lesser refractive index, n, will be totally reflected when the angle of incidence is such that its sine is greater than $\frac{n}{N}$. If the limiting value, ϕ, of the angle of total reflection is observed, and the index of refraction of one substance is known, the other index may be found from the relation

$$\sin \phi = \frac{n}{N}.$$

Various instruments using this principle have been devised, which permit the determination to the fourth or fifth decimal place of the index of refraction of any liquid, and of such solids as have indices less than the index of a liquid used in the apparatus. It is especially useful for liquids in small quantities, for solids in thin plates or in powdered form, for crystals, for bodies with only one small reflecting face, and for opaque bodies. When the face of the body is of small reflecting power, or is only approximately flat, the method is still applicable.

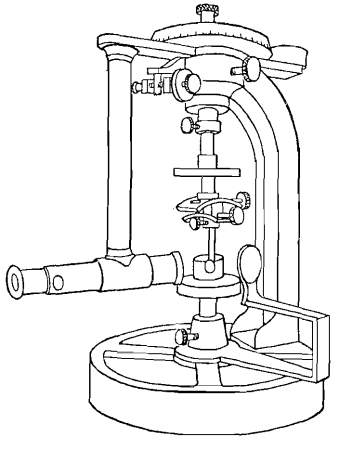

Fig. 99. Total-Reflectometer

183. Total-Reflectometer (Kohlrausch). *Solids.* — The body to be investigated is supported in a small flask filled with a liquid of large and known refractive index. Such liquids are carbon bisulphide (1.63), α-bromonaphthalene (1.65), and methyl iodide (1.74). The index of the liquid may be determined from tables or with a hollow prism and the spectrometer, as described in Art. 178. The support of the body passes through the center of a divided circle and is provided with centering devices for placing the face of the body in the axis of rotation, as shown in Fig. 99.

The front of the flask is a plane-parallel plate through which a small telescope may receive light reflected from the body.

The telescope should be perpendicular to the axis of rotation, and the face of the body and of the flask parallel to this axis. A Gauss eyepiece or a collimator is provided for facilitating these adjustments, which may be made according to principles explained in Art. 172.

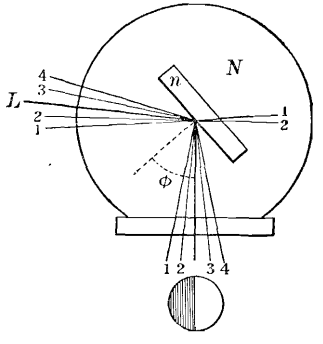

Fig. 100. Kohlrausch Total-Reflectometer

The flask is illuminated by sodium light, L (Fig. 100), and by the interposition of a piece of oiled tissue paper, diffused light from many directions will fall upon the body. All rays having a smaller angle of incidence than the angle of total reflection (rays 1 and 2) will be partially refracted and partially reflected, while all rays having a greater angle of incidence (3 and 4) will be totally reflected. By turning the support of the body, the dividing line corresponding to the angle of total reflection may be brought to the center of the field of view of the telescope. When this position has been found, a condensing lens may be employed to increase the illumination. Read the position of the body as indicated by the divided circle. Illuminate the opposite side of the flask, and rotate the body until the limiting angle for this position is observed. The difference between the two positions is twice the limiting angle, ϕ, of total reflection, and the index of refraction of the body is

$$n = N \sin \phi,$$

N being the index of the liquid.

The index of refraction of carbon bisulphide is 1.6277 at 20°, which changes 0.00080 per degree, being less for higher temperatures. The light may be placed forty or more centimeters from the flask, and a screen with a glass-covered aperture may be used to protect the liquid from the heat of the flame.

Liquids. — If a solid plane plate having a known index of refraction n is suspended in the liquid of unknown but greater

index N, and the angle of total reflection is observed, the index of refraction of the liquid is

$$N = \frac{n}{\sin \phi}.$$

If the liquid has an index less than that of the solid plane-parallel plate, the plate is to be so immersed that there is a film of air behind it. This is secured by making the plate one side of a small box. The index of the plate should be large, but it need not be known. Making the observations as before, the index of refraction of the liquid in which the box is immersed is

$$N = \frac{1}{\sin \phi}.$$

If the glass box is immersed in a liquid of known index, less than that of its plane-parallel side but greater than that of an unknown liquid, the index of the latter may be determined by filling the box with it and proceeding exactly as for the index of a solid. If only a small quantity of the unknown liquid is available, instead of filling the box with it, a single small drop placed on the inner surface of the plane-parallel plate suffices for the experiment.

Crystals. — The refraction of a crystal can only be described with the aid of two, or sometimes three, indices of refraction. In order to determine them the direction of the axis of the crystal must be known, and then the limiting angles of reflection corresponding to the various indices may be distinguished with the aid of an analyzing Nicol prism. The method of procedure is described in special treatises and in the references.

