Precision evaluation of lens systems using a nodal slide/MTF optical bench

Victor J. Doherty, Philip D. Chapnik

Eidolon Corporation
3 Erie Drive
Natick, MA 01760

ABSTRACT

A compact, self-contained production instrument designed to permit the rapid and precise performance characterization of a wide variety of lenses and optical systems has been developed by Eidolon Corporation. The **Eidolon Production Nodal Slide/MTF Measurement System** can be used to measure effective focal length (EFL), distortion, field curvature, chromatic aberration, spot size, and modulation transfer function (MTF).

1. INTRODUCTION

The characteristics of a lens system may be obtained without the need to know specifics about its internal structure. This is true for all lenses. It is this fact which allowed the Eidolon Corporation to design an instrument which measures some fundamental properties related to imaging lenses. This instrument is known as a Nodal slide.

Virtually all positive lens systems with focal lengths from 0.1 mm to 300 mm can be measured. Some of the typical lens systems characterized with the Eidolon production nodal slide/MTF measurement system include:

singlets
zoom lenses
GRIN lenses
holographic elements
ophthalmic lenses
near-IR lenses
cemented doublets
eyepieces
binary optics
endoscopes
aspheric lenses
micro-lenses & arrays
photographic lenses
microscope objectives
diamond turned lenses
molded plastic lenses
night vision devices
multi-focal lenses

2. THEORY OF OPERATION

In order to understand how the Eidolon Nodal Slide works, it is necessary to understand some fundamental properties of imaging lenses. These fundamentals are described in this section.

A light ray will in general exit a lens in a direction different from that with which it entered. If an entering ray and an exiting ray are extended on the side near the lens, they will intersect at some point in the vicinity of the lens. When this process is performed for all the rays in a parallel ray bundle, the locus derived from each of the intersection points defines an equivalent refracting

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surface for the lens. In a well corrected optical system, the equivalent surface is in the form of a sphere. In the paraxial approximation, this sphere becomes a plane and is referred to as the **principal plane.**

If parallel ray bundles are traced from the left and from the right hand side of the lens system, two distinct principal planes are formed. The intersection of the principal planes with the optical axis form the two **principal points** in the system. Principal points P2 and P1 reference the principal plane for light traveling to the left and to the right, respectively. Generally, there is a separation between the two nodal points. In the "thin lens" approximation, the principal points are coincident.

Paraxial rays which enter an imaging lens parallel to the optical axis and emerge from the lens intersect one another on the optical axis at the **focal point.** Parallel light which enters from the left and right form focal points F2 and F1, respectively. For parallel ray bundles which form a small angle (paraxial) to the optical axis, the emerging rays will intersect on a **focal plane** with the optical axis as its normal. **The distance from a principal point to its corresponding focal point is defined as the focal length.**

In a lens system, there exists a pair of **nodal points** N_1 and N_2 which lie on the optical axis such that a paraxial ray which intersects N_1 at an arbitrary angle will emerge from N2 at the same angle as shown in Figure 1. In the thin lens approximation, the two nodal points are coincident, hence the rule that a ray which intersects a thin lens at the optical axis is not deviated. In the paraxial region, there exist nodal planes, the positions of which are defined by the nodal points. These are conjugate planes of unit magnification such that a ray aimed at one plane at a given height above the optical axis will emerge from the conjugate nodal plane at the same height. **When the refractive index is the same on both sides of the lens, then the nodal points are coincident with the principal points.**

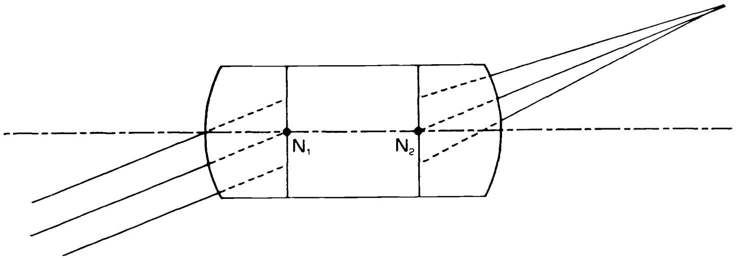

FIGURE 1 – DEFINITION OF THE NODAL POINT

When a parallel bundle of light enters a positive focal length lens from the left, it comes to a focus at a plane centered on focal point F_2. If the lens is mounted such that it can be rotated around an axis perpendicular to the ray bundle, there will always exist within the bundle a ray which intersects nodal point N_1. This ray will then emerge from nodal point N_2 parallel to the incoming ray. The ray bundle will come to a focus at a point along this outgoing ray one focal length from the principal plane. For an arbitrary rotation point, N_2 will move with respect to a fixed external point and therefore the focus will also move, as shown in Figure 2.

