Determination of singlet lens paraxial optical properties

Introduction: It is often necessary to determine singlet lens paraxial optical properties for such different reasons as incoming inspection to see if the lenses meet specification, precise determination of some property not controlled by the tolerance so lens spacings can be re-calculated or for reverse engineering a lens assembly. This note shows how an autostigmatic microscope can be used to find the lens properties by locating five conjugates of the lens, the axial locations of the two vertices, the two centers of curvature and the back focal length.

The five conjugates can all be located with the microscope located on one side of the lens provided the microscope has a working distance larger than the longest convex radius. If a convex radius is longer than the working distance of the objective the properties can still be found by reversing the lens and measuring from the other side. This is particularly useful when there is a long convex radius because turning the lens around makes the convex surface into a concave one that can be reached.

First the paraxial formulas for the conjugates will be derived and applied to an example catalog singlet lens. It will be shown that the formulas are independent of which side of the lens is being viewed. Then matters become more interesting. The formulas assume you know the radii, center thickness and index from which you can find the conjugates. Particularly if the wish is to reverse engineer the lens then it is desired to determine the index, center thickness and radii from the conjugates.

This is possible using either a multi-configuration lens design model and asking the optimizer to find the four parameters that match the conjugates, or use the optimizer (the “Solver” add-in) in a spreadsheet such as Excel to do the same. It is often easier to use the spreadsheet than to set up the four configurations needed to solve for the four lens parameters. The spreadsheet example will be illustrated although the model has been verified using Zemax.

One thing that should be kept in mind is that the solutions are paraxial. The algebraic solutions are exact paraxial ones whereas the Zemax solutions are only paraxial in the limit of a very small cone angle or beam diameter. This is pointed out for those who want to test the algebraic formulas against Zemax or other lens design packages. The answers will only be the same in the limit of a paraxial model.

Formulae: The formulae for the five conjugates are most easily discussed with the aid of Fig. 1 that shows the five axial settings of the autostigmatic microscope. The Figure assumes an objective with a 10 mm working distance and this is illustrated in the central configuration where the objective is focused on the upper vertex of the lens to produce a Cat’s eye reflection. All distances calculated will be with respect to the front vertex with positive distances extending into the lens. The upper surface radius will be designated R_1, the physical center thickness as t, and the index of refraction as n.

For the particular example given it is a Philips molded plastic lens CAY033 and the design is in the Zemax lens catalog and the Thorlabs catalog. Both surfaces are aspheric but this doesn’t matter since it is a paraxial model. In the design $R_1 = 2.237$, $R_2 = -3.484$, $t = 2.7$ and $n = 1.489$.

The simplest conjugate to find (assuming it is less than the objective working distance) is the first surface radius as in the configuration second from the left. Since the radius R_1 is +2.237 the distance from the objective to the front surface is 7.763 when the objective focus is at the center of curvature of the front surface. In other words, the center of curvature is 2.237 below the front surface that acts like a convex mirror of that radius.

The next least complicated conjugate is the optical center thickness (ct) of the lens and depends on R_1, n and t. Here the ray starts at R_2 with a height of zero and heads toward R_1 at an angle u_0. It reaches R_1 at a height y_1 where it is refracted at angle u_1. The optical thickness, t_O, is the distance from R_1 that the refracted ray intersects the x axis as in Fig. 2.

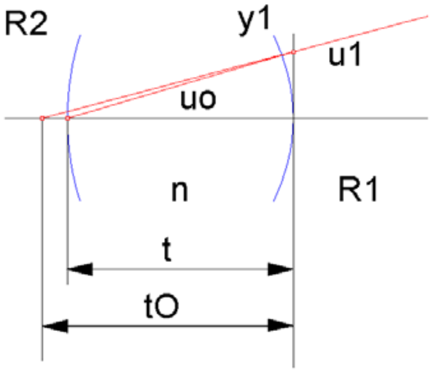


Fig. 2 Paraxial diagram of rays to find the optical center thickness

Doing the first order ray trace we find that

$$t_O = {}^{y_1}\!\big/_{u_1} = \frac{-tR_1}{nR_1 + t(n-1)} = \frac{-2.7 * -2.237}{-1.489 * 2.237 + 2.7 * (1.489 - 1)} = -3.004$$

