

2.0 QUALIFICATION OF OPTICAL MATERIAL

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Materials for optical parts are generally given some inspection before they are set up for grinding because the cost of the optical work is often quite large compared with the cost of the material.

2.1 Internal Defects

Bubbles, seeds, stones, and striae can be detected by illuminating the sample with a bright light and observing the scattered light. The use of a shadow graph or Foucault knife-edge (Schlieren) is very useful in detecting internal defects. In the Schlieren test, shown in Fig. 2-1, the irregularities become visible since they deviate light past the stop. If the surfaces are not polished, they may be "indexed out" by use of an index matching fluid and glass plates of trusted quality. Internal defects must be a minimum for systems having low scatter requirements and elements used near a focal plane such as a field lens or reticles.

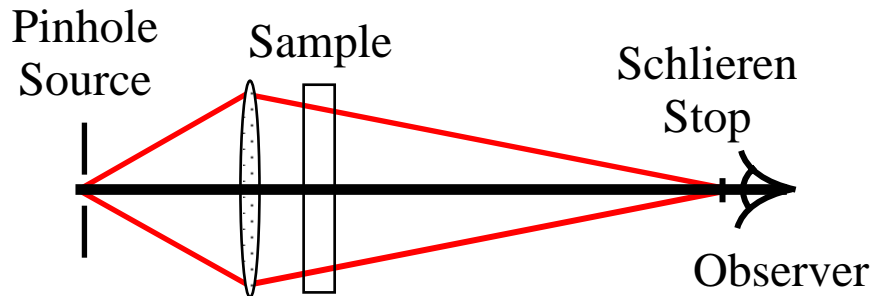


Fig. 2-1. Schlieren test for observing bubbles, seeds, stones, and striae.

The refractive index uniformity often must be known to better than one part in the fifth decimal place, and in some cases one or two parts in the sixth place. These measurements can be performed using a Mach-Zehnder or Twyman-Green interferometer. The surfaces must either have good quality or be "indexed out" with plates of trusted surface quality and uniform index. The oil layer of index close to that of the test piece is so thin as not to affect the results and yet it eliminates optical path variations introduced by surface irregularity.

Table 1 gives the Schott bubble and refractive index uniformity specifications.

Table 1. Schott Bubble and index uniformity specifications

Bubble classification (includes all bubbles and inclusions $\geq 0.06\text{mm}$)

Bubble Group	Total bubble cross-section per 100cm^3 volume of glass
0	0 – 0.029mm^2
1	$0.03 – 0.10\text{mm}^2$
2	$0.11 – 0.25\text{mm}^2$
3	$0.26 – 0.50\text{mm}^2$
4	$0.51 – 1.00\text{mm}^2$
5	$1.01 – 2.00\text{mm}^2$

Index uniformity

(A) Normal Quality (N) tested for striae and birefringence in one direction. Normal quality \Rightarrow variation of $n_d \leq \pm 1 \times 10^{-4}$, within one melt.

NH1 - $\Delta n_d \leq \pm 2 \times 10^{-5}$ within one melt

NH2 - $\Delta n_d \leq \pm 5 \times 10^{-6}$ within one blank

(B) Precision Quality (P) tested in one or more directions

$\Delta n_d \leq \pm 5 \times 10^{-6}$ within one blank

2.2 Refractive Index

In many optical systems, not only is the refractive index homogeneity of interest, but the absolute refractive index must be precisely known. The two most common refractometers are spectrometric systems and critical angle systems.

2.2.1 Spectrometers

The refractive index of a substance is a ratio. This implies the knowledge of two pieces of information. In the case of a spectrometric system used for measuring the refractive index of a sample in prismatic form, these two pieces of information are the angle of the sample prism and the angle through which a beam of light is deviated by the prism, under known conditions. For very accurate work the tolerances on both of these angles are small. This carries the implication that the sample prism must be very good in an optical sense.

For the case of a prism of angle A ., producing for some particular wavelength a minimum deviation angle D , the refractive index of the prism relative to air is given by

$$n = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A} \quad (2.1)$$

Partial differentiation of this expression with regard to A and D shows that for typical values of the angle and deviation, errors in measurement of A are more significant than errors in the measurement of D . Recognition of this result is important in the design and use of spectrometers for refractometry purposes.

The derivation of Eqn. 2.1 is shown in the appendix.

2.2.1.1 Basic Spectrometer Technique

Figure 2-2 shows the preferred methods for measuring prism and deviation angles. (A drawing of an autocollimator, an essential component in the basic spectrometer technique, is shown in the appendix.) Two precautions should be taken in performing the measurements. First, to reduce effects of aberrations in the telescope objective, the optical axis of the telescope must intersect the two refracting faces of the sample centrally; and second, the two prism faces should be perpendicular to the optical axis of the telescope in the vertical plane.

Apart from taking precautions against thermal and like errors, if each angle is measured to 1 arc second, n can be measured to one part in the fifth decimal place. With a well-designed instrument, accuracies approaching one part in the sixth decimal place can be reached.

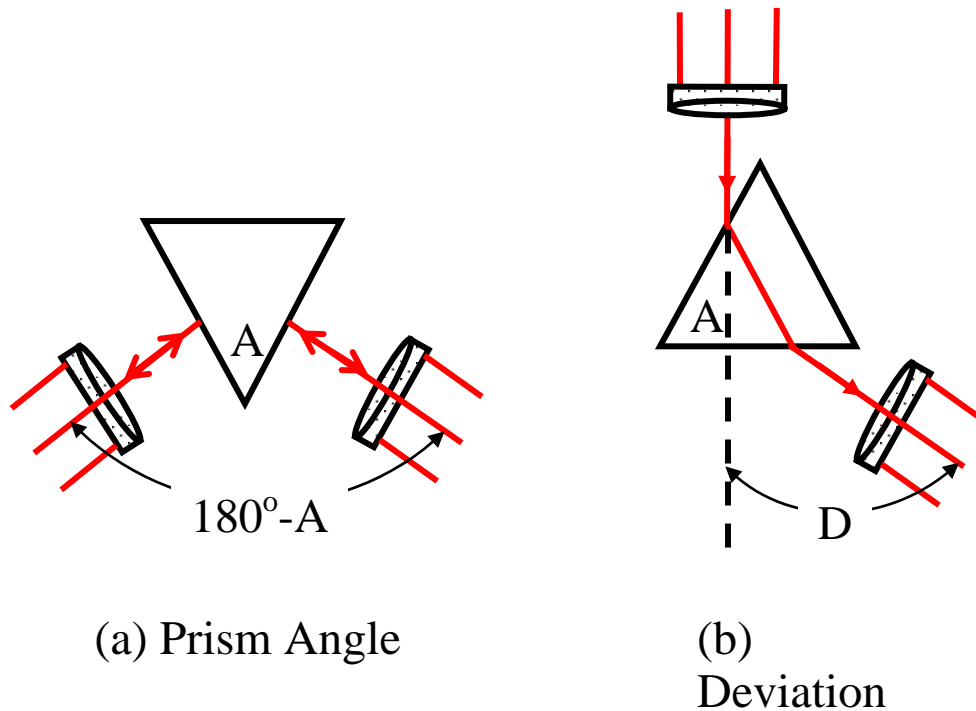


Fig. 2-2. Preferred methods for measuring prism and deviation angles.

2.2.1.2 Autocollimating Goniometer

The collimator and telescope can be combined into one stationary auto collimating telescope in which only the prism table rotates as in Fig. 2-3. The angular distance between the refracting-face normal and the direction of a beam normally reflected from the second surface of the test piece gives the angle of incidence corresponding to an internal angle of refraction equal to the prism angle. The angular distance between the normals to the two surfaces of the test prism gives directly the prism angle itself. The refractive index is then obtained from the ratio of the sines of these two angles. Sixth-place accuracy is achievable.

2.2.1.3 Hilger Chance Refractometer

In making refractive-index measurements with a spectrometric apparatus, the major time involved is spent in preparing the samples. The normal preparation consists of grinding and polishing two plane refracting faces to a reasonable degree of flatness, and then grinding the base of the prism perpendicular to the refracting surfaces. The samples for measurement on the Chance refractometer require only the grinding and smoothing of two faces at 90° to each other. Neither the angle nor the flatness is critical. This saving in preparation time is the feature of major value.

