Manipulating the critical temperature for the superfluid phase transition in trapped atomic Fermi gases

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We examine the effect of the trapping potential on the critical temperature T_C for the BCS transition to a superfluid state in trapped atomic gases of fermions. T_C for an arbitrary power-law trap is calculated in the Thomas-Fermi approximation. For anharmonic traps, T_C can be increased by several orders of magnitude in comparison to a harmonic trap. Our theoretical results indicate that, in practice, one could manipulate the critical temperature for the BCS phase transition by shaping the traps confining the atomic Fermi gases.

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Since the observation of Bose-Einstein condensation in trapped atomic gases [1], there has been increasing interest in the possibility of observing the Bardeen-Cooper-Schrieffer (BCS) phase transition to the superfluid state [2] in dilute fermionic alkali-metal gases [3–11]. Currently, experimental efforts in cooling fermionic atoms of ^{6}Li [12,13] and ^{40}K [14,15] to the quantum degenerate regime have made significant progress, reaching temperatures as low as $0.2T_F$ where T_F is the Fermi temperature. However, these temperatures are still far beyond the critical temperature required for the BCS phase transition, which is *at least* an order of magnitude lower [3–7].

On the other hand, the current techniques used to cool fermionic atoms lose their utility when the temperature becomes much less than T_F. For example, evaporative cooling, which has been used for ^{40}K, becomes ineffective for temperatures much less than T_F due to Pauli blocking [16] while sympathetic cooling of the fermions using bosons, as done with ^{6}Li [12], loses its effectiveness when the heat capacity of the bosons falls below that of the fermions. Due to these technical obstacles in cooling fermionic atoms, it seems that one must seek other avenues to reach the BCS superfluid state.

Recently there have been a number of proposals to study the possibility of achieving a higher T_C by increasing the strength of the attractive interactions, i.e., the scattering length between the fermions using either a Feshbach resonance [9,10] or photoassociation [11]. Since T_C is a function of the attractive interaction between fermions and the density of states at the Fermi energy ρ_F, higher transition temperatures can also be achieved by increasing the density of states for the fermions at the Fermi surface. In bulk superconductors, the only way to modify the density of states is by changing the spatial dimensionality of the system. However, in atomic systems the density of states can be controlled through the shape and strength of the external trapping potential.

The ability of an external trapping potential to change the critical temperature for a phase transition by modifying the density of states $\rho(\epsilon)$ [see Eq. (8) below], was first considered by Bagnato *et al.* for Bose-Einstein condensation [17]. They showed that the critical temperature for Bose-Einstein condensation T_{BEC} as a function of the number of atoms confined in a power-law trap N_B had the power-law form $T_{BEC}\sim N_B^{\eta}$ where $2/9\leqslant\eta\leqslant 2/3$. In a similar manner, we will show that for a trapped gas of N_F fermions the density of states at the Fermi surface has the form $\rho_F\sim N_F^{\beta}$ where β varies from 1/9 to 1/3 depending on the type of trap. Since T_C depends *exponentially* on ρ_F, small changes in β can have a dramatic effect on T_C. In all the experiments with trapped fermions, the trapping potential is well approximated as being harmonic. Our results indicate, however, that higher values of T_C/T_F can be obtained using anharmonic power-law traps. This indicates that the use of anharmonic traps could place T_C within the range of current experiments.

At the ultracold temperatures achieved in fermion experiments, *p*-wave collisions between atoms are highly suppressed and *s*-wave collisions between atoms in the same internal state are forbidden by the Pauli exclusion principle. Since the BCS transition requires an attractive interaction between fermions in order to form Cooper pairs, the most likely candidate for Cooper pairing in alkali-metal gases is an attractive *s*-wave interaction between atoms in different hyperfine states. Fortunately, both ^{6}Li and ^{40}K appear to be very promising candidates. ^{6}Li possesses an anomalously large and negative *s*-wave scattering length equal to $-2160a_0$ [3], where a_0 is the Bohr radius, while for ^{40}K, a Feshbach resonance exists for two of the hyperfine states, which can be used to create the required large attractive interaction [9].

