- Reference – Born and Wolf

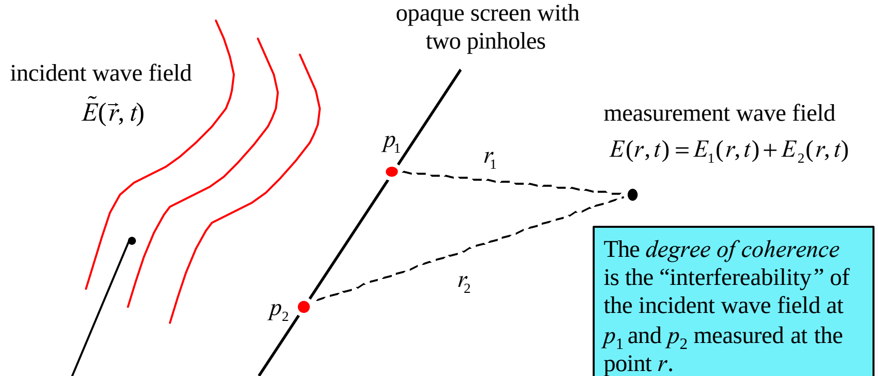
The Diffractometer



The Diffractometer

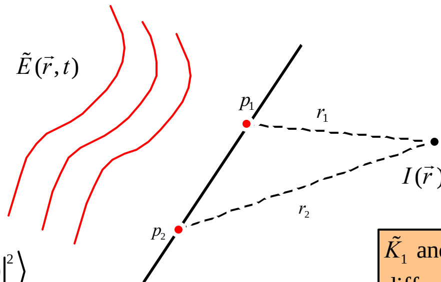


Formal Coherence Development



- Analytic – square integrable and well defined.
- Ergodic – the ensemble average equals the time average from a typical member.
- Stationary – ensemble averages are independent of the time origin.

Formal Coherence Development



$$I(\vec{r}) = \left\langle \left| E_1(\vec{r},t) + E_2(\vec{r},t) \right|^2 \right\rangle$$

$$\tau = \frac{r_2 - r_1}{c} = \frac{OPD'}{c}$$

$\tilde{K}_1$ and $\tilde{K}_2$ are amplitude diffraction constants from the pinholes to the observation point.

$$I(\vec{r}) = \left\langle \left| E_1(\vec{r},t) + E_2(\vec{r},t) \right|^2 \right\rangle$$

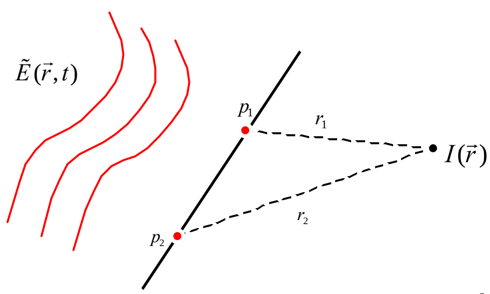
$$= \left\langle \left| \sqrt{K_1}\,\tilde{E}\left(\vec{r}_{ph1}, t + \frac{r_1}{c}\right) + \sqrt{K_2}\,\tilde{E}\left(\vec{r}_{ph2}, t + \frac{r_2}{c}\right) \right|^2 \right\rangle$$

$$= \left\langle \left| \sqrt{K_1}\,\tilde{E}(\vec{r}_{ph1}, t) \right|^2 \right\rangle + \left\langle \left| \sqrt{K_2}\,\tilde{E}(\vec{r}_{ph2}, t) \right|^2 \right\rangle + 2\,\mathrm{Re}\left\{ \left\langle \sqrt{K_1}\,\tilde{E}\left(\vec{r}_{ph1}, t + \frac{r_1}{c}\right) \sqrt{K_2^*}\,\tilde{E}^*\left(\vec{r}_{ph2}, t + \frac{r_2}{c}\right) \right\rangle \right\}$$

$$= K_1 \tilde{I}_1(\vec{r}_{ph1}) + K_2 \tilde{I}_2(\vec{r}_{ph2}) + 2\sqrt{K_1 K_2}\,\mathrm{Re}\left\{ \left\langle \tilde{E}(\vec{r}_{ph1}, t)\,\tilde{E}^*(\vec{r}_{ph2}, t+\tau) \right\rangle \right\}$$

$$= K_1 \tilde{I}_1 + K_2 \tilde{I}_2 + 2\sqrt{K_1 K_2}\,\mathrm{Re}\left\{ \left\langle \tilde{E}_1(t)\,\tilde{E}_2^*(t+\tau) \right\rangle \right\}$$