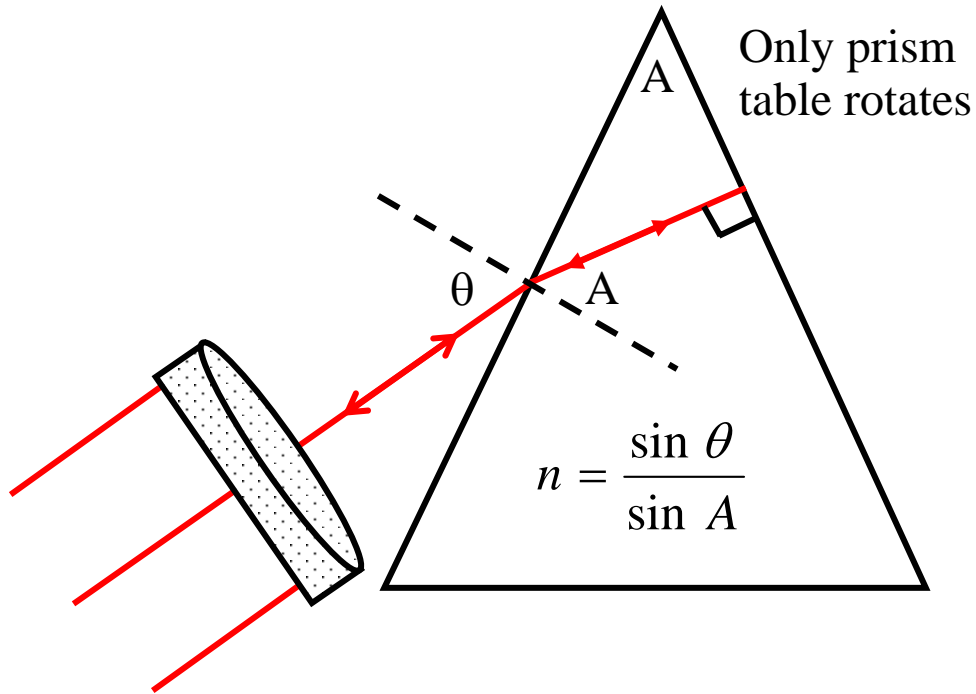


Fig. 2-3. Autocollimating goniometer.

The instrument is effectively a goniometer working in the vertical plane (see Fig. 2-4). A fixed horizontal collimator has in its focal plane a glass slit consisting of a horizontal clear slot in an opaque background, with a fine opaque line running along the axis of the slot.

Following the collimator is a glass V-block sample holder. This consists of two 90°-45° glass prisms, optically contacted together with heat treatment. The V-block forms a 90° prismatic cavity into which is placed the test piece. The gray surfaces of the test piece are made to be transmitting by use of immersion oil, the refractive index of which is near that of the test prism.

The relationship between the refractive index n_1 of the sample and the refractive index n of the V-block is given by

$$n_1 = \left[n^2 - \sin \theta \left(n^2 - \sin^2 \theta \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad (2.2)$$

(A derivation of Eqn. 2.2 is given in the appendix.) Under normal conditions, an accuracy of two units in the fifth decimal place can be readily achieved. In the case of liquids (these being contained in a V-block with fused sides), this accuracy is more difficult to reach because of temperature control problems.

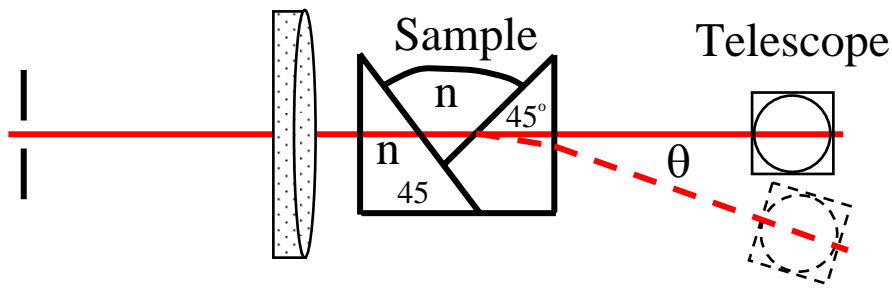


Fig. 2-4. Schematic diagram of Hilger Chance spectrometer refractometer.

2.2.2 Critical Angle Systems

The critical angle condition at which total internal reflection commences is put to good use in a large number of instruments. There are three primary reasons for critical angle refractometers being popular. First, the problem of measuring the angle of the sample prism is avoided and is finally transferred to the maker of the instrument. Second, recognition of the critical angle boundary completely specifies the angle of incidence. The third advantage is that the sample, be it a liquid or a solid, can be colored or even opaque since with many of these instruments the illumination can be arranged to be reflected from the sample. There is a corresponding disadvantage: when a critical angle measurement is made, only the skin refractive index is measured, and this may be different from the bulk refractive index.

The basic principle of critical angle systems is illustrated in Fig. 2-5. If a solid is being measured the fluid is a high index oil such that $n_2 > n_1$; also a necessary condition is that $n_3 > n_1$. θ_1 can go to 90° , in which case $n_1 = n_3 \sin\theta_3$, and if n_3 is known and θ_3 is measured, n_1 can be determined. If the refractive index of a fluid is measured, the solid should have a high index such that $n_1 > n_2$. A second requirement is that $n_3 > n_2$. θ_2 can go to 90° , in which case n_2 is given by $n_2 = n_3 \sin\theta_3$.

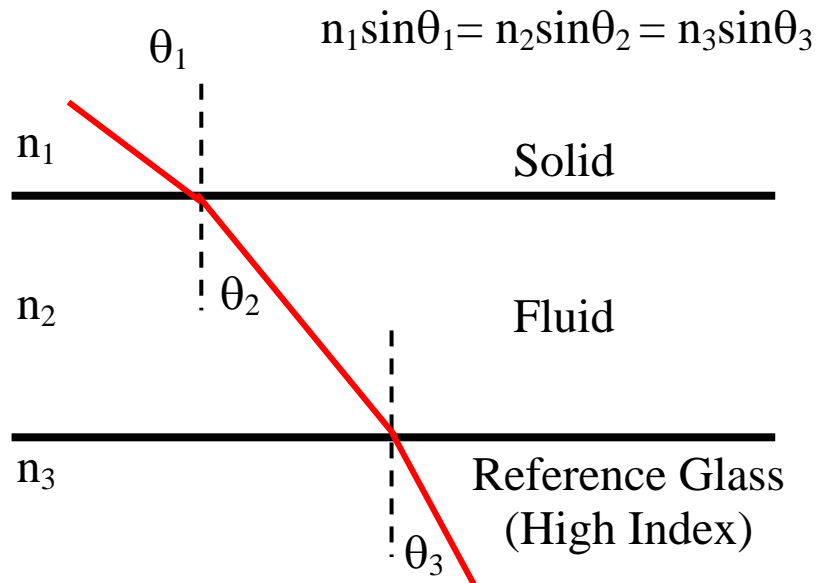


Fig. 2-5. Basic principle of critical angle measurement of refractive index.

2.2.2.1 Abbe Refractometer

Of the critical angle refractometers in common use, the so-called Abbe refractometer is by far the most important. In the classical form of the Abbe refractometer the essential components are a pair of prisms, one being a measuring prism (the second being used for illumination), and a simple inverting telescope in front of which is placed a pair of counter rotating Amici prisms.

The two prisms are usually of 30°-60°-90° construction, made from glass, the refractive index of which is higher than the upper limit of the instrument range. The prisms are housed, one in each half of a hinged water-jacket. The two halves of the water jacket, when brought together enclose a thin film of the liquid being measured. When a solid is being measured the water jacket is left open and the solid is placed on the measuring prism with a contact liquid of high refractive index placed between (see Fig. 2-6).

In use, the telescope is rotated to pick up the critical angle boundary, which in white light normally appears colored. The Amici prisms are rotated in order that they may impose a dispersion equal and opposite to the dispersion of the measuring prism plus the sample system. When this condition is reached the boundary becomes almost achromatic; it can then be set accurately on the telescope cross webs. The rotational passage of the telescope relative to the sector scale then gives the refractive index of the test sample.

"White light" Abbe refractometers are normally calibrated to read n_D , the sample refractive index for sodium yellow radiation. The Amici prisms used to annul the dispersion of the sample-plus-prism combination are designed to allow sodium D light to pass undeviated; hence, the direction of the observed achromatic boundary coincides with that due to sodium D light conditions, and the instrument will read directly in n_D .



Fig. 2-6. Typical classical Abbe refractometer opened to show prism system.

The sector scale of an Abbe refractometer is normally direct reading in refractive index. The dispersion of the sample is derived from tables by use of the refractive index and the rotation of each Amici prism.

Normally, laboratory forms of Abbe refractometers can give an accuracy of from one to two units in the fourth decimal place and an indication of the dispersion of one unit in the fourth decimal place.

As shown in the appendix, the Abbe refractometer can also be used to measure the thickness and refractive index of a thin film coated on a substrate.

2.2.2.2 Pulfrich Refractometer

The Pulfrich refractometer is a critical angle refractometer, generally used for more accurate work than is attempted with the Abbe refractometer. The general arrangement has a superficial similarity to the Chance refractometer. The main differences are that a condensing system, in place of the collimator, is used to provide a horizontal converging beam, and a critical angle prism system is used in place of the V-block prism of the Chance instrument (see Fig. 2-7).

Solid samples require two polished surfaces at a right angle; the intersecting edge must be clean and free from chips, while the surface in contact with the block must be flat. As usual, an immersion liquid is used between the sample and the block, but here the sample surface must be set parallel to the block surface, at least in the direction of the illuminating beam. Interference fringes set parallel to the axis of the system provide the criterion.

If the block has an angle of 90° , the refractive index n_1 of the specimen is obtained from

$$n_1 = (n^2 - \sin^2 \theta)^{\frac{1}{2}} \quad (2.3)$$

in which n is the refractive index of the block and θ is the final angle of emergence obtained from the telescope setting.