Therefore, we discuss the case of *s*-wave pairing between atoms of mass m in two hyperfine states with an *s*-wave scattering length $a<0$ for collisions between atoms in different states. Furthermore, we assume that the number of atoms in each of the hyperfine states, N_F, is the same since this is the optimal condition for Cooper pairing. In this case, the transition temperature for a dilute homogeneous gas is given by

$$\frac{T_C}{T_F}=\alpha\exp\left(-\frac{1}{g\rho_F}\right), \tag{1}$$

where $g=4\pi\hbar^2|a|/m$ is the coupling constant for the two-body interactions and $\alpha=2^{7/3}e^{C-7/3}/\pi\approx 0.28$ with C being the Euler's constant [8,18,19]. The density of states at the Fermi surface, in terms of the Fermi momentum $\hbar k_F$, is

$$\rho_F = mk_F/2\pi^2\hbar^2.$$

From Eq. (1) we can see that by raising the density of states at the Fermi surface (or equivalently, by raising k_F), there will be an exponential increase in the transition temperature. Note that even though we restrict ourselves to the case of *s*-wave pairing, our results can easily be extended to pairing via higher partial waves since in those cases the critical temperature has a similar exponential dependence on ρ_F [19].

The goal of this paper is to determine the critical temperature for the inhomogeneous Fermi gas where the atoms in both hyperfine states are subject to the trapping potential,

$$U(\mathbf{r}) = \varepsilon_1 \left|\frac{x}{a}\right|^p + \varepsilon_2 \left|\frac{y}{b}\right|^l + \varepsilon_3 \left|\frac{z}{c}\right|^q, \tag{2}$$

and p, l, and q are positive integers. We refer to the magnitude of the exponents p, l, and q as the confining power of the trap. For simplicity, we assume that the trapping potential is independent of the hyperfine state, hence the density of fermions is the same for both states, $n(\mathbf{r})$. If $n(\mathbf{r})$ varies sufficiently slowly, then we can make a local-density (or Thomas-Fermi) approximation by assuming that at each point in space the Fermi gas can be treated as being homogeneous [20]. In that case, a local value of the critical temperature, $T_C(\mathbf{r})$, can be calculated using Eq. (1). It should be emphasized that $T_C(\mathbf{r})$ represents the BCS transition temperature for the corresponding homogeneous gas with density $n(\mathbf{r})$. The actual transition temperature for the trapped gas, T_C, which is the temperature at which Cooper pairing first appears as the temperature is lowered, is independent of $\mathbf{r}$. For temperatures $T \ll T_F$, the local-density approximation is valid provided the average distance between atoms, $\sim k_F^{-1}$, is much less than the distances over which $n(\mathbf{r})$ changes significantly. Since the changes in $n(\mathbf{r})$ are determined by $U(\mathbf{r})$ for which the characteristic length scales are a, b, and c, we have the condition $k_F^{-1} \ll (abc)^{1/3}$. In addition, for $T \ll T_C$, the coherence length for the Cooper pairs, $\xi_0 = \hbar^2 k_F/\pi m \Delta_0$ where $\Delta_0 = (\pi e^{-C}) k_B T_C$ is the BCS gap at $T=0$, should also be much less than $(abc)^{1/3}$ [3,4].

In the Thomas-Fermi approximation, the chemical potential μ is replaced by a local value, $\mu(\mathbf{r}) = \mu - U(\mathbf{r}) + gn(\mathbf{r})$ so that the fermions can be treated at each $\mathbf{r}$ as an ideal homogeneous gas with chemical potential $\mu(\mathbf{r})$. The term $gn(\mathbf{r})$ represents the Hartree-Fock potential experienced by each atom due to the atoms in the opposite hyperfine state. Since we are interested in temperatures $T \ll T_F$, the chemical potential may be approximated by the Fermi energy, $\mu \simeq E_F = k_B T_F = \hbar^2 k_F^2/2m$, since for low temperatures $\mu = E_F + O(T/T_F)^2$ [21]. Correspondingly, we can express $\mu(\mathbf{r})$ in terms of a local Fermi wave number, $k_F(\mathbf{r})$, defined as

$$\frac{\hbar^2 k_F^2(\mathbf{r})}{2m} = E_F - U(\mathbf{r}) + gn(\mathbf{r}). \tag{3}$$