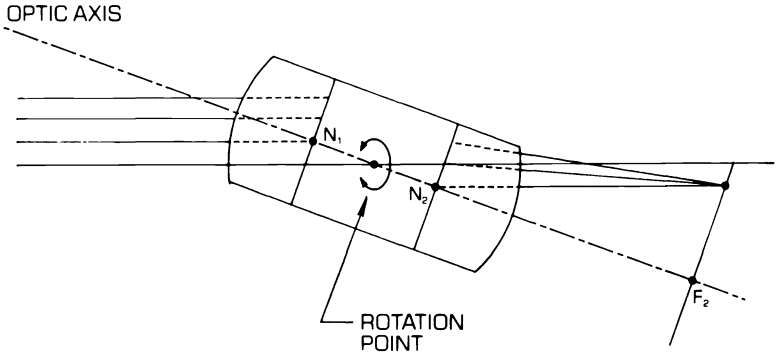

Figure 2 - Motion of the focus for an arbitrary rotation point

When N_2 coincides with the rotation axis, there is no motion of N_2 as the lens is rotated. The ray which emerges from this point must also be stationary, as shown in Figure 3. Therefore, the focus which lies along this emerging ray must be stationary with respect to a fixed external point.

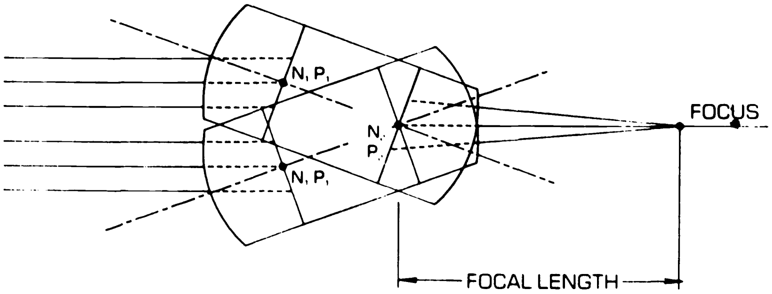

Figure 3 - Measuring the focal length by rotating the lens about its nodal point

The principal points coincide with the nodal points when the lens is in air. **Thus, when the nodal point N_2 coincides with the rotation axis through the lens system, the distance between the nodal point and the focus is, by definition, the effective focal length of the lens system.**

3. THE NODAL SLIDE

The Eidolon production nodal slide/MTF measurement system positions the nodal point of an lens over a fixed rotation point while accurately monitoring the separation of the focus with respect to that rotation point. This allows the precise determination of the focal length. The instrument as shown in Figure 4 consists of several main components. They are:

- Collimated light source
- Lens mount and carriage
- T-Bar
- Nodal bearing
- Microscope assembly
- Encoder/digital display
- Base