where R_1 is negative because its center of curvature lies to the left of the surface. However, we have drawn the whole figure backwards to our original diagram to make light travel from left to right as we do the ray trace. When the diagram is reversed we get a positive 3.004 that matches the left hand diagram in Fig. 1. Although it is not obvious here because we've left out a couple steps in the derivation the angle u_1 drops out of the equation as it should for a paraxial calculation because y_1 is proportional to u_1.

Finding the rear surface radius becomes a bit more complex but the calculation is very useful because it permits the measurement of convex radii longer than the working distance of available objectives. Here, in Fig. 3, a ray starts at an arbitrary height y_0 normal to R_2 and intersects R_1 at y_1. The angle $u_0 = y_0/R_2$ so $y_1 = y_0 - ty_0/R_2 = y_0(1 - t/R_2)$ while $u_1 = n(u_0 - (y_1/R_1))$. Then since both y_1 and u_1 are proportional to y_0, and y_0 is arbitrary it drops out of both numerator and denominator so

$$T = {y_1}/{u_1} = \frac{\left(1 - {t}/{R_2}\right)}{\frac{n}{R_2} - \left[\frac{(n-1)\left(1 - \left({t}/{R_2}\right)\right)}{R_1}\right]} = \frac{.225}{.427 - \left(.489 * .225/{-2.237}\right)} = \frac{.225}{.4766} = .472$$

Again Fig. 3 is reversed relative to Fig. 1 so T is -.472. Notice that T is a function of all four lens parameters.

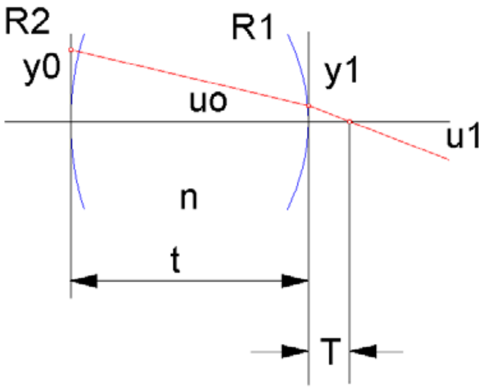
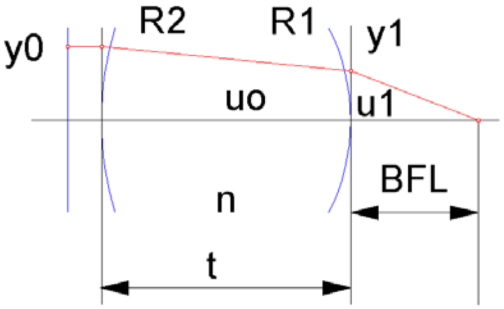


Fig. 3 Paraxial diagram of rays to find the optical rear surface center of curvature

Lastly we look at the back focal length as shown in Fig. 4.



Here a ray starts at an arbitrary height y_0, is refracted at angle u_0 by R_2 to strike R_1 at y_1. Here it is refracted again at angle u_1 and crosses the axis at a distance BFL from R_1. Following the method above we find $u_0 = -y_0(n - 1)/nR_2$ so $y_1 = y_0 - ty_0(n - 1)/nR_2 = y_0[1 - t(n - 1)/nR_2]$. On refraction $u_1 = nu_0 - y_1(1 - n)/R_1$. On substitution

$$u_1 = -y_0(n-1)\left[\frac{1}{R_2} - \frac{1}{R_1} + \frac{t(n-1)}{nR_1R_2}\right]$$

Those who are really familiar with paraxial optics will recognize that $-u_1/y_0$ is the reciprocal focal length so once we calculate the BFL we will also know the efl. The advantage of finding the efl this way is that a nodal slide (read expensive) is not needed.