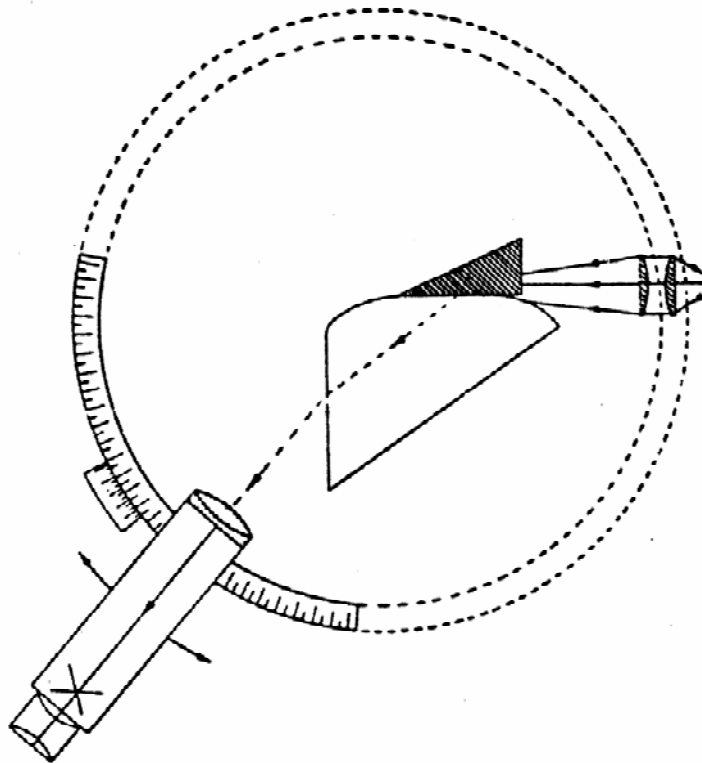


Fig. 2-7. Optical diagram of Pulfrich refractometer.

A well made instrument will give an accuracy of about two or three units in the fifth decimal place, if it is used with care and with a well prepared sample. Temperature control is also important.

2.2.3 Ellipsometry (See the appendix)

2.3 Strain

If a flat piece of glass is subjected to a steady longitudinal tension, the glass becomes slightly elongated (strained); as a consequence it becomes birefringent. If the strain is small, the retardation is proportional to strain. The directions of fast and slow axes indicate principal axes of strain. In most substances, rays vibrating in a plane parallel to direction of compressive strain travel more rapidly than rays vibrating in the direction of tensile stress.

Birefringence is expressed in milli-micrometers (or nanometers) per centimeter of thickness. The best annealing, even in large pieces, reduces the strain to less than 5 nm per centimeter, while for less exacting applications the strain may be as high as 30 nm per centimeter. In round blanks, higher strains can be tolerated if the strain pattern is symmetrical, but non-symmetrical strain should be carefully avoided.

A polariscope, as shown in Fig. 2-8, is often used to measure the amount of strain introduced into a sample. The sample is placed between crossed polarizers as shown. The amount of light restored and its color at any given point depend upon the directions and magnitudes of principal strain at that point. If the directions of principal strains are parallel and perpendicular to the plane of polarization, the field remains dark. If the directions of principal strain make some angle with the plane of polarization, the light is partially restored and the color of the field depends upon the strain difference. As the strain increases, the color changes from black to orange, red, blue-green, etc., the colors repeating themselves until they fade out in the fourth or fifth order. The precision of the method can be increased considerably by introducing a sensitive-tint plate, T, in front of the analyzer. The sensitive tint plate retards green light by a full wavelength and therefore causes the field to appear magenta where the specimen is free of strain. In regions where strain is present, however, the path difference between the ordinary and extraordinary beam is increased or decreased, and some color other than green is extinguished. More retardation changes the color toward blue, while less retardation changes the color toward red. The eye is very sensitive to color changes around this region and a change in path length of 10 nm ($\sim \lambda/50$) produces a perceptible change in color. The system can also be calibrated by using a Babinet or Soleil compensator instead of a sensitive tint plate.

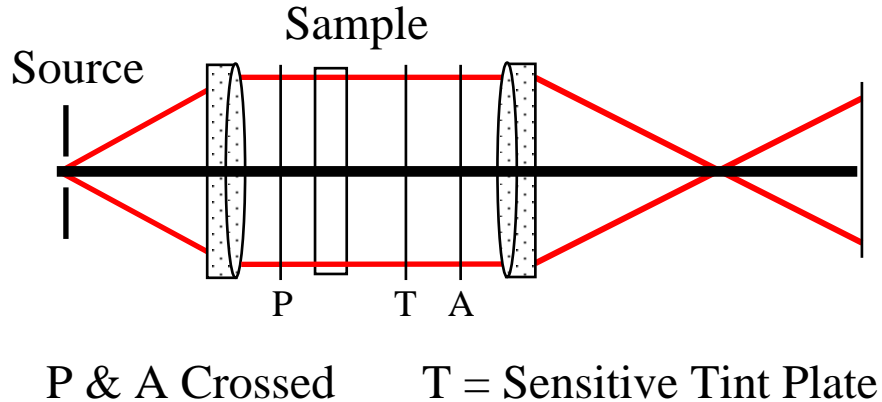


Fig. 2-8. Polariscope.

(See the appendix for a description of measuring birefringence and measuring the Stokes Parameters.)

2.4 Mechanical and Thermal Properties

Strain introduced by mechanical stress or temperature gradients can be measured by use of a polariscope as described above. Also, a Twyman-Green or Mach-Zehnder interferometer can be used to measure optical path differences introduced by mechanical stress or temperature gradients. If the surfaces are not flat, the surfaces must be "indexed out."

Holographic interferometry can be used to determine the mechanical and thermal properties of large unworked mirror blanks before any time and money are spent putting an optical surface on the blank.

Let a hologram be made of an unworked mirror blank, as illustrated in Fig. 2-9. It is convenient to coat the front surface of the test blank with a retro-reflective paint, such as Coddington reflective liquid, so a relatively large percentage of the illuminating flux is directed back in the direction of the source to reduce the required exposure time and give a uniform reflectance across the blank, even over large cone angles. After suitable exposure and processing, let the recording medium be replaced in exactly the same position it occupied during the exposure. When the hologram is illuminated with the original reference wavefront, a reconstructed wavefront is produced that, apart from a constant amplitude and phase factor, is an exact replica of the original object wavefront. If at the same time the mirror blank is illuminated in the same manner as during the hologram exposure, there will be two wavefronts emerging from the hologram plane, a wavefront emanating from the mirror blank, and the reconstructed wavefront produced by the hologram. The amplitudes of the two interfering wavefronts should be made equal, which can be achieved, for example, by use of a variable transmittance-reflectance beamsplitter.

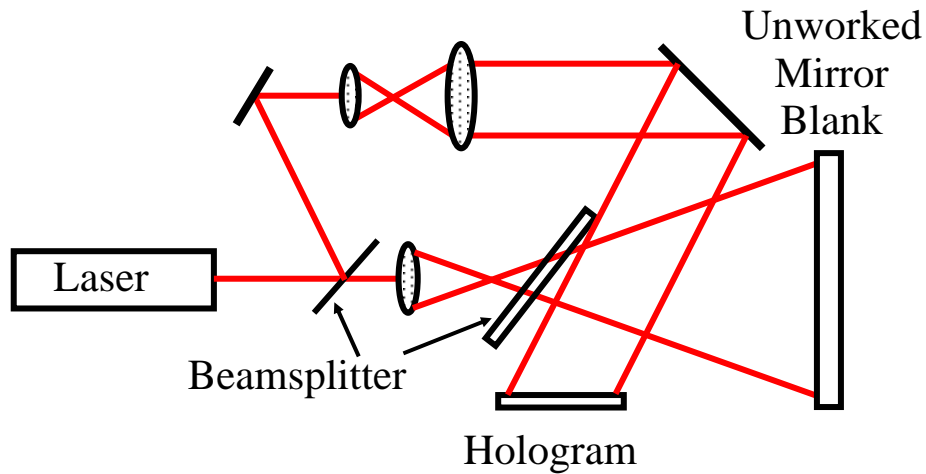


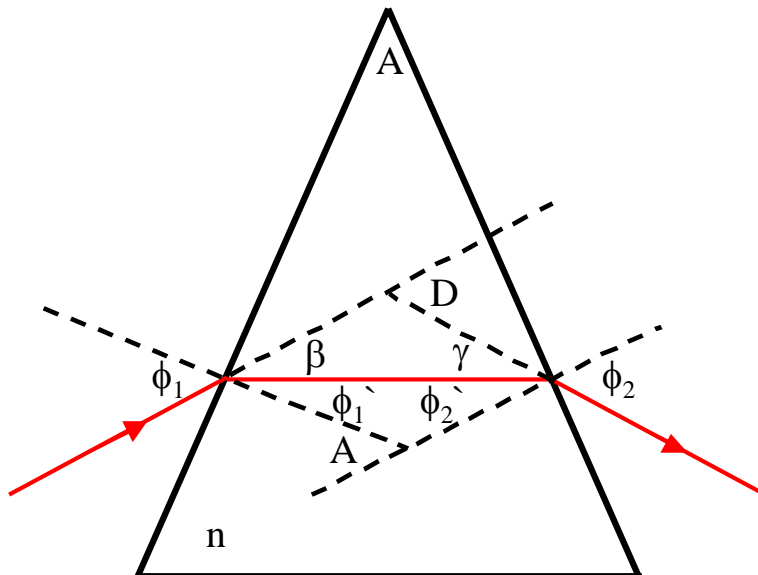
Fig. 2-9. Hologram interferometric test system.

