Homework 4 solutions

Monday, September 21, 2009
12:47 PM

1) $D = 20\,mm$, $k = 0$, $R = 250 \pm 1\,mm$

$$sag = \frac{\rho^2}{2R} = \frac{(10\,mm)^2}{2(250 \pm 1)} \Rightarrow$$

$.2\,mm \pm .0008\,mm$

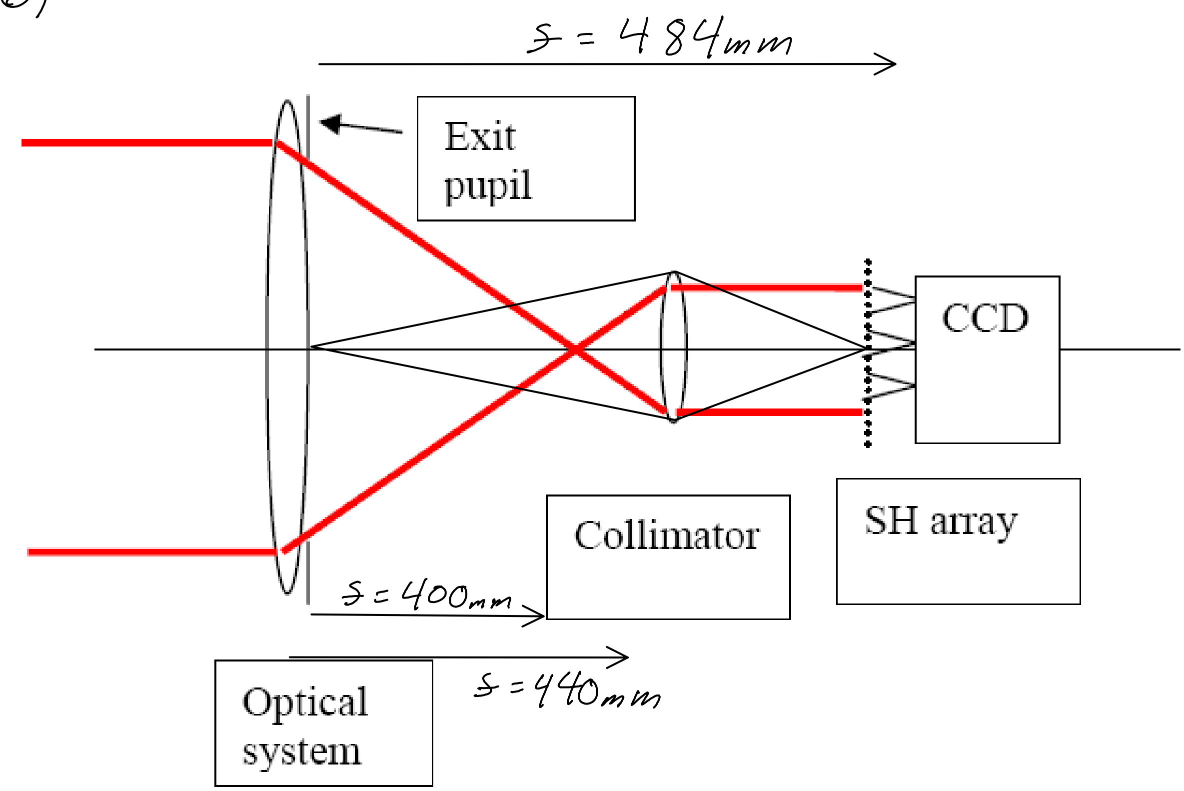
this is tight

2) $f/4$ $f = 400\,mm$ $\lambda = .5\,\mu m$

a) $$m = \frac{h'}{h} = \frac{1}{10} = \frac{f_{collimate}}{f_{obj}} = \frac{40\,mm}{400\,mm}$$

$f_{collimate} = 40\,mm$

b)



$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

To image the EP $\left(\frac{1}{-440} + \frac{1}{40}\right)^{-1}$

$$z_{ep} = 44\,mm \Rightarrow 484\,mm$$

c) $D_{lens} = .2\,mm \qquad f_{SH} = 5\,mm$

$D_{pixel} = 0.01\,mm$

$$D_{Airy} = 2.44\,\lambda\, f/\# = 2.44(.5)\left(\frac{5000}{200}\right)$$

$$\boxed{= 30.5\,\mu m \simeq 3 \text{ pixels}}$$

d) $z_4 = .1\,\mu m \qquad m_T = 10$

$$W = z_4 \sqrt{3}(2\rho^2 - 1)m_T \text{ but...}$$

We want the shift at the image plane i.e. Rayfront Error

$$\varepsilon_r = \frac{\partial W}{\partial \rho} = -\frac{R}{r_c} z_4 \sqrt{3}(4\rho)m_T$$

$$\text{if } \rho \to 1 \quad \varepsilon_r = -\frac{10\,mm}{0.1\,mm}(4\sqrt{3})(.1\,\mu m) = \boxed{692\,\mu m}$$

e) $z_9 = 0.1\,\mu m$

$$W = z_9 \sqrt{5}(6\rho^4 - 6\rho^2 + 1)m_T$$

$$\varepsilon_r = -\frac{R}{r_c}\sqrt{5}\, z_9 (24\rho^3 - 12\rho)m_T \text{ let } \rho \to 1$$

$$\varepsilon_r = -\frac{10\,mm}{0.1\,mm}(12\sqrt{5})(0.1\,\mu m)m_T = \boxed{2680\,\mu m}$$