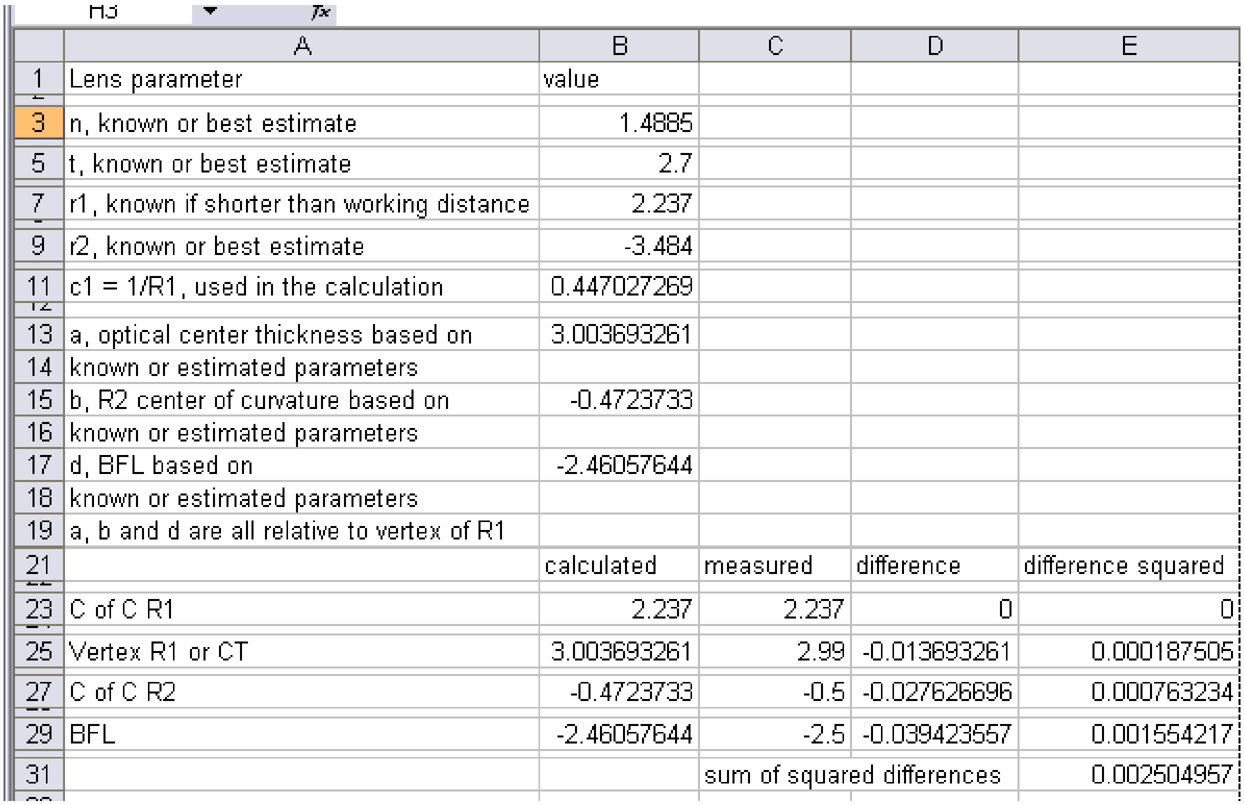
To finish off

$$BFL = \frac{y_1}{u_1} = \frac{\left[1 - \frac{t(n-1)}{nR_2}\right]}{(n-1)\left[\frac{1}{R_2} - \frac{1}{R_1} + \frac{t(n-1)}{nR_1R_2}\right]} = \frac{.7455}{.489(.287 + .447 - .114)} = \frac{.7455}{.3032} = 2.459 .$$

These measurements are good to have and very useful to see if an optical component meets specifications from a vendor. However, in certain cases it would be useful to know R_2, n and t given the optical R_2, center thickness and back focal length. Because the variables are so thoroughly mixed up in the three equations there is no algebraic solution. An iterative solution is the only possibility aside from a three or four configuration lens design model.