If the mirror blank's shape is unchanged, the two interfering wavefronts should produce a single interference fringe. If the blank's shape is distorted, for example by application of a mechanical load or a temperature gradient, the wavefront coming directly from the object will be different from the stored wavefront, and the interference of the two nearly identical wavefronts will produce interference fringes from which the object's deformation can be determined. For an on-axis test setup, each interference fringe corresponds to half a wavelength surface deformation in the illumination and viewing direction.

Double-exposure holographic interferometry can also be used. If two holographic exposures of a given object are superimposed on a single recording medium, and between exposures the object is deformed, when the hologram is illuminated with the original reference wavefront, two virtual images of the object are reconstructed. One image is of the object in the original shape, whereas the second image shows the object in the deformed shape. Just as for the single-exposure case, interference fringes give the amount of deformation.

The Appendix follows.

Minimum Angle of Deviation



n is the refractive index of the prism, A is the prism angle, and D is the angle of deviation.

$\phi_1 = \phi_2$, otherwise by reversibility of light rays there would be two different angles of incidence giving a minimum angle of deviation.

$$\phi_1' = \phi_2'; \quad A = 2 \phi_1';$$

$$\beta = \gamma; \quad D = 2 \beta;$$

$$\phi_1 = \phi_1' + \beta$$

Therefore,

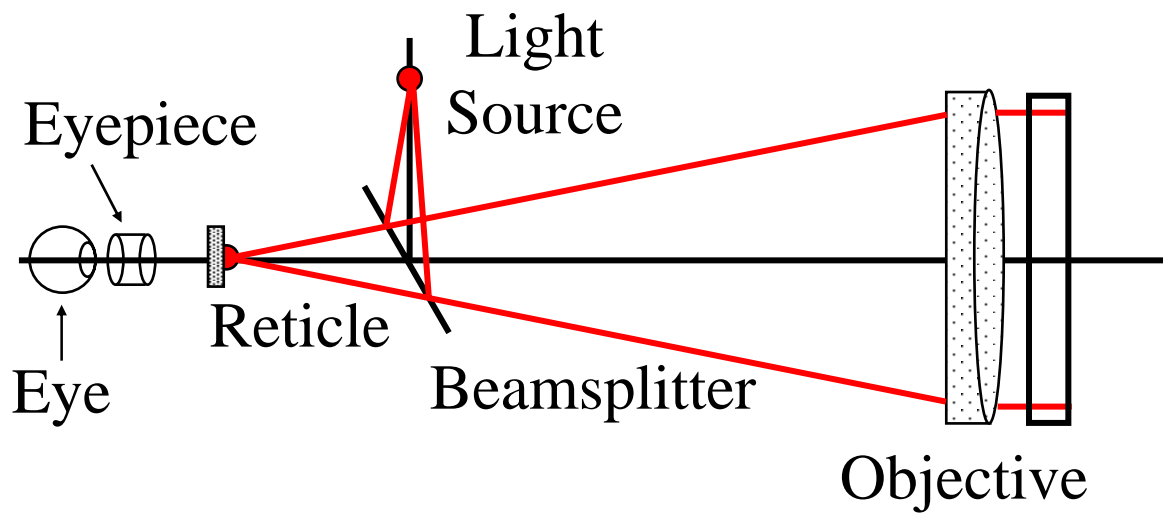
$$\phi_1' = \frac{1}{2} A; \quad \phi_1 = \frac{1}{2} (A + D);$$

From Snell's law

$$n \sin[\phi_1'] = \sin[\phi_1]$$

$$n = \frac{\sin[\frac{1}{2} (A + D)]}{\sin[\frac{1}{2} A]}$$

Autocollimator

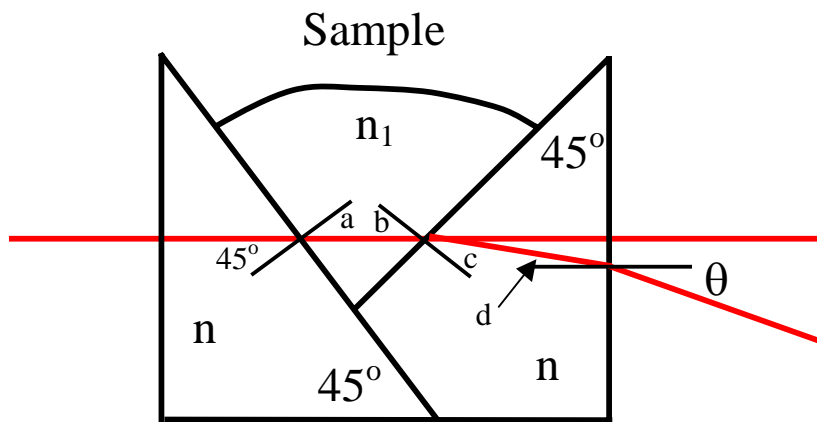


Hilger-Chance Refractometer

Show that for the Hilger-Chance refractometer the relationship between the refractive index n_1 of the sample and the refractive index n of the V-block is given by

$$n_1 = \left(n^2 - \sin[\theta] \left(n^2 - \sin[\theta]^2 \right)^{1/2} \right)^{1/2}.$$

Solution



$$n \sin[45^\circ] = n_1 \sin[a] \Rightarrow \sin[a] = \frac{n}{n_1} \sin[45^\circ]$$

$$a + b = 90^\circ, \quad c + d = 45^\circ, \quad \sin[\theta] = n \sin[d]$$

$$n_1 \sin[b] = n \sin[c] \Rightarrow n_1 \cos[a] = n \sin[45^\circ - d] \quad (1)$$

From (1)

$$n_1 \sqrt{1 - \frac{n^2}{n_1^2} \sin[45^\circ]^2} = n \frac{\cos[d] - \sin[d]}{\sqrt{2}}$$

$$\sqrt{1 - \frac{n^2}{2 n_1^2}} n_1 = n \frac{-\frac{\sin[\theta]}{n} + \sqrt{1 - \frac{\sin[\theta]^2}{n^2}}}{\sqrt{2}}$$

$$\left(\sqrt{1 - \frac{n^2}{2 n_1^2}} n_1 \right)^2 = \left(n \frac{-\frac{\sin[\theta]}{n} + \sqrt{1 - \frac{\sin[\theta]^2}{n^2}}}{\sqrt{2}} \right)^2$$

$$-\frac{n^2}{2} + n_1^2 = \frac{1}{2} n^2 \left(-\frac{\sin[\theta]}{n} + \sqrt{1 - \frac{\sin[\theta]^2}{n^2}} \right)^2$$

$$\text{Solve}\left[-\frac{n^2}{2} + n_1^2 == \frac{1}{2} n^2 \left(-\frac{\sin[\theta]}{n} + \sqrt{1 - \frac{\sin[\theta]^2}{n^2}}\right)^2, n_1\right]$$

$$\left\{ \left\{ n_1 \rightarrow -\sqrt{n^2 - n \sin[\theta]} \sqrt{1 - \frac{\sin[\theta]^2}{n^2}} \right\}, \left\{ n_1 \rightarrow \sqrt{n^2 - n \sin[\theta]} \sqrt{1 - \frac{\sin[\theta]^2}{n^2}} \right\} \right\}$$

$$n_1 = \sqrt{n^2 - \sin[\theta]} \sqrt{n^2 - \sin[\theta]^2}$$

Abbe Thin Film Measurement

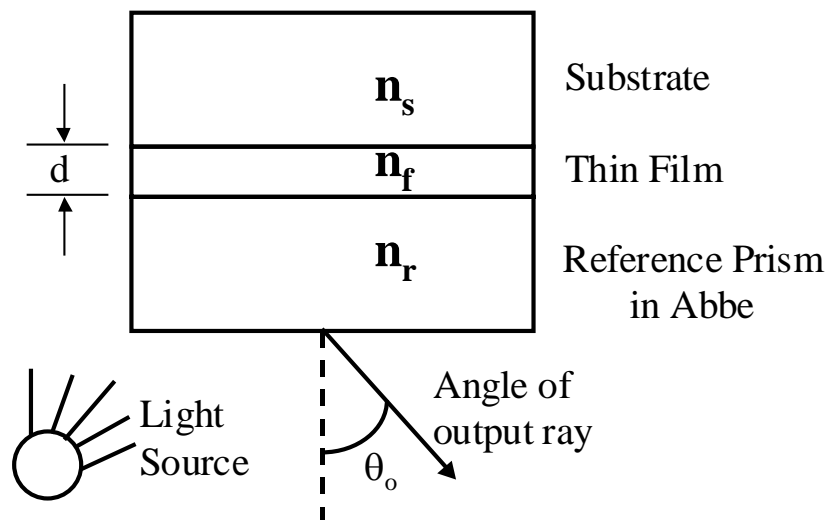
The Abbe refractometer is used to measure the thickness and refractive index of a thin film coated on a substrate having a higher refractive index than the film. When the thin film is contacted to the measuring prism and light is transmitted up through the measuring prism, the light reflected off the two surfaces of the thin film is observed. Interference fringes are seen. The first two dark interference fringes are designated n_1 and n_2 respectively, where n_1 is the one with apparently higher index of refraction. Show that n_f , the index of the film, and d , the film thickness, are given by

$$n_f = \sqrt{\frac{(4 n_1^2 - n_2^2)}{3}}, \quad \text{and} \quad d = \frac{\lambda}{2} \sqrt{\frac{3}{(n_1^2 - n_2^2)}}.$$

Solution

The Abbe thin film measuring technique described below is useful for looking at film thicknesses between 0.05 and 0.0001 mm. The refractive index of the film must be within the range of the instrument and the sample must be flat and uniform in thickness. The sample is looked at in reflection.