By using the result $n(\mathbf{r}) = k_F^3(\mathbf{r})/(6\pi^2)$ for an ideal Fermi gas, one obtains an expression for the density,

$$n(\mathbf{r})\left(1 - \frac{g}{A^{2/3}} n^{1/3}(\mathbf{r})\right)^{3/2} = \frac{1}{A}[E_F - U(\mathbf{r})]^{3/2}, \tag{4}$$

where $A \equiv 6\pi^2(\hbar^2/2m)^{3/2}$. For a dilute gas where $|a| n(0)^{1/3} \ll 1$, the Hartree-Fock term in Eqs. (3) and (4) can be neglected in a first approximation. The Fermi energy is then determined by the requirement that the total number of atoms in each spin component be conserved,

$$N_F = \frac{1}{A}\int_{V(E_F)} [E_F - U(\mathbf{r})]^{3/2} d^3r, \tag{5}$$

where the integration volume $V(E_F)$ is the volume available to a classical particle with total energy E_F, i.e., $E_F \geqslant U(\mathbf{r})$. Equation (5) can easily be solved for E_F for the case of the power-law trap (2) and gives

$$E_F = \left[\frac{A\varepsilon_1^{1/p}\varepsilon_2^{1/l}\varepsilon_3^{1/q}\Gamma(\delta+1)}{\Gamma(1+1/p)\Gamma(1+1/l)\Gamma(1+1/q)\Gamma(5/2)}\left(\frac{N_F}{8abc}\right)\right]^{1/\delta}, \tag{6}$$

where $\Gamma(x)$ is the gamma function and

$$\delta \equiv 3/2 + 1/p + 1/l + 1/q.$$

Note that the limit $p,l,q \to \infty$ corresponds to a box with volume $8abc$ [17]. For a harmonic oscillator, $p=l=q=2$, $8abc$ is equal to $2^{9/2}\ell_x \ell_y \ell_z$ where $\ell_i = \sqrt{\hbar/m\omega_i}$ is the harmonic oscillator length along the axis of the trap with frequency ω_i. Therefore, $\overline{n} = N_F/(8abc)$ can be used to define a characteristic atomic density. Note that, with the exception of the rigid box, $\overline{n}$ does not correspond to the average density of atoms in the trap. In what follows, we will assume that $\overline{n}$ is fixed and independent of δ.

Equation (6) along with Eq. (3) can be used to calculate $T_C(\mathbf{r})$. It is clear from Eq. (3) that $k_F(\mathbf{r})$ is a maximum at the center of the trap and hence, the density of states at the local Fermi surface, $\rho_F(\mathbf{r}) = mk_F(\mathbf{r})/2\pi^2\hbar^2$, is a maximum there. Consequently, $T_C(\mathbf{r})$ is largest at $\mathbf{r}=0$. As a result, T_C for the trapped gas is equal to $T_C(0)$ since this is the temperature at which Cooper pairing first occurs. As the temperature is further lowered below $T_C(0)$, Cooper pairing spreads out from the center of the trap to the edges. Therefore at $\mathbf{r}=0$ and neglecting the Hartree-Fock term in Eq. (3), the transition temperature is simply

$$\frac{T_C(0)}{T_F} = \alpha \exp\left(-\frac{\hbar\pi}{|a|\sqrt{8mE_F}}\right). \tag{7}$$

Evaluating the critical temperature at the center of the trap has the added benefit that the Thomas-Fermi approximation is expected to be most accurate here.

From Eqs. (6) and (7) we see that for a given value of $\overline{n}$, $T_C(0)/T_F$ is an increasing function of $1/\delta$. Furthermore, $T_F = E_F/k_B$ is also an increasing function of $1/\delta$. Therefore, increasing the confining power of the trap increases not only the ratio of the critical temperature to the Fermi temperature but also the absolute value of the critical temperature. More

TABLE I. T_F and $T_C(0)$ as a function of the confining power of the isotropic power-law trap, for ^{6}Li.