Fortunately, Microsoft Excel has an optimizer as an add-in under the Tools tab called “Solver”. If the Solver does not appear when the tab is clicked it will have to be downloaded from the Office disk or from the MS online support.

There are probably a dozens ways of setting up the solution but the one given here works even if it does not look pretty.



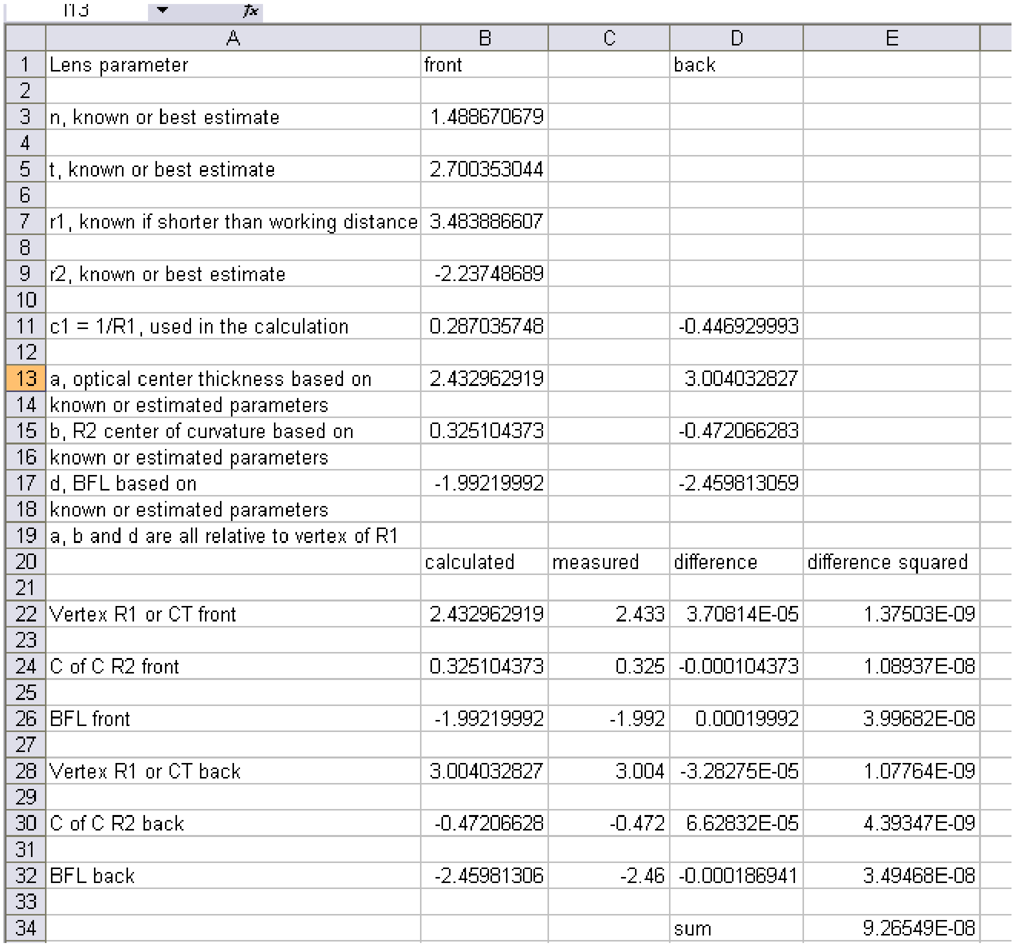
	A	B	C	D	E
1	Lens parameter	value			
3	n, known or best estimate	1.4885			
5	t, known or best estimate	2.7			
7	r1, known if shorter than working distance	2.237			
9	r2, known or best estimate	-3.484			
11	c1 = 1/R1, used in the calculation	0.447027269			
13	a, optical center thickness based on	3.003693261			
14	known or estimated parameters				
15	b, R2 center of curvature based on	-0.4723733			
16	known or estimated parameters				
17	d, BFL based on	-2.46057644			
18	known or estimated parameters				
19	a, b and d are all relative to vertex of R1				
21		calculated	measured	difference	difference squared
23	C of C R1	2.237	2.237	0	0
25	Vertex R1 or CT	3.003693261	2.99	-0.013693261	0.000187505
27	C of C R2	-0.4723733	-0.5	-0.027626696	0.000763234
29	BFL	-2.46057644	-2.5	-0.039423557	0.001554217
31			sum of squared differences		0.002504957