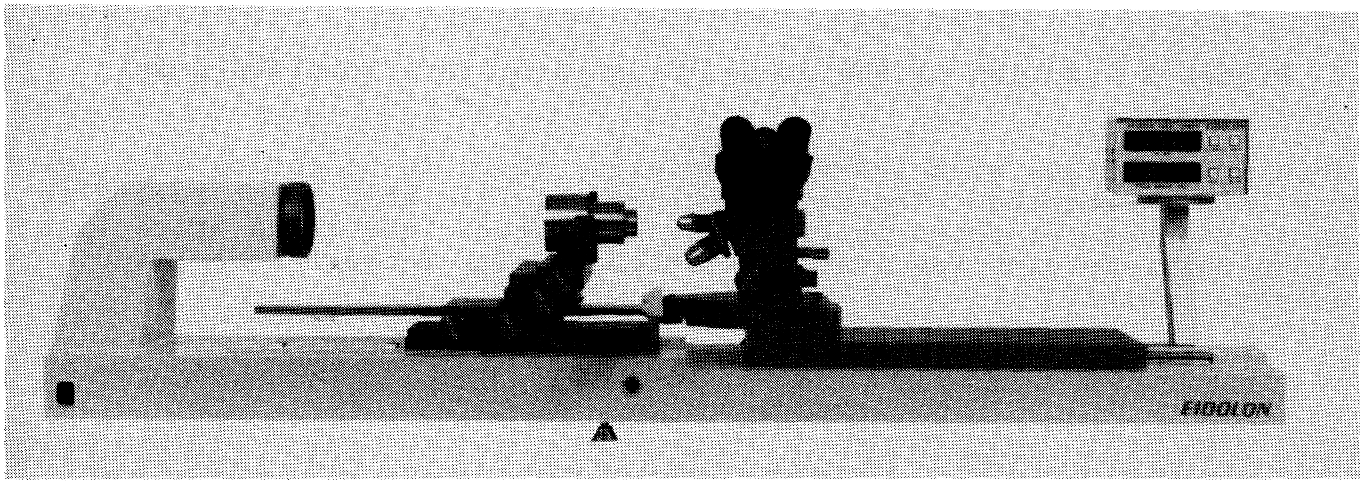

Figure 4 - The Eidolon production nodal slide/MTF measurement system

The following is a discussion of each of these components.

Collimated light source

This component creates collimated light with rays parallel to the translation axis of the nodal slide base. The light from a low voltage incandescent source is collected by a condenser lens and presented to the filter plane. Positioned at the filter plane is a four-position filter wheel. The user is free to install any combination of filters required. A second lens collects the light exiting the filter wheel and focuses it to the pinhole plane. A five-position self-aligning pinhole wheel allows the user to rapidly change pinhole size to optimize the test conditions as they relate to the lens under test. The pinhole plane is located precisely at the focus of the collimator lens. The standard system is equipped with a 600 mm focal length achromatic collimator lens with an aperture of 75 mm. An apochromatic collimator is available as an option.

Lens mount/carriage

The lens mount assumes multiple functions. It holds the lens system firmly in place on the instrument. The mount as shown in Figure 5 is designed such that the optical axis of an arbitrary rotationally symmetric lens is automatically positioned at the correct height in the instrument. Provisions are made to translate the lens laterally such that the optical axis of the lens lies directly over the nodal bearing. The lens mount also contains a slide to allow the lens to be translated along its optical axis in order to place the nodal point directly over the nodal bearing.

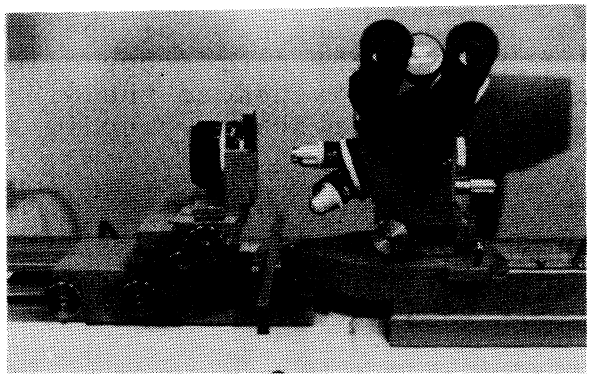

Figure 5 - Lens Mount and Carriage

T-bar

The T-bar is a precision ground component which slides in and out of the lens holder to define the focal length of the lens under test. The perpendicular of the T-bar defines a reference plane parallel to the focal plane of the lens.

Nodal bearing

The nodal bearing is a precision bearing on which the lens holder/T-bar assembly rotates. The rotation axis of the bearing is used to define the position of the nodal point in the lens system.

Microscope assembly

The microscope assembly consists of a microscope system for viewing the focal point of the lens under test. In this assembly there exists a pivot bearing on which the T-Bar rests. The center of this bearing defines the object plane for the microscope system. Additional mechanisms on this assembly allow the microscope to be positioned in both the vertical and lateral directions.

The microscope assembly translates along the length of the nodal slide base.

Encoder scale/digital display

An encoder is used to measure the focal length of the lens under test. It monitors the distance between the nodal bearing center and the pivot bearing center. A rotary encoder mounted in the base of the instrument monitors the angle of the nodal bearing. The encoder signals are processed by the system electronics and digitally displayed.

Base

The base of the nodal slide holds all components in their proper locations. A spring located in the base maintains contact between the T-bar and the pivot bearing. The microscope assembly rides along a precision translation stage on the base of the instrument. As the T-bar is extended or retracted, the microscope slides along the base. The scale reading head that is attached to the microscope assembly moves along the stationary encoder scale which is attached to the base.

4. OPERATION OF THE NODAL SLIDE

4.1 Determining effective focal length

The effective focal length of a lens may be determined very precisely with the Eidolon production nodal slide/MTF measurement system using the following procedure:

Install the lens to be tested in the lens chuck. Zero the field angle of the T-bar. While looking through the microscope eyepiece, extend the T-bar until the focal spot is seen. By extending or retracting the T-bar, the spot size should be minimized. Adjust the microscope laterally and vertically until the spot is centered in the field of view. This will compensate for lateral misplacement or vertical tilt of the lens in the mount.