When the thin film is contacted to the measuring prism the usual critical angle dividing line disappears and a series of interference fringes are seen. The first two dark interference fringes are used. Figure 1 shows a schematic of the thin film and reference prism.



While the question only asked for the case where $n_s > n_f$, we will look at both the case where $n_s > n_f$ and the case where $n_s < n_f$. For both cases $n_r > n_f$.

■ **Case I: $n_s > n_f$**

$$n_f \sin[\theta_f] = n_r \sin[\theta_r] = n_o \sin[\theta_o] = n_1$$

= index we read on Abbe scale

$$\cos[\theta_f] = \frac{\sqrt{n_f^2 - n_1^2}}{n_f}$$

Since $n_s > n_f$ we have a π phase change upon reflection.

For dark fringe

$$2 n_f d \cos[\theta_f] = m \lambda, \quad m \text{ is an integer}$$

For first dark fringe (other than $\theta_f = 90^\circ$)

$$2 n_f d \cos[\theta_{f1}] = \lambda \quad \text{where} \quad \cos[\theta_{f1}] = \frac{\sqrt{n_f^2 - n_1^2}}{n_f}$$

For second dark fringe we have

$$2 n_f d \cos[\theta_{f2}] = 2 \lambda \quad \text{where} \quad \cos[\theta_{f2}] = \frac{\sqrt{n_f^2 - n_2^2}}{n_f}$$

Therefore,

$$2 d \sqrt{n_f^2 - n_1^2} = \lambda \quad \text{and} \quad 2 d \sqrt{n_f^2 - n_2^2} = 2 \lambda$$

$$\text{filmIndex} = n_f /. \text{Solve}[4 (n_f^2 - n_1^2) == n_f^2 - n_2^2, n_f]$$

$$\left\{ -\frac{\sqrt{4 n_1^2 - n_2^2}}{\sqrt{3}}, \frac{\sqrt{4 n_1^2 - n_2^2}}{\sqrt{3}} \right\}$$

$$n_f = \text{filmIndex}[[2]];$$

$$n_f = \frac{\sqrt{4 n_1^2 - n_2^2}}{\sqrt{3}}$$

Next we will find the film thickness, d.

$$d = \text{FullSimplify}\left[\frac{\lambda}{2} \frac{1}{\sqrt{n_f^2 - n_1^2}}\right];$$

$$d = \frac{\sqrt{3} \lambda}{2 \sqrt{n_1^2 - n_2^2}}$$

■ **Case II: $n_s < n_f$ so no π phase change upon reflection.**

For dark fringe

$$2 n_f d \cos[\theta_f] = \left(m - \frac{1}{2}\right) \lambda, \quad m \text{ is an integer}$$

For first dark fringe (other than $\theta_f = 90^\circ$)

$$2 n_f d \cos[\theta_{f1}] = \frac{\lambda}{2}$$

For second dark fringe we have

$$2 n_f d \cos[\theta_{f2}] = \frac{3 \lambda}{2}$$

Therefore,

$$2 d \sqrt{n_f^2 - n_1^2} = \frac{\lambda}{2} \quad \text{and} \quad 2 d \sqrt{n_f^2 - n_2^2} = \frac{3 \lambda}{2}$$

$n_f = .;$

filmIndex = n_f /. **Solve**[$9 (n_f^2 - n_1^2) == n_f^2 - n_2^2, n_f$]

$$\left\{ -\frac{\sqrt{9 n_1^2 - n_2^2}}{2 \sqrt{2}}, \frac{\sqrt{9 n_1^2 - n_2^2}}{2 \sqrt{2}} \right\}$$

$n_f = \text{filmIndex}[[2]];$

$$n_f = \frac{\sqrt{9 n_1^2 - n_2^2}}{2 \sqrt{2}}$$

Next we will find the film thickness, d.

$$d = \text{FullSimplify}\left[\frac{\lambda}{4} \frac{1}{\sqrt{n_f^2 - n_1^2}}\right];$$

$$d = \frac{\lambda}{\sqrt{2} \sqrt{n_1^2 - n_2^2}}$$

Ellipsometry

Introduction

Ellipsometry is the measurement of the effect of reflection on the state of polarization of light. The result of an ellipsometric measurement can be the complex refractive index of the reflecting material, or if the reflecting material is a film-covered substrate, the thickness and optical constants of the film can be determined. Ellipsometry is particularly attractive because it does not perturb the sample being measured and it is extremely sensitive to minute interfacial effects and can be applied to surface films having a thickness as small as monoatomic to as large as several microns. Any substrate-film-ambient combination that provides reasonably specular reflection of the incident light beam can be measured. Scattering during the reflection process causes partial depolarization of the incident beam and, consequently, reduced precision and accuracy.

Since ellipsometry essentially measures the state of polarization of reflected or transmitted light it can be thought of as polarimetry. The state of polarization is defined by the phase and amplitude relationships between the two component plane waves into which the electric field is resolved. The wave having the electric field in the plane of incidence is called the p wave, and the wave having the electric field normal to the plane of incidence is called the s wave. If the p and s components are in phase, or 180 degrees out of phase, the resultant wave is plane polarized. A difference of phase, other than 180°, corresponds to elliptical polarization. In general, reflection causes a change in relative phases of the p and s waves and a change in the ratio of their amplitudes. The change in phase is characterized by the angle Δ , and the amplitude ratio change is characterized by $\text{Tan}[\psi]$. If the amplitudes of the incident and reflected beams are designated e and r, respectively, and phases of the incident and reflected beams are α and β , respectively

$$\text{Tan}[\psi] = \frac{|r_p|}{|r_s|} \frac{|e_s|}{|e_p|}$$

$$\Delta = (\beta_p - \beta_s) - (\alpha_p - \alpha_s)$$

Ellipsometry is the measurement of ψ and Δ .

Measurement Principles

The principle of the measurement of ψ and Δ is explained with the help of Figure 1, which is a schematic representation of an ellipsometer. The incident monochromatic beam is collimated and transmitted through a linear polarizer and compensator (retarder). (In some ellipsometers a broad spectral band source is used and ψ and Δ are measured as a function of wavelength. In this discussion we will consider only monochromatic illumination.) The azimuthal orientations of the polarizer and compensator determine the relative amplitudes and phase difference between the p and s components of the beam incident upon the substrate. These orientations are adjusted so the difference in phase just compensates that which results from reflection off the sample. The plane polarized beam reflected off the sample is transmitted by the analyzer to a telescope and detector and the analyzer is oriented to extinguish the reflected beam. Δ and ψ are determined from the orientation of the polarizer and analyzer for extinction.

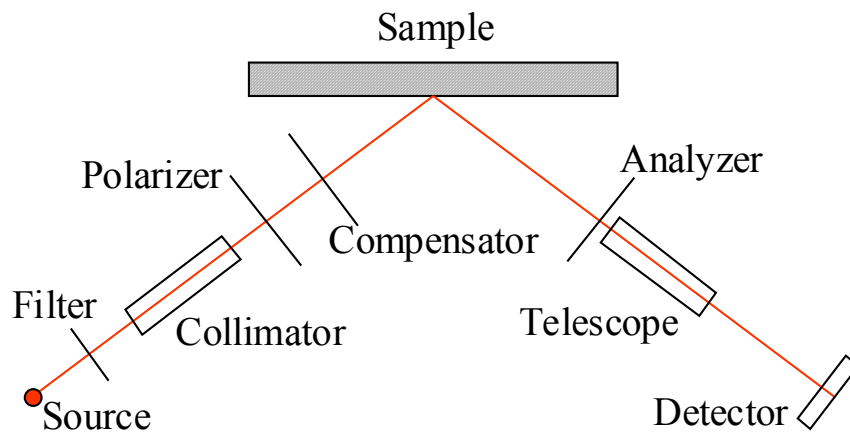


Figure 1. Schematic representation of ellipsometer.

In the discussion below it will be assumed that the polarizer, analyzer, and compensator are ideal. It is assumed that the compensator is a wave-plate introducing a retardation of δ and no attenuation. The orientation of the wave-plate is selected so the slow axis is inclined at 45° to the plane of incidence. Any angle can be used, but the compensator is generally used at $\pm 45^\circ$. Let p be the angle between the polarizer transmission axis and the x-axis which is taken to be the direction for p polarization as illustrated in Figure 2.