Formal Coherence Development – Mutual Coherence



$$I(\vec{r}) = K_1 \tilde{I}_1 + K_2 \tilde{I}_2 + 2\sqrt{K_1 K_2}\,\mathrm{Re}\left\{ \left\langle \tilde{E}_1(t)\,\tilde{E}_2^*(t+\tau) \right\rangle \right\}$$

$$= K_1 \tilde{I}_1 + K_2 \tilde{I}_2 + 2\sqrt{K_1 K_2}\,\mathrm{Re}\left\{ \Gamma_{12}(\tau) \right\}$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}\left\{ \gamma_{12}(\tau) \right\}$$

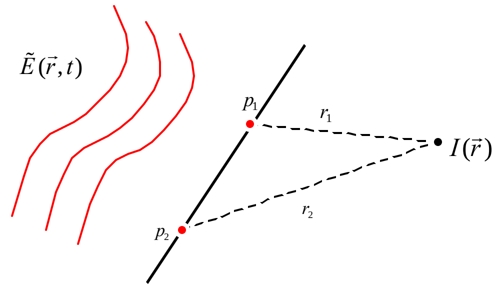
mutual coherence

$$\Gamma_{12}(\tau) = \left\langle \tilde{E}_1(t)\,\tilde{E}_2^*(t+\tau) \right\rangle$$

complex degree of coherence

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\tilde{I}_1 \tilde{I}_2}}$$

Formal Coherence Development – Degree of Coherence



$$|\gamma_{12}(\tau)| = \text{degree of coherence}$$
$$0 \le |\gamma_{12}(\tau)| \le 1$$

$$I(\vec{r}) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}\{\gamma_{12}(\tau)\}$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma_{12}(\tau)|\cos[\alpha_{12}(\tau) - \delta]$$

where $\alpha_{12}(\tau) = \text{phase}\{\gamma_{12}(\tau)\}$

Formal Coherence Development – Limits

mutual coherence	$\Gamma_{12}(\tau) =$	$\langle \tilde{E}_1(t)\,\tilde{E}_2^*(t+\tau)\rangle$
self coherence	$\Gamma_{11}(\tau) =$	$\langle \tilde{E}_1(t)\,\tilde{E}_1^*(t+\tau)\rangle$
irradiance	$\Gamma_{11}(0) =$	$\langle \tilde{E}_1(t)\,\tilde{E}_1^*(t)\rangle = I_1$
mutual intensity	$\Gamma_{12}(0) =$	$\langle \tilde{E}_1(t)\,\tilde{E}_2^*(t)\rangle = J_{12}$

temporal coherence

$$m_{12}\left(\frac{OPD'}{c}\right) = m_{12}(\tau) = |\gamma_{12}(\tau)| = \left|\frac{\Gamma_{12}(\tau)}{\sqrt{\tilde{I}_1 \tilde{I}_2}}\right|$$
$$\beta_{12}\left(\frac{OPD'}{c}\right) = \beta_{12}(\tau) = \text{phase}\{\gamma_{12}(\tau)\}$$

spatial coherence

$$\mu_{12} = |\gamma_{12}(0)| = \left|\frac{J_{12}(0)}{\sqrt{\tilde{I}_1 \tilde{I}_2}}\right| \quad \text{normalized mutual intensity}$$
$$\beta_{12} = \text{phase}\{\gamma_{12}(0)\}$$

YDPI – Combination Effects