While viewing the focal spot, swing the lens mount/T-bar assembly side to side. In general, when this procedure is initiated, the user will see the spot move back and forth across the field-of-view. Adjust the lens mount to translate the lens forward or backward. The T-bar will automatically be carried with the lens mount, thereby maintaining focus at the microscope. In doing so, it will be seen that one direction will lessen the motion of the focal spot. If the lens is continuously translated in this direction, a point will be reached at which the spot motion has reached a minimum. When this occurs, the nodal plane is close to being directly over the nodal bearing center.

When the nodal point is directly over the bearing center, there will be no motion of the focal spot. To achieve this condition, it is necessary to translate the lens laterally until its optical axis is directly above the bearing center. Once again, swing the lens mount/T-bar assembly while viewing the focal spot. If the spot continues to move, longitudinally move the lens mount. It may take several iterations of lateral and longitudinal translations of the lens mount before the spot motion is zeroed. If a null in the spot motion cannot be achieved, then distortion is present in the lens under test (See section below). If this is the case, the lens carriage should be translated such that the spot motion is minimized.

When the nodal point of the lens has been properly positioned over the rotation axis of the nodal bearing and the lens under test has been focused, then the digital display will read the effective focal length.

4.2 Determining Field Curvature - Most common lens systems are designed for flat field imaging. The flat T-bar has been designed with this fact in mind. When a flat field lens is swung to an off-axis position, the focus will be maintained over the pivot bearing center.

Field curvature is a third-order (non-paraxial) aberration in which the focus occurs over a curved field for off-axis field angles.

If the focal spot goes out of focus for off-axis imaging, then field curvature is present in the lens under test. The amount of field curvature can be measured by extending or retracting the T-bar until the focal spot size is once again minimized. The focal length scale will now show the distance from the nodal point to the focus for the off-axis field angle. By measuring the focal length for a series of off-axis angles, it is possible to find an exact curve representing the focal surface of the lens under test. Figure 6 depicts the measurement of field curvature.

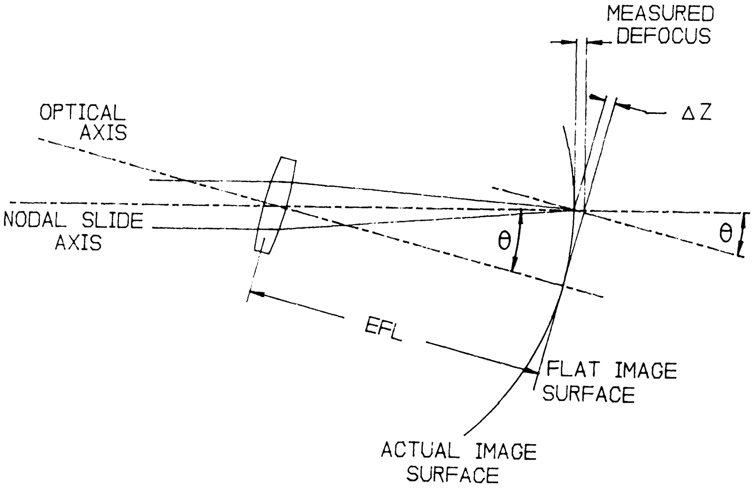

Figure 6 - Measuring Field Curvature

To find the distance of the focal surface from the flat field plane defined by the on-axis focal point, make the following calculations: 1) Create a difference value by subtracting the off-axis focal length from the on-axis focal length. 2) Multiply the cosine of the off-axis field angle by the difference value using the following equation:

$$\Delta Z = \text{Defocus} \, (\cos \Theta) \qquad (1)$$

4.3 Measuring Distortion - Distortion is a higher order aberration in which the principal ray emerging from the lens is deviated from the angle dictated by first-order imaging. As the focus must occur along the principal ray, distortion has the effect of laterally translating the focal spot. When the focal spot is closer to the optical axis than dictated by first-order imaging, pincushion distortion is present. When the focal spot is further from the optical axis than dictated by first order imaging, then barrel distortion is present.

If the on-axis focal length of a lens has been properly found on the nodal slide and distortion is present, then the focal spot will translate when the lens is swung for off-axis imaging. This fact indicates that it is impossible to obtain zero focal spot motion even when the nodal point of the lens lies directly over the nodal bearing center.

To measure distortion, center the focal spot in the microscope field of view when the lens is imaging on-axis. Swing the lens to the desired field angle. For a flat field lens, this action will cause the focal spot to shift position but remain in focus. Laterally translate the microscope until the focal spot is once again centered. The distortion is equal to the lateral translation divided by the cosine of the field angle. Figure 7 depicts the measurement of distortion.