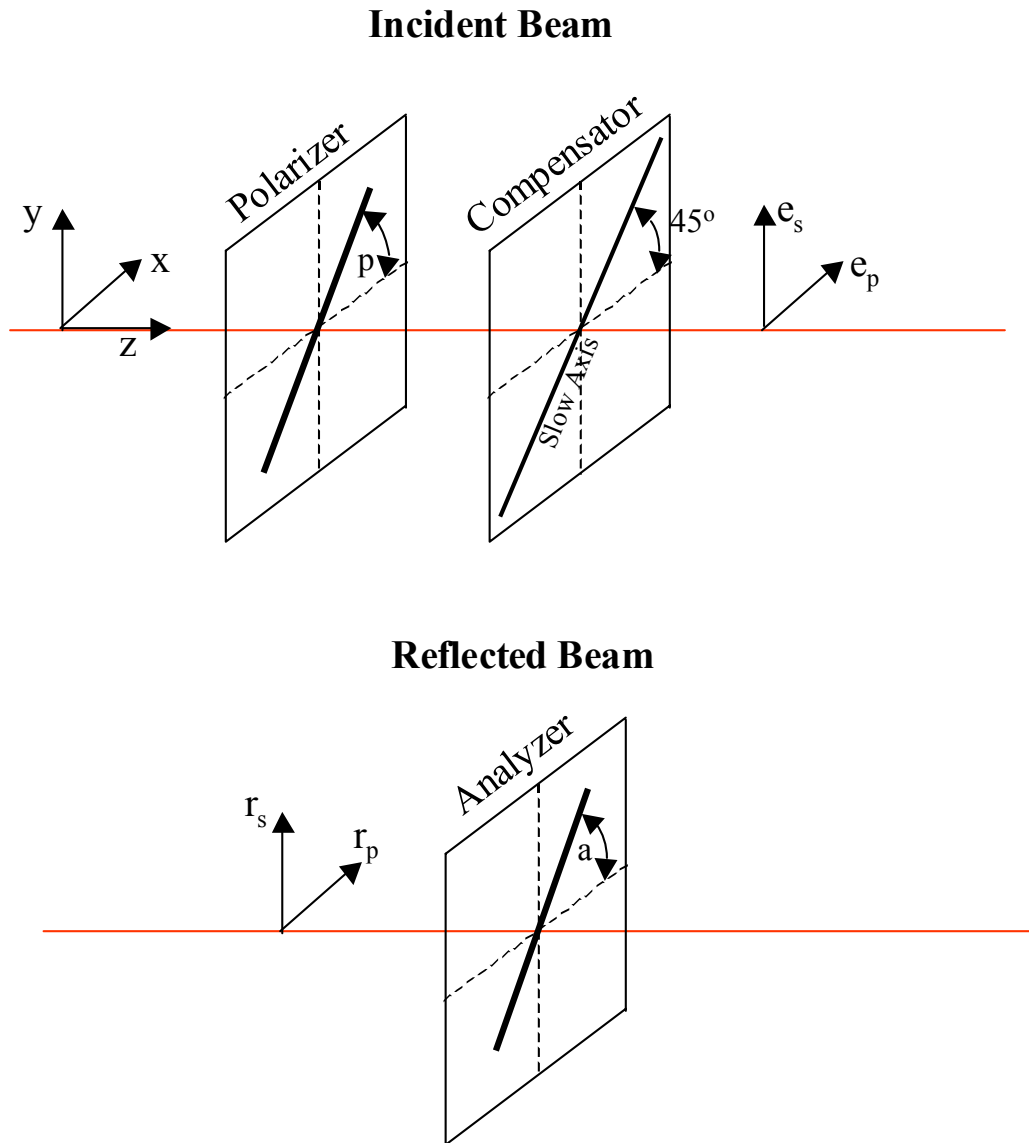


Figure 2. Orientation of polarizer, compensator, and analyzer.

The light transmitted through the polarizer can be written in the form of a Jones vector as

$$\mathbf{lightLinear} = \begin{pmatrix} \cos[p] \\ \sin[p] \end{pmatrix};$$

A retarder with the fast axis horizontal can be written in terms of a Jones Matrix as

$$\mathbf{rfah}[\delta_] := e^{-i\delta/2} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$$

A rotation matrix can be written as

$$\mathbf{rot}[\theta_] := \begin{pmatrix} \cos[\theta] & \sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{pmatrix}$$

A retarder of retardation δ having a fast axis at an angle of θ from the horizontal can be written as

$$\mathbf{rrot}[\delta, \theta] := \mathbf{rot}[-\theta] \cdot \mathbf{rfah}[\delta] \cdot \mathbf{rot}[\theta]$$

Thus, the light transmitted through the wave plate and incident upon the sample can be written as

$$\mathbf{lightIncident} = \mathbf{FullSimplify}[\mathbf{rrot}[\delta, -45^\circ] \cdot \mathbf{lightLinear}] // \mathbf{MatrixForm}$$

The p-component (x) can be written as

$$\mathbf{pComponent} = \mathbf{Cos}[p] \mathbf{Cos}\left[\frac{\delta}{2}\right] + \mathbf{i} \mathbf{Sin}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right];$$

and the s-component (y) can be written as

$$\mathbf{sComponent} = \mathbf{Cos}\left[\frac{\delta}{2}\right] \mathbf{Sin}[p] + \mathbf{i} \mathbf{Cos}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right];$$

Phase determination

The tangent of the phase of the p component can be written as

$$\mathbf{tanpComponent} = \frac{\mathbf{Sin}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right]}{\mathbf{Cos}[p] \mathbf{Cos}\left[\frac{\delta}{2}\right]};$$

The tangent of the phase of the s component can be written as

$$\mathbf{tansComponent} = \frac{\mathbf{Cos}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right]}{\mathbf{Cos}\left[\frac{\delta}{2}\right] \mathbf{Sin}[p]};$$

The goal is to find the tangent of the phase difference between the p and s components. Remembering that

$$\mathbf{Simplify}\left[\frac{\mathbf{Tan}[\alpha] - \mathbf{Tan}[\beta]}{1 + \mathbf{Tan}[\alpha] \mathbf{Tan}[\beta]}\right] = \mathbf{Tan}[\alpha - \beta]$$

we can write

$$\mathbf{tan}\Delta\mathbf{Incident} = \mathbf{Factor}\left[\mathbf{TrigExpand}\left[\mathbf{FullSimplify}\left[\frac{\frac{\mathbf{Sin}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right]}{\mathbf{Cos}[p] \mathbf{Cos}\left[\frac{\delta}{2}\right]} - \frac{\mathbf{Cos}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right]}{\mathbf{Cos}\left[\frac{\delta}{2}\right] \mathbf{Sin}[p]}}{1 + \frac{\mathbf{Sin}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right]}{\mathbf{Cos}[p] \mathbf{Cos}\left[\frac{\delta}{2}\right]} \frac{\mathbf{Cos}[p] \mathbf{Sin}\left[\frac{\delta}{2}\right]}{\mathbf{Cos}\left[\frac{\delta}{2}\right] \mathbf{Sin}[p]}}\right]}\right]\right] =$$

$$-\frac{1}{2} \mathbf{Sin}[\delta] (\mathbf{Cot}[p] - \mathbf{Tan}[p])$$

But,

$$\mathbf{FullSimplify}\left[\mathbf{TrigToExp}\left[-\frac{1}{2} (\mathbf{Cot}[p] - \mathbf{Tan}[p])\right]\right] = -\mathbf{Cot}[2p]$$

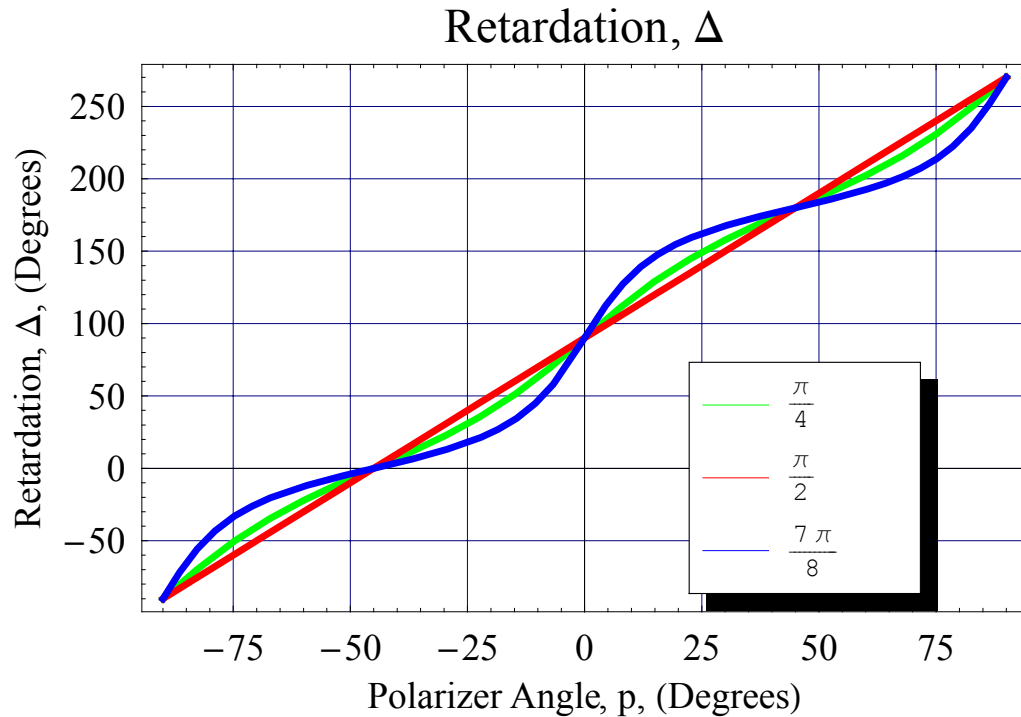
Furthermore,

$$\mathbf{Tan}[2p - 90^\circ] = -\mathbf{Cot}[2p]$$

Therefore,

$$\tan\Delta_{\text{Incident}} = \text{Sin}[\delta] \text{Tan}[2p - 90^\circ];$$

It is interesting to look at a plot of Δ as a function of p . To correct for a discontinuity in the ArcTan function 180 degrees will be added for $p \geq 0$ to make the function continuous.