3) $\Delta W_{PV} = \frac{\lambda}{10}$ (defocus)

$\Delta W = -\varepsilon_z \ldots \qquad \boxed{\varepsilon_z = 4\lambda(f/\#)^2}$

$$\Delta W = \frac{-\varepsilon_z}{8(f/\#)^2} \Rightarrow \boxed{\varepsilon_z = \frac{4}{5}\lambda (f/\#)^2}$$

$$SR \simeq 1 - \sigma^2 \qquad \sigma = \frac{2\pi}{\lambda} W_{RMS}$$

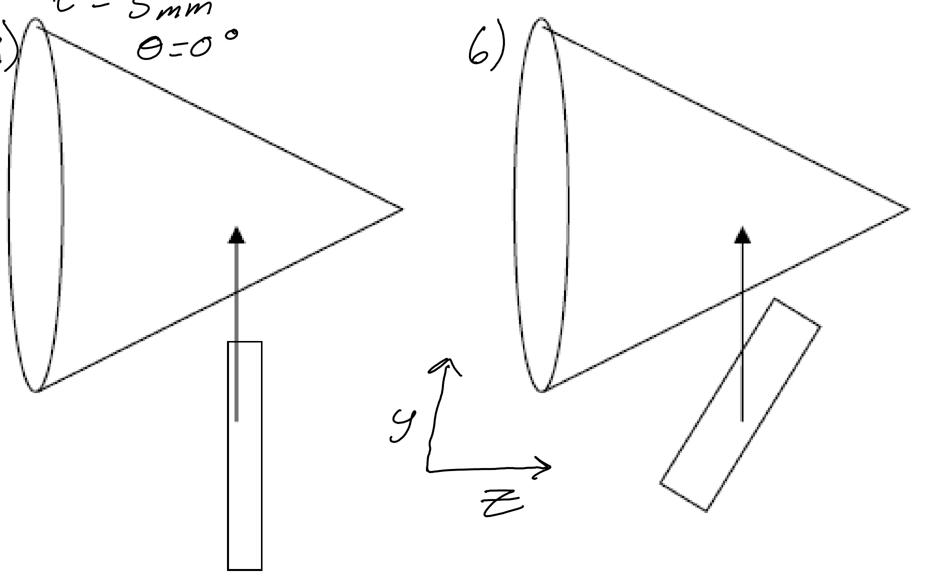
$$W_{RMS} = .289 (W_{P-V})$$

$$SR = 1 - (2\pi \cdot .0289)^2 = \boxed{0.967}$$

4) $n = 1.5$ $\qquad \theta = 45°$

$t = 5mm$

a) $\theta = 0°$ $\qquad$ b)



$$\text{axial shift} \simeq \frac{(n-1)t}{n} \qquad \text{lateral shift} \simeq \left(\frac{n-1}{n}\right) t\,\theta_{radians}$$

a) $\Delta y = 0$
$\Delta z \simeq \frac{t}{3} = 1.7mm$

b) $\Delta y = \frac{5}{3}\left(\frac{\pi}{4}\right) = 1.3mm$
$\Delta z = t/3 = 1.7mm$

3rd order aberrations

a) $W_{040} = \frac{t(n^2-1)}{(f/\#)^4 128 n^3}$

b) $W_{040}, W_{\lambda z}$
$W_{131} = t\theta(n^2-1)$

$$W_{\lambda z} = -\frac{t(n-1)\lambda}{n^2 c}$$

$$\ldots \frac{}{(S/\#)^3\, 16 n^3}$$

$$W_{220} = \frac{t^2\theta^2(n^2-1)}{(S/\#)^2\, 8 n^3}$$

$$W_{\lambda y} = \frac{t\theta(n-1)\lambda}{n^2 c}$$

5) a) From the notes

Definition of alignment for a rotationally symmetric system

A system is aligned if the center-of-curvature of each element is positioned on the optical axis at its correct location.

http://www.optics.arizona.edu/fab%26test/Fall09/Notes/Alignment%20I%20Lecture%20-%20revised.pdf

b) Also from the notes

http://www.optics.arizona.edu/fab%26test/Fall09/Notes/Alignment%20I%20Lecture%20-%20revised.pdf

Alignment is really a two step process -

A first order step to control object & image location and mag

A second step to zero out aberrations using remaining DoF

If only the first step is done, then light is focused at the correct location but the image quality will be poor, in general

The second step must be done to eliminate aberrations due to alignment while holding the first order properties constant