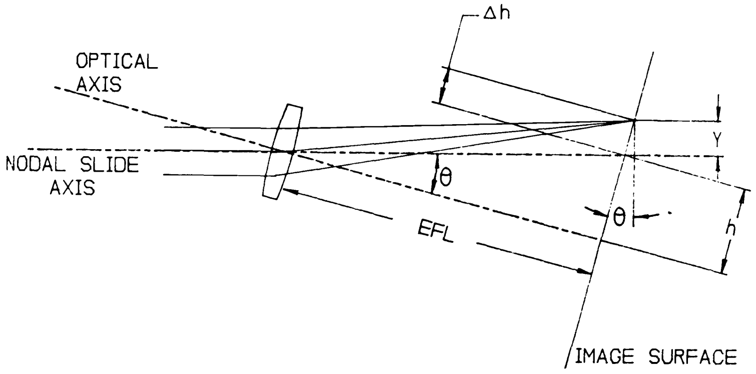

Figure 7 - Measuring Distortion

Using the nodal slide the Y displacement that is determined can be related to the distortion of the lens by using the following equations:

- Calculate

$$\Delta h = \frac{Y}{\cos \Theta} \qquad (2)$$

- Since

$$h = EFL(\tan \Theta) \qquad (3)$$

- Substituting

$$\% \text{ DISTORTION} = \frac{\Delta h}{h} 100 = \frac{Y}{EFL(\sin \Theta)} 100 \qquad (4)$$

Distortion is independent of all other aberrations and may occur in a lens with or without field curvature. When field curvature and distortion are both present, it is necessary to plot out the position of the focus position from the on-axis point as a function of the of the field angle. These two aberrations can then be derived from the resulting graph.

4.4 Other Aberrations

Spherical Aberration - Spherical aberration is, by definition, the variation in focal length as a function of ray height at the entrance aperture of the lens. Spherical aberration is independent of field angle. It is rotationally symmetric about the principal ray and causes the focal spot to be poorly defined. Presence of this aberration tends to causes a shift in value of the focal length obtained on the nodal slide compared to the paraxial focal length.

Astigmatism - Astigmatism is an aberration which causes the sagittal (horizontal) and tangential (vertical) ray fans to come to a focus at different distances from the nodal point. Most lenses exhibit astigmatism for large field angles. This aberration has a quadratic dependence with field angle. Astigmatism is sometimes seen on-axis. When astigmatism is present in a lens system for the on-axis field position, the cause is manufacturing/alignment error of one or more of the lens elements comprising the lens system. When astigmatism is not seen for the on-axis field position, but is instead seen off-axis, it is usually in the design of the lens system under test.

As the lens under test is brought in and out of focus, astigmatism appears as a vertical line which transforms into a rotationally symmetric blur and then into a horizontal line. The focus defined by the rotationally symmetric blur is referred to as the circle of least confusion.

Coma - Coma is an off-axis aberration which causes the focal spot to flare out in a direction away from the principal ray. As viewed, the spot appears somewhat like the tail of a comet, hence the name.

Axial Color - Axial color is a symmetric aberration due to the variation of focal length with wavelength. As viewed, this aberration appears as series of centered colored rings. Figure 8 depicts how axial color can be measured with the nodal slide.

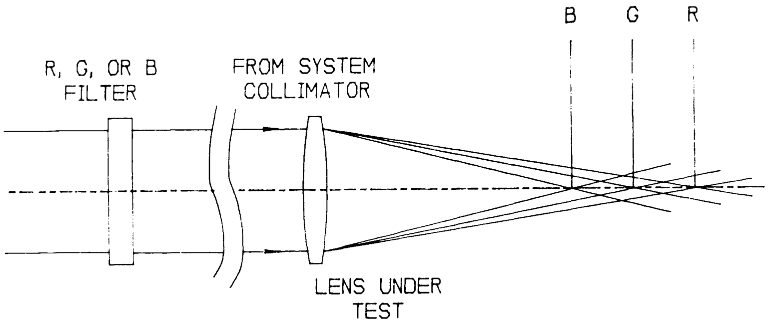

Figure 8 - Measuring axial color

Lateral Color - This is an off-axis aberration due to the variation of lens properties with wavelength. As viewed, lateral color appears as a spread of color away from the optical axis. When lateral color is seen on-axis, its' presence indicates manufacturing/alignment error of one or more of the lens elements comprising the lens system.