A 180 degree rotation of the polarizer introduces a 360 degree change in the retardation. If the compensator is a quarter-wave plate ($\delta=\pi/2$) there is a linear relationship between Δ_{Incident} , the phase difference between the p and s components of the light incident upon the sample, and p , the orientation angle of the polarizer.

Amplitude determination

Next we will look at the ratio of the amplitudes of the s and p components of the electric field incident upon the sample. Let

$$\tan L = \frac{|e_p|}{|e_s|}$$

Then

$$\tan L^2 = \frac{\text{ComplexExpand}[p\text{Component Conjugate}[p\text{Component}]]}{\text{ComplexExpand}[s\text{Component Conjugate}[s\text{Component}]]};$$

Remembering that

$$\text{Simplify}\left[\frac{1 - \tan[\alpha]^2}{1 + \tan[\alpha]^2}\right] = \cos[2\alpha]$$

$$\cos 2L = \text{Simplify}\left[\frac{1 - \tan L^2}{1 + \tan L^2}\right] = -\cos[2p] \cos[\delta];$$

Similar relationships are obtained with appropriate changes in sign if the slow axis is oriented at -45° to the plane of incidence.

It is interesting to note that if the compensator is a quarter-wave plate the orientation of the polarizer has no effect upon the ratio of the amplitudes of the s and p components of the electric field incident upon the sample.

Measurement procedure

The measuring procedure consists of adjusting the polarizer and analyzer so the detected beam is extinguished. There are two orientations of the polarizer which lead to plane polarized light. The two conditions are

$$\Delta_{\text{Incident}} = -\Delta_{\text{Sample}}$$

and

$$\Delta_{\text{Incident}} = -\Delta_{\text{Sample}} + 180^\circ$$

It follows from the equation for $\tan \Delta_{\text{Incident}}$ that the two conditions for plane polarized light being reflected off the sample are

$$\tan \Delta_{\text{Sample}} = \sin[\delta] \tan[90^\circ - 2p_1]$$

and

$$\tan \Delta_{\text{Sample}} = \sin[\delta] \tan[270^\circ - 2p_2]$$

At extinction, the analyzer transmission axis orientation angle, a , is equal to $r \pm 90^\circ$, where r is the angle of the reflected linear polarization relative to the plane of incidence.

$$\tan[r] = \frac{|r_s|}{|r_p|}$$

$$\tan[\psi] = \frac{|r_p|}{|r_s|} \frac{|e_s|}{|e_p|}$$

$$\tan[\psi] = \frac{\tan[-a_1]}{\tan[L_1]}$$

and for the second set of angles

$$\tan[\psi] = \frac{\tan[a_2]}{\tan[L_2]}$$

$$\text{Since } \cot[L_1] = \tan[L_2]$$

$$\tan[\psi]^2 = \tan[-a_1] \tan[a_2]$$

If the compensator is a quarter-wave plate, $\delta = 90^\circ$, the relationships between Δ and ψ and the extinction settings are especially simple.

$$\Delta = 90^\circ - 2 p_1 = 270^\circ - 2 p_2$$

$$\psi = -a_1 = a_2 .$$

Interpretation of data

Using the measured values of Δ and ψ it is possible to determine the complex refractive index of substrates and the thickness and refractive index of thin films, however the equations are extremely complicated and their solution and use for interpreting ellipsometric data requires electronic computation. Details on the specific computations are beyond the scope of these notes and they can be found in reference 1.

References

- 1) Azzam, R.M.A. and Bashara, N.M., (1988). "Ellipsometry and Polarized Light", North-Holland, New York.
- 2) Archer, R.J. "Manual on Ellipsometry", Gaertner Scientific, Skokie, IL.
- 3) Spanier, R., (September, 1975). "Ellipsometry, A Century Old New Science", Industrial Research.
- 4) Hecht, E., (1998). "Optics", Addison Wesley, New York.
- 5) Born, M. and Wolf, E., (1959). "Principles of Optics", Pergamon Press, New York.

Fresnel Equations

Bare substrates

Fresnel first derived equations for the reflection coefficients of bare surfaces in terms of the angle of incidence, angle of refraction, and the complex refractive index. The results for the amplitude reflection coefficient and amplitude transmission coefficient are given below. The sign convention used is not standardized. For our equations the sign convention used in reference 4 (Hecht) is followed.

$$r_s = \frac{n_i \cos[\theta_i] - n_t \cos[\theta_t]}{n_i \cos[\theta_i] + n_t \cos[\theta_t]}$$

$$t_s = \frac{2 n_i \cos[\theta_i]}{n_i \cos[\theta_i] + n_t \cos[\theta_t]}$$

$$r_p = \frac{n_t \cos[\theta_i] - n_i \cos[\theta_t]}{n_i \cos[\theta_t] + n_t \cos[\theta_i]}$$

$$t_p = \frac{2 n_i \cos[\theta_i]}{n_i \cos[\theta_t] + n_t \cos[\theta_i]}$$

For an optically absorbing medium the complex index of refraction of the substrate is given by

$$n_t = n - i k;$$

From the definitions given above it follows that

$$\frac{r_p}{r_s} = \tan[\psi] e^{i\Delta}$$

The algebra for solving for n and k from ψ and Δ is extremely messy and will not be given here. The details can be found in references 1 and 2.

Phase change at normal incidence

$$r = \frac{1 - n_t}{1 + n_t}$$

$$\frac{1 + i k - n}{1 - i k + n}$$

$$r_{\text{Bottom}} = \text{Simplify}[\text{Denominator}[r] \text{Conjugate}[\text{Denominator}[r]], \{k > 0, n > 0\}]$$

$$k^2 + (1 + n)^2$$

$$r_{\text{Top}} = \text{Simplify}[\text{Numerator}[r] \text{Conjugate}[\text{Denominator}[r]], \{k > 0, n > 0\}]$$

$$1 + 2 i k - k^2 - n^2$$

$$\text{phase}[n_, k_] := \text{ArcTan}\left[\frac{2 k}{1 - n^2 - k^2}\right]$$

Thin films

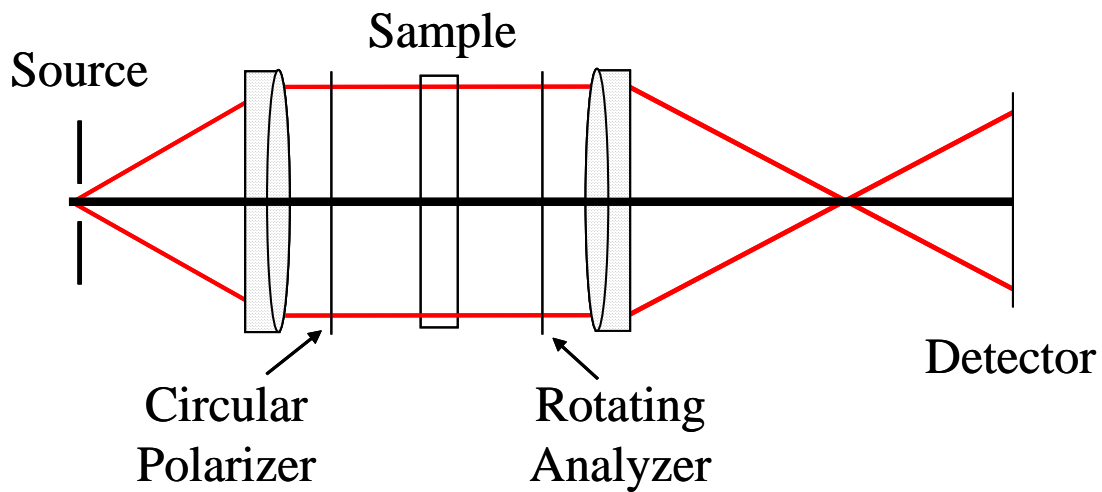
The reflectance of substrates having a coating of thin films can be calculated using the characteristic matrix approach as described in references 4 and 5. D and y can be calculated in terms of the angle of incidence, the wavelength, the optical constants of the film and substrate and the thickness of the film. The equations are extremely complicated and their solution and use for interpreting ellipsometric data requires electronic computation. References 1 and 2 give additional information.