$p=l=q$	E_F(J)	T_F(K)	$T_C(0)/T_F$	$T_C(0)$(K)
1	1.90×10^{-30}	1.38×10^{-7}	1.62×10^{-4}	2.24×10^{-11}
2	5.56×10^{-30}	4.03×10^{-7}	3.58×10^{-3}	1.44×10^{-9}
3	1.06×10^{-29}	7.69×10^{-7}	0.0115	8.84×10^{-9}
4	1.63×10^{-29}	1.18×10^{-6}	0.022	2.60×10^{-8}
5	2.20×10^{-29}	1.59×10^{-6}	0.031	4.93×10^{-8}
∞	1.36×10^{-28}	9.82×10^{-6}	0.116	1.14×10^{-6}

importantly, from the perspective of current experiments, anharmonic traps for which $p,l,q\geqslant3$ will result in higher values of T_C [assuming that all the other terms in Eq. (6) are the same]. This is the central result of this paper.

To illustrate the effect of varying the confining power of the trap, we consider the case of $p=l=q$ for the values of 1 through 5 and ∞. We choose values of ε_i and *a*, *b*, and *c* that correspond to parameters used in current experiments with harmonic traps. In Tables I and II we calculate the Fermi temperature T_F as well as $T_C(0)$ for ^{6}Li and ^{40}K, respectively. For ^{6}Li we consider an isotropic trap with $N_F=10^5$ and $\varepsilon_i=\hbar\omega=\hbar(2\pi\times100\ \mathrm{s}^{-1})$, which gives $a=b=c=\sqrt{2\hbar/m\omega}=5.8\ \mu$m and $\overline{n}=6.4\times10^{13}\ \mathrm{cm}^{-3}$. For ^{40}K, we use values from the experiment by DeMarco and Jin [14]. This gives $N_F=3.5\times10^5$ and $\varepsilon_1=\varepsilon_2=\hbar\omega_r=\hbar(2\pi\times127\ \mathrm{s}^{-1})$ and $\varepsilon_3=\hbar\omega_z=\hbar(2\pi\times19.5\ \mathrm{s}^{-1})$. This gives values for the characteristic lengths of $a=b=2\ \mu$m and $c=5.09\ \mu$m and a characteristic density of $\overline{n}=2.15\times10^{15}\ \mathrm{cm}^{-3}$. Bohn has recently showed that there exists an experimentally accessible Feshbach resonance for two of the hyperfine states of ^{40}K that could be accessed to create a scattering length of $a=-1000a_0$ [9]. We, therefore, adopt this value for *a* since in the absence of a Feshbach resonance, the scattering lengths for ^{40}K would result in unreasonably small values of T_C.

Tables I and II illustrate the dramatic effect that *p* has on T_F and $T_C(0)$. For both ^{6}Li and ^{40}K, there is a two order of magnitude increase in the Fermi temperature as *p* is increased from 1 to ∞. Furthermore, in going from a harmonic trap $(p=2)$ to a rigid box $(p=\infty)$ the Fermi temperatures increase by factors of 24 and 36 for ^{6}Li and ^{40}K, respectively. Even more striking is the change in $T_C(0)/T_F$, which increases by three orders of magnitude over the full range of *p*. Altogether, this implies that by increasing the confining power of the trap (*p*) one could, in principle, increase $T_C(0)$ by as much as three orders of magnitude in comparison to a harmonic potential. Note that in our calculation, we take E_F as the Fermi energy of an ideal Fermi gas. Including the attractive atom-atom interaction will decrease E_F. However, one must now include the Hartree-Fock term $gn(0)$ in Eq. (7), which raises the critical temperature. We checked this effect numerically and found that for the parameters used in the paper, including the atom-atom interactions, increase $T_C(0)$ by a factor of 2–3.

Physically, the increase in T_F and $T_C(0)$ with increasing confining power is a result of the trap being able to confine the atoms to a smaller total volume, which thereby increases the local density. For a rigid box, the atoms are confined to a volume of $8abc$ regardless of the value of E_F. For a harmonic trap, the total volume occupied by the gas corresponds to the extent of the wave function for an atom in the highest occupied state with energy $E_F\gg\hbar\omega$, which has a volume much larger than the volume of the ground-state wave function given by $\sim abc$. In general, the total volume is given by $V(E_F)$. For the isotropic form of the power-law potential, $U(\mathbf{r})=\varepsilon(r/a)^p$, the volume scales as $V(E_F)\sim N_F^{2/(p+2)}$. This explains the increase in T_F and T_C since, in the limit of a homogeneous gas, they depend only on the density of the fermions.