5. THE MODULATION TRANSFER FUNCTION

5.1 MTF theory

An adaptation of the nodal slide allows us to characterize lens performance in terms of the Modulation Transfer Function (MTF). The MTF depicts image contrast as a function of spatial frequency in the image plane.

The pinhole, as imaged by the lens under test, is essentially the PSF (Point Spread Function) of the lens. The PSF is defined in terms of light intensity as a function of position. Mathematical theory shows that if the Fourier transform is performed on the point spread function of a lens, the result is directly proportional to the MTF of the lens. With the proper scaling factors, the result can be made to equal the MTF exactly.

In order to obtain the true PSF of a lens, it is essential that the object that produces the image to have zero extent, i.e., the input light to the lens should be perfectly collimated. White light sources, such as incandescent bulbs, have a rather low radiance as compared to laser sources. Thus, in practice, the pinhole must be of a finite diameter. The finite extent of the object is manifested in the lens image as a spot larger than the PSF of an object of zero extent. The actual spot is equal to the PSF convolved with the geometrical image of the pinhole. If the finite extent of the pinhole is not accounted for, the result is a false decrease in the MTF, thus indicating lens performance below its full capability.

When the intensity profile of a finite pinhole image is Fourier transformed, the geometrical image of the pinhole is also carried along in the calculation. This fact makes it rather easy to remove the effect of the pinhole from the final MTF calculation. Mathematical theory states that a convolution in the image domain is equivalent to a multiplication in the frequency domain. As stated previously, the actual spot image is given by the convolution of the PSF with the geometric pinhole image. The Fourier transform is thus proportional to the MTF multiplied by the Fourier transform of the geometric pinhole image. As the Fourier transform of the pinhole is a known function, it can be divided out from the original transform function leaving a function directly proportional the actual MTF.

Mathematically,

$$\mathrm{Int} = \mathrm{PSF} * \mathrm{Pinhole} \qquad (5)$$

$$\begin{aligned} F\{\mathrm{Int}\} &= F\{\mathrm{PSF} * \mathrm{Pinhole}\} \qquad (6) \\ &= F\{\mathrm{PSF}\}\, F\{\mathrm{Pinhole}\} \\ &= k(\mathrm{MTF})\, F\{\mathrm{Pinhole}\} \end{aligned}$$

where, $*$ represents the convolution function
k is a scaling constant
$F\{g\}$ represents the Fourier transform of function g.

Therefore,

$$\mathrm{MTF} = \frac{F\{\mathrm{Int}\}}{k\, F\{\mathrm{Pinhole}\}} \qquad (7)$$

If one is concerned with the MTF in only one direction, then a slightly different course is taken in the calculation. If a line source is imaged by a lens, the result is known as the LSF (Line Spread Function). The LSF is a one-dimensional function measuring intensity as a function of position in a direction perpendicular to the line image. The Fourier transform of the LSF yields a function directly proportional to the MTF measured in a direction perpendicular to the line image. Mathematically, the LSF is equivalent to the PSF convolved with a geometric line. Equivalently, the value of the LSF at any point can be determined by slicing the PSF and adding up the intensity values along that slice. Again, the finite extent of either a pinhole or a slit must be accounted for as explained above in order to obtain the proper MTF.

In practice, the Fourier transform, as performed on a computer, is calculated by the DFT (Discrete Fourier Transform). The FFT (Fast Fourier Transform) is a particular algorithm for computing the DFT in as few calculations as possible. The number of calculations required for the FFT is on the order of

$$N \log_2 N \tag{8}$$

where N is the number of data values in a one-dimensional array, such as the LSF. For a two-dimensional array, i.e., the PSF, the number of computations is on the order of

$$N^2 \log_2 N^2 \tag{9}$$

The FFT algorithm requires N to be equal to a power of 2. Typical values for N are 512 or 256. If the MTF is needed in only a single direction, there is a decrease in the number of required calculations by a factor of 2N. This represents a significant decrease in the time needed to obtain the MTF.

5.2 MTF in Practice: Eidolon FastMTF

The Eidolon Nodal Slide comes with an option to allow measurement of lens MTF. The option runs a custom Microsoft Windows application, known an **FastMTF**, on a high performance 486 based PC.

In the nodal slide, the pinhole, as imaged by the lens under test, is relayed to a solid state camera by a high performance microscope objective. This objective serves to magnify the pinhole as imaged by the test lens. The camera is a CCD device consisting of a two dimensional array of minute solid state detectors. The camera thus serves to sample the PSF over a discreet set of fixed points. The camera we have chosen for our instrument exhibits good linearity and low noise, even at room temperature.