References

- 1) Azzam, R.M.A. and Bashara, N.M., (1988). "Ellipsometry and Polarized Light", North-Holland, New York.
- 2) Archer, R.J. "Manual on Ellipsometry", Gaertner Scientific, Skokie, IL.
- 3) Spanier, R., (September, 1975). "Ellipsometry, A Century Old New Science", Industrial Research.
- 4) Hecht, E., (1998). "Optics", Addison Wesley, New York.
- 5) Born, M. and Wolf, E., (1959). "Principles of Optics", Pergamon Press, New York.

Measuring Birefringence

Illuminate sample with circularly polarized light. Put rotating analyzer between sample and detector and measure light transmitted thru sample and analyzer.



Basic Definitions

Circular polarization

$$\text{stokes} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

Horizontal linear polarizer

$$\text{hlpMueller} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

Linear retarder of retardation δ with fast axis horizontal

$$\text{retarderHorizontal}[\delta_] := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \text{Cos}[\delta] & \text{Sin}[\delta] \\ 0 & 0 & -\text{Sin}[\delta] & \text{Cos}[\delta] \end{pmatrix}$$

Rotation Matrix

$$\text{rotMueller}[\theta_] := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Cos}[2\theta] & \text{Sin}[2\theta] & 0 \\ 0 & -\text{Sin}[2\theta] & \text{Cos}[2\theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Calculation of matrix of a retarder of retardation δ having a fast axis at an angle θ from the horizontal

$$\text{rrot}[\delta_ , \theta_] := \text{rotMueller}[-\theta] . \text{retarderHorizontal}[\delta] . \text{rotMueller}[\theta]$$

Measuring the birefringence

Rotate polarizer in front of detector and determine the birefringence δ and the angle of the birefringence, θ . ωt is the angle of the polarizer.

■ Measure phase and amplitude of signal as polarizer rotates

$$\text{polarimeter} = \text{rotMueller}[-\omega t] . \text{hlpMueller} . \text{rotMueller}[\omega t] . \text{rrot}[\delta, \theta] . \text{stokes};$$

$$\text{polarimeter}[[1]][[1]] // \text{Simplify}$$

$$\frac{1}{4} (2 + \text{Cos}[\delta + 2\theta - 2t\omega] - \text{Cos}[\delta - 2\theta + 2t\omega])$$

$$\text{signal} = \frac{1}{4} (2 + \text{Simplify}[\text{Cos}[\delta + 2\theta - 2t\omega] - \text{Cos}[\delta - 2\theta + 2t\omega]]) // \text{Simplify}$$

$$\frac{1}{2} (1 - \text{Sin}[\delta] \text{Sin}[2(\theta - t\omega)])$$

The amplitude of the $\text{Sin}[2\omega t]$ signal is given by the $\text{Sin}[\text{birefringence}]$ and the phase of the signal is given by 2 times the angle of the birefringence, θ .

■ Measure signal for discrete positions of polarizer

$$\text{signal1} = \text{signal} /. t\omega \rightarrow 0$$

$$\frac{1}{2} (1 - \text{Sin}[\delta] \text{Sin}[2\theta])$$

$$\text{signal2} = \text{signal} /. t\omega \rightarrow \pi/4 // \text{FullSimplify}$$

$$\frac{1}{2} (1 + \text{Cos}[2\theta] \text{Sin}[\delta])$$

$$\text{signal3} = \text{signal} /. t\omega \rightarrow \pi/2$$

$$\frac{1}{2} (1 - \text{Sin}[\delta] \text{Sin}[2(-\frac{\pi}{2} + \theta)])$$

```

signal3 = signal3 /. Sin[2 (- $\frac{\pi}{2}$  +  $\theta$ )] -> TrigReduce[Sin[2 (- $\frac{\pi}{2}$  +  $\theta$ )]]
 $\frac{1}{2}$  (1 + Sin[ $\delta$ ] Sin[2  $\theta$ ])

```

```

signal4 = signal /. t  $\omega$  -> 3  $\pi$  / 4 // Simplify
 $\frac{1}{2}$  (1 - Cos[2  $\theta$ ] Sin[ $\delta$ ])

```

```

 $\frac{\text{signal3} - \text{signal1}}{\text{signal2} - \text{signal4}}$  // Simplify
Tan[2  $\theta$ ]

```

If we take an ArcTan we obtain the orientation of the birefringence.

```

(signal3 - signal1) // Simplify
Sin[ $\delta$ ] Sin[2  $\theta$ ]

```

```

(signal2 - signal4) // Simplify
Cos[2  $\theta$ ] Sin[ $\delta$ ]

```

```

(signal3 - signal1)2 + (signal2 - signal4)2 // Simplify
Sin[ $\delta$ ]2

```

If we take the ArcSin of the square root we get the magnitude of birefringence. Since we know 2θ , the sign of $(\text{signal3} - \text{signal1})$ gives us the sign of the birefringence.

Measuring the Stokes Parameters

One method for measuring the Stokes parameters is to measure the intensity of the light after it passes through a rotating quarter-wave plate followed by a horizontal linear polarizer. The following four intensity measurements are required:

Fast-axis of the quarter-wave plate at

- a) 0° ,
- b) 30° ,
- c) 60° , and
- d) 135° .

Basic Definitions

Stokes Vector

$$\text{stokes} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix};$$

Horizontal linear polarizer

$$\text{hlpMueller} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

Linear retarder of retardation δ with fast axis horizontal

$$\text{retarderHorizontal}[\delta_] := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \text{Cos}[\delta] & \text{Sin}[\delta] \\ 0 & 0 & -\text{Sin}[\delta] & \text{Cos}[\delta] \end{pmatrix}$$

Rotation Matrix

$$\text{rotMueller}[\theta_] := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Cos}[2\theta] & \text{Sin}[2\theta] & 0 \\ 0 & -\text{Sin}[2\theta] & \text{Cos}[2\theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Quarter-wave plate at angle θ

$$\text{qwp}[\theta_] := \text{rotMueller}[-\theta] \cdot \text{retarderHorizontal}\left[\frac{\pi}{2}\right] \cdot \text{rotMueller}[\theta]$$

Polarimeter Output

Rotating quarter-wave plate and horizontal linear polarizer

Fast axis of quarter-wave plate at 0° .

$$\begin{aligned} \text{output1} &= \text{hlpMueller} \cdot \text{qwp}[0] \cdot \text{stokes}; \text{MatrixForm}[\text{output1}] \\ &\begin{pmatrix} \frac{s_0}{2} + \frac{s_1}{2} \\ \frac{s_0}{2} + \frac{s_1}{2} \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

Fast axis of quarter-wave plate at 30° .

$$\begin{aligned} \text{output2} &= \text{hlpMueller} \cdot \text{qwp}\left[\frac{\pi}{6}\right] \cdot \text{stokes}; \text{MatrixForm}[\text{output2}] \\ &\begin{pmatrix} \frac{s_0}{2} + \frac{s_1}{8} + \frac{\sqrt{3}}{8} s_2 - \frac{\sqrt{3}}{4} s_3 \\ \frac{s_0}{2} + \frac{s_1}{8} + \frac{\sqrt{3}}{8} s_2 - \frac{\sqrt{3}}{4} s_3 \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

Fast axis of quarter-wave plate at 60° .

$$\begin{aligned} \text{output3} &= \text{hlpMueller} \cdot \text{qwp}\left[\frac{\pi}{3}\right] \cdot \text{stokes}; \text{MatrixForm}[\text{output3}] \\ &\begin{pmatrix} \frac{s_0}{2} + \frac{s_1}{8} - \frac{\sqrt{3}}{8} s_2 - \frac{\sqrt{3}}{4} s_3 \\ \frac{s_0}{2} + \frac{s_1}{8} - \frac{\sqrt{3}}{8} s_2 - \frac{\sqrt{3}}{4} s_3 \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

Fast axis of quarter-wave plate at 135° .

$$\begin{aligned} \text{output4} &= \text{hlpMueller} \cdot \text{qwp}\left[\frac{3}{4}\pi\right] \cdot \text{stokes}; \text{MatrixForm}[\text{output4}] \\ &\begin{pmatrix} \frac{s_0}{2} + \frac{s_3}{2} \\ \frac{s_0}{2} + \frac{s_3}{2} \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

The four Stokes parameters are calculated as follows:

```
ans = Solve[{output1[[1, 1]] == reading1, output2[[1, 1]] == reading2,
  output3[[1, 1]] == reading3, output4[[1, 1]] == reading4}, {s0, s1, s2, s3}];
```

```
{s0 /. ans, s1 /. ans, s2 /. ans, s3 /. ans} // MatrixForm
```

$$\begin{pmatrix} \frac{2(-\sqrt{3} \text{reading1} + 2\sqrt{3} \text{reading2} + 2\sqrt{3} \text{reading3} + 6 \text{reading4})}{3(2+\sqrt{3})} \\ \frac{4(3 \text{reading1} + 2\sqrt{3} \text{reading1} - \sqrt{3} \text{reading2} - \sqrt{3} \text{reading3} - 3 \text{reading4})}{3(2+\sqrt{3})} \\ \frac{4}{3} (\sqrt{3} \text{reading2} - \sqrt{3} \text{reading3}) \\ \frac{2(\sqrt{3} \text{reading1} - 2\sqrt{3} \text{reading2} - 2\sqrt{3} \text{reading3} + 3\sqrt{3} \text{reading4})}{3(2+\sqrt{3})} \end{pmatrix}$$