References. — *Kohlrausch*, Physical Measurements, p. 161; *Leiss*, Die optischen Instrumente, pp. 52–61; *Drude*, Theory of Optics, pp. 339–344.

184. Refractometer (Pulfrich). — This instrument uses not total reflection but refraction at grazing incidence. A glass prism of known index of refraction, greater than that of any substance to be investigated, has attached to its horizontal face a glass cylinder to contain the liquid to be tested (Fig. 101).

Behind the instrument in the plane of the prism is placed a sodium light, which is condensed upon the surface of the prism through the cylinder of liquid. The rays at grazing incidence will be refracted at the angle of total reflection, while all other rays will be partially refracted at less angles and partially reflected. A telescope moving in a vertical plane about the axis of a divided circle measures the angle of emergence a by being first set normal to the vertical face of the prism (Art. 172), and then so that the field of view is half bright and half dark. If N is the index of the glass prism, the index of refraction of the liquid in the cylinder is

$$n = \sqrt{N^2 - \sin^2 a}.$$

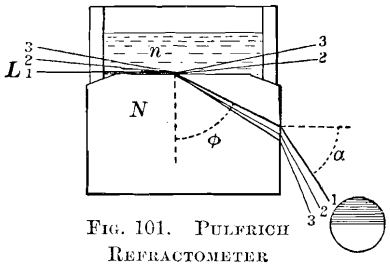

Fig. 101. Pulfrich Refractometer

Tables are usually provided which for a given prism show the values of n for given values of a. The index of refraction of water may be measured to show that the circle and prism are properly set. For water at 20°, $n = 1.3332$.

The index of refraction of liquids varies considerably with the temperature, hence the temperature of the observations must be noted; usually this should be made 20°. If the room temperature is higher than this, the liquid may be cooled a small amount, and the observation made when its temperature passes through 20°; and vice versa.

The cylinder may be surrounded by a hollow case through which hot water or steam may be passed, or a coil of wire carrying a current of electricity may be used, and the variations of the index of refraction with temperature, or the index of a substance which is transparent only when melted, may be thus investigated. In these cases a correction to the index of the prism is required.

A partition of black glass may be cemented in the cylinder, dividing it into two cells, permitting the convenient measurement of the differential refraction of two liquids.

A solid substance may be investigated if it has two surfaces approximately at right angles, one of which is plane and well polished, and the other polished sufficiently to transmit light. The edge in which the surfaces intersect must be perfect. The plane face is joined to the surface of the prism with a liquid of high refractive index, the cylinder being removed if necessary. The index is then determined as described above.

If the solid is in the form of powder, its index of refraction may be determined with moderate precision, provided its index is intermediate in value between that of two available liquids (alcohol, ether, or acetone, 1.36; benzene, 1.50; *a*-bromonaphthalene, 1.65) which when mixed will not dissolve the powder. The cylinder of the refractometer is filled with the powder and the mixed liquids; the proportions of the latter are so altered that the index of the mixture is the same as that of the powder. If the index of the liquid is too great, a bright band appears at the boundary between the light and dark parts of the field of view; while if the index of the liquid is too small, the boundary is indistinct. When the mixture of the liquids is correct the dividing line indicating the limiting angle is quite sharp, and then the index computed is that of both solid and liquid.

Reference. — *Pulfrich*, Astrophysical Journal, Vol. **3**, p. 259, 1896.

185. Crystal-Refractometer (Abbe). — In this method a hemisphere of glass is substituted for the flask of liquid of the Kohlrausch method (Fig. 102). It requires but a small quantity of liquid to join the crystal to the flat surface of the hemisphere, and by rotating the hemisphere on an axis perpendicular to the surface of the crystal it permits a convenient observation of the relations of the several limiting curves of total reflection. It is not possible to adjust the surface of the plate as accurately as in Kohlrausch's method, and the construction of the hemisphere and its adjustment introduce difficulties.

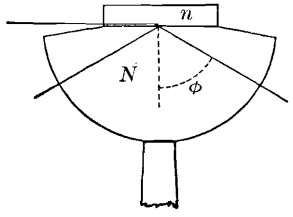

Fig. 102. Crystal-Refractometer

Furthermore, it is not so well adapted to the investigation of small and imperfectly reflecting surfaces. It is used according to the principles explained for Kohlrausch's method, and is applicable for a solid whose index is less than that of the hemisphere and of the liquid with which it is joined to the hemisphere, and for any liquid with an index less than that of the hemisphere. It may be used with light at grazing incidence, or by total reflection. The reference may be consulted for the method of use.

Reference. — *Leiss*, Die optischen Instrumente, pp. 38–49.