A frame grabber on the computer captures the camera image by converting the analog camera output to a series of 8-bit digital values. Conversion occurs for a single camera frame when indicated by the software. The digital values are stored in an on-board memory buffer and are then available for use by the CPU.

Initializing the Instrument

In the instrument, software controls most aspects of data acquisition. Certain aspects are under user control. These are found under the **CONTROL** window, depicted in Figure 9. The first three inputs control the calibration of the camera. Essentially, these controls set the voltage output of the camera to the voltage input of the frame grabber board, thereby maximizing the dynamic range of the system.

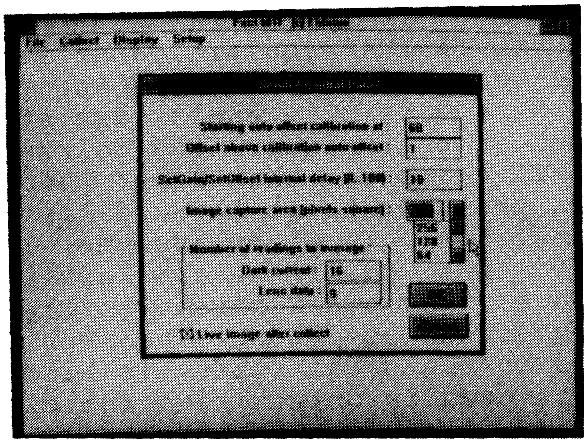

Figure 9 - FastMTF software CONTROL window

The next input in the **CONTROL** window sets the image capture area, defined in terms of the number of pixels squared. The reason for this input will be discussed in an upcoming section.

The last inputs in the **CONTROL** window determine the number of frames which are averaged during calibration and during image acquisition.

Offset, either in terms of electrical noise or in background light has the effect of decreasing the actual MTF. The software has a calibration feature which allows the user to subtract off fixed offsets from the actual electrical signal. As electrical noise introduces randomly occurring offsets on a pixel by pixel basis, it is best to determine the offset by averaging over many frames of data. The **CALIBRATION** window determines this frame averaged offset from a "dark" frame, that is a frame with no PSF image. The number of frames to be averaged is set in the **CONTROL** window.

As with the offset, averaging many frames of PSF data reduces the effects of electrical noise, thereby resulting in a more precise MTF calculation. The number of PSF frames to be averaged is also set in the **CONTROL** window.

Before actual data is taken, the user will go into the **SETUP** window in order to tell the program physical parameters of the nodal slide. Specifically, the wavelengths, the pinhole diameter, and the objective magnification are input. The values are used in the calculation and display of the MTF curves. Through this window, the user may also set the units of display for the MTF curves.

Collecting Data

At present, FastMTF performs two one-dimensional FFTs on LSFs derived from the PSF. The LSF is computed by adding up the pixel data along either rows or columns over the entire collection window. The direction of the MTFs are for the sagittal and tangential direction in the image plane. Future options will allow the computation of a full two-dimensional MTF.

The **COLLECT** window controls the acquisition of PSF data. As shown in Figure 10, lens title and serial number are input at this window. These inputs tag the data when MTF results are stored. The lens F-number is also input at this window, and is used in determining the diffraction-limited MTF curve for that lens. Typically, a user will be interested in lens performance at three different field-angles. The specific field angles are input by the user in their proper places in the window. These positions are labeled for field-angle as a matter of convenience. Any other parameter may be input in this option such as through-focus position. The lens effective focal length, as determined by the nodal slide, is also input in this window. This value is used in determining the diameter of the geometric pinhole image which in turn is used to correct the MTF for the finite pinhole diameter.

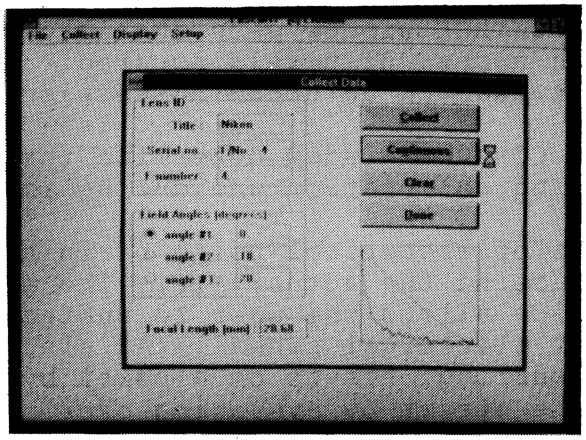

Figure 10 - FastMTF software COLLECT window

Three option buttons are available in the **COLLECT** window. One of these, "CLEAR", clears out previously collected data and is typically used when a new lens is to be tested. A second button, "COLLECT" is used to collect multiple frames of PSF data for MTF calculation. The number of frames averaged during a collection is set in the **CONTROL** as discussed previously.