Alternatively, the increase in T_C can also be explained by examining the density of states for an atom with *total* energy between ϵ and $\epsilon+d\epsilon$ [17],

$$\rho(\epsilon)=\frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_{V(\epsilon)}\sqrt{\epsilon-U(\mathbf{r})}\,d^3r. \tag{8}$$

Increasing the confining power of the trap reduces the volume of phase space available to an atom with energy ϵ, $V(\epsilon)$, and as a result, the number of states is reduced. In fact, it is easy to show that $\rho(\epsilon)\sim\epsilon^{\delta-1}$. The reduction in the

TABLE II. Same as Table I for ^{40}K.

$p=l=q$	E_F(J)	T_F(K)	$T_C(0)/T_F$	$T_C(0)$(K)
1	1.71×10^{-30}	1.24×10^{-7}	3.93×10^{-4}	4.87×10^{-11}
2	5.76×10^{-30}	4.17×10^{-7}	7.8×10^{-3}	3.26×10^{-9}
3	1.19×10^{-29}	8.62×10^{-7}	0.023	1.98×10^{-8}
4	1.93×10^{-29}	1.40×10^{-6}	0.04	5.54×10^{-8}
5	2.72×10^{-29}	1.97×10^{-6}	0.054	1.06×10^{-7}
∞	2.12×10^{-28}	1.54×10^{-5}	0.155	2.37×10^{-6}

density of states causes E_F to be increased since the fermions are forced to occupy higher-energy states in order to accommodate all N_F atoms. In the Thomas-Fermi approximation, the density of states on the *local* Fermi surface at the center of the trap is $\rho_F(0)=mk_F(0)/2\pi^2\hbar^2\sim\sqrt{E_F}$. Consequently, an increase in the Fermi energy E_F increases the local density of states and therefore $T_C(0)$, as can be seen from Eq. (7).

Both Eq. (1) and Eq. (5) are valid in the limit of a dilute gas, $|a|n(0)\ll 1$. This approximation begins to break down for $p>5$ when $|a|n(0)\gtrsim 0.2$ for both cases considered here. There are, however, two reasons why this need not be of any great concern. First, the neglect of the Hartree-Fock mean field in Eq. (4) underestimates the actual density of the gas at the center of the trap because the interactions are attractive. The fact that attractive interactions increase the local density for trapped gases is well known for Bose-Einstein condensates [22] and has been previously noted for fermions [3]. Second, Heiselberg has shown that in the regime of intermediate densities, $|a|n>1$, T_C becomes a finite fraction of T_F with values of $T_C/T_F\gtrsim 0.1$ [23]. Therefore, for large confining powers, our results underestimate the actual value of $T_C(0)$.

Finally, we remark that from an experimental point of view, changing the confining power of the trapping potential appears to be realistic. The generation of power-law traps would most easily be accomplished using far-detuned optical dipole traps, which have the added benefit of producing a confining potential that is independent of the hyperfine state of the atoms. In particular, the generation of higher-order Bessel beams with radial intensity profiles proportional to $J_l^2(k_r r)$, where J_l is a Bessel function of integer order and k_r is the radial component of the wave vector, has recently been achieved using an axicon [24]. Bessel beams with $l=1$ to 4 and radii for the hollow core of the beam on the order of tens of micrometers were created. For blue-detuned beams and small $k_r r$, the radial optical dipole potential experienced by the atoms is proportional to $(k_r r)^{2l}$. Two perpendicular intersecting Bessel beams with $l>1$ could be used to create an anharmonic potential. It is worth noting that evaporative cooling of a two-component gas of ^{6}Li to temperatures below T_F in an optical trap has recently been demonstrated experimentally [13].

In conclusion, we have examined the effect that a power-law trapping potential has on the BCS transition temperature. We have shown that by increasing the confining power of the trap, one can obtain values of T_C that are several orders of magnitude larger than the corresponding harmonic trap. The source of the increase is the ability of tighter traps to confine the atoms to a smaller total volume.

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