The first values in B3 – B9 are taken from the design or are estimates based on spherometer and indicator readings. Obviously n has to be from the design. It is these values that will be varied in the Solver to find a solution based on the measured data.

In elements B11 – B17 the values are calculated based on the design numbers or estimates in B3 – B9 using the formulae derived above. In B23 – B29 the values are just transferred from B11 – B17. In C23 – C29 are the values measured using an autostigmatic microscope. These measurements should equal the calculated values if the correct values have been entered for n, t, and the two radii.

To find a least squares solution the differences between the calculated and measured values are entered in column D. These differences are squared in column E and then summed in E31. When the solver button is clicked the "Target Cell" is E31 and it should be set to zero. Enter B3, B5, B7 and B9 in the "By changing cells" line. Then hit "Solve". The options box is used to set a tolerance on how close an acceptable answer is. The solution appears instantaneously.

Of course, the radius of R_1 is known to the depth of focus of the microscope and the accuracy of the axial scale assuming the radius is less than the working distance of the objective; it is the other three parameters that are of real interest. If R_1 is longer than the working distance then the lens has to be measured from both sides so the long radius convex curve appears as though it is concave. This situation can also be solved using the spreadsheet set up slightly differently.



	A	B	C	D	E
1	Lens parameter	front		back	
2					
3	n, known or best estimate	1.488670679			
4					
5	t, known or best estimate	2.700353044			
6					
7	r1, known if shorter than working distance	3.483886607			
8					
9	r2, known or best estimate	-2.23748689			
10					
11	c1 = 1/R1, used in the calculation	0.287035748		-0.446929993	
12					
13	a, optical center thickness based on	2.432962919		3.004032827	
14	known or estimated parameters				
15	b, R2 center of curvature based on	0.325104373		-0.472066283	
16	known or estimated parameters				
17	d, BFL based on	-1.99219992		-2.459813059	
18	known or estimated parameters				
19	a, b and d are all relative to vertex of R1				
20		calculated	measured	difference	difference squared
21					
22	Vertex R1 or CT front	2.432962919	2.433	3.70814E-05	1.37503E-09
23					
24	C of C R2 front	0.325104373	0.325	-0.000104373	1.08937E-08
25					
26	BFL front	-1.99219992	-1.992	0.00019992	3.99682E-08
27					
28	Vertex R1 or CT back	3.004032827	3.004	-3.28275E-05	1.07764E-09
29					
30	C of C R2 back	-0.47206628	-0.472	6.62832E-05	4.39347E-09
31					
32	BFL back	-2.45981306	-2.46	-0.000186941	3.49468E-08
33					
34				sum	9.26549E-08

The upper left of the spreadsheet looks the same as the previous one but now in D11 – D17 are the calculated values for the center thickness, rear center of curvature and bfl looking through the lens from the other side. Now the calculated values are transferred down into elements B22 – B32 from all six measurements while the measured values are entered into C22 – C32. Now we have six equations for finding four unknowns. The solution is handled the same way. The difference between calculated and measured are squared and summed in E34. In the Solver this is now the Target cell and B3 – B9 are still the cells to change.

There are a few cautions; signs must be correct and the estimates have to be somewhat close since there are at least three possible solutions to the three equations. If the guess is too far off another solution may be found. Common sense should indicate if this is the case.