The third option button is "CONTINUOUS". **This option allows the user to view a real-time MTF curve**, as shown in Figure 10. This option is especially useful for optimizing the focus of a lens as the lens may refocused as the MTF curve is monitored. Instead of averaging, a single frame of data is used for this option unlike the "COLLECT" option. The use of a single frame greatly increases the real-time capability of the system.

An option for "DATA COLLECTION WINDOW SIZE" exists in the **CONTROL** window. In the default, the system will collect data from the largest available window size; 512 x 512 pixels. This value is used in both the "COLLECT" and "CONTINUOUS" options in the **COLLECT** window. As the data collection window size is decreased, the number of required computations decreases, thereby increasing the update rate of the MTF curve in the continuous mode. The collection window is displayed on the monitor indicating the required position of the PSF. As long as the data collection window size is chosen to be large enough to enclose most of the energy of the pinhole image, the real-time MTF will indicate the optimum focus position. A larger window size can then be used during the "COLLECT" option to ensure that no data is lost outside the collection window. Figure 11 depicts a typical spot image on the monitor with a data collection window size of 128 x 128.

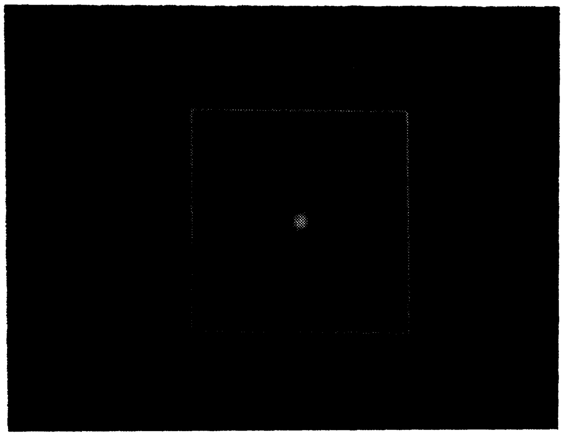

Figure 11 - Data collection window size of 128 x 128

Update rates for some data collection window sizes are as follows. The table is given for a 486 based PC running at 25 MHz.

Window Size (Pixels Squared)	Update Rate (Frames/second)
512	0.5
256	1.3
128	2.5
64	3.1

As each "COLLECT" is performed, the system will automatically step through each of the field angle positions. At any time, any field angle may be manually chosen to either skip ahead or recollect data for a given field angle.

Displaying results

The **DISPLAY** option provides MTF output in graphic format, Figure 12 or tabular format. As depicted in Figure 12, MTF results are displayed for up to three field angles with data for both sagittal and tangential directions for each field angle. The theoretical diffraction limited curve is also displayed for reference purposes.

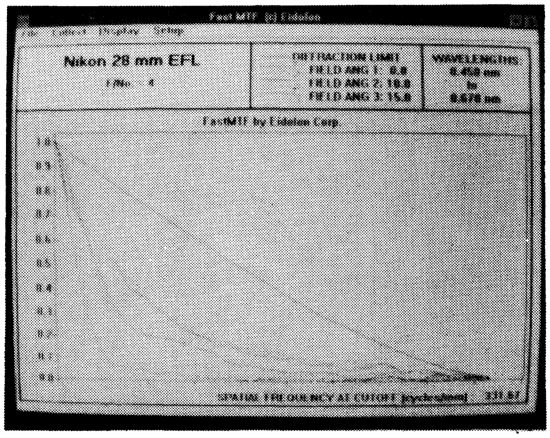

Figure 12 - FastMTF software graphical MTF output

Another **DISPLAY** option displays the LSF determined from the PSF. We have found this option useful in determining the validity of a questionable MTF. Often times, a PSF is perceived to appear better than it actually is. Low level light values, though not seen visually, will appear under the scrutiny of a multiple frame averaged LSF.

6. CONCLUSION

The **Eidolon Production Nodal Slide/MTF Measurement System** is a unique self-contained instrument that was designed to measure the first-order properties of a wide variety of lens systems. The performance characteristics of a lens including effective focal length (EFL), distortion, field curvature, chromatic aberration, and spot size can be rapidly determined. With Eidolon's **FastMTF** software, the 2-D modulation transfer function (MTF) can be acquired at sub-second rates.