

# Opti 415/515

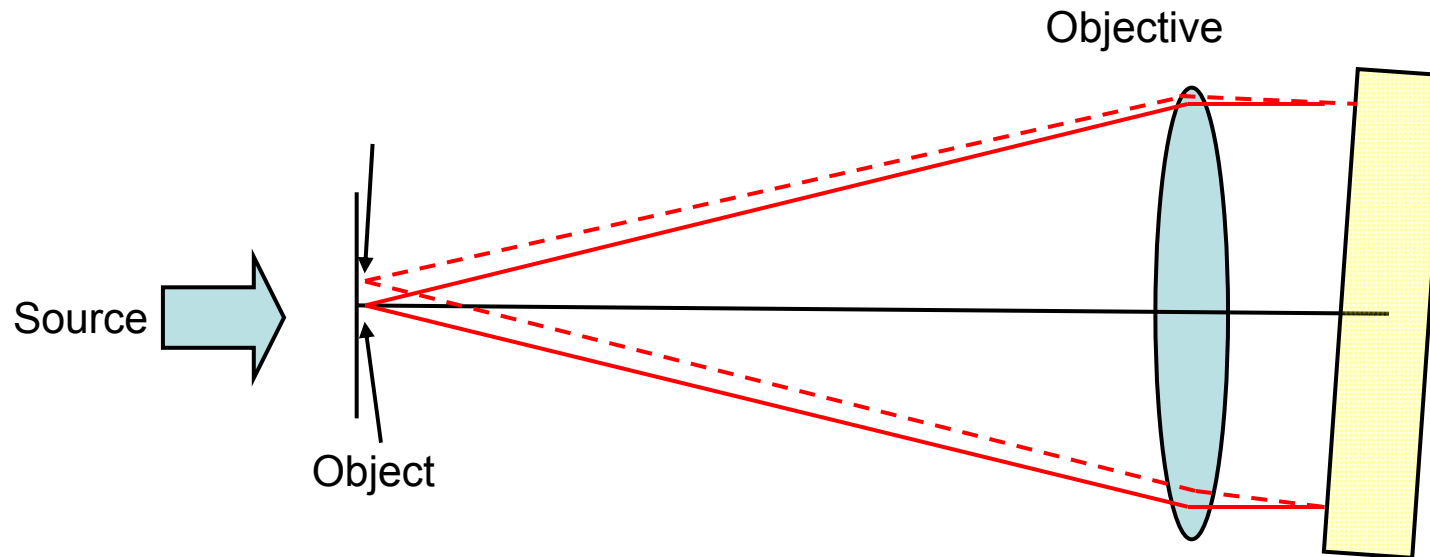
## Measurement of focal length & distortion

# Purpose

- Methods to measure:
  - Focal point location
  - Focal length
  - Distortion – the first non-ideal characteristic of a lens
- Accuracy of measurements:
  - Acceptable level of measurement uncertainty
  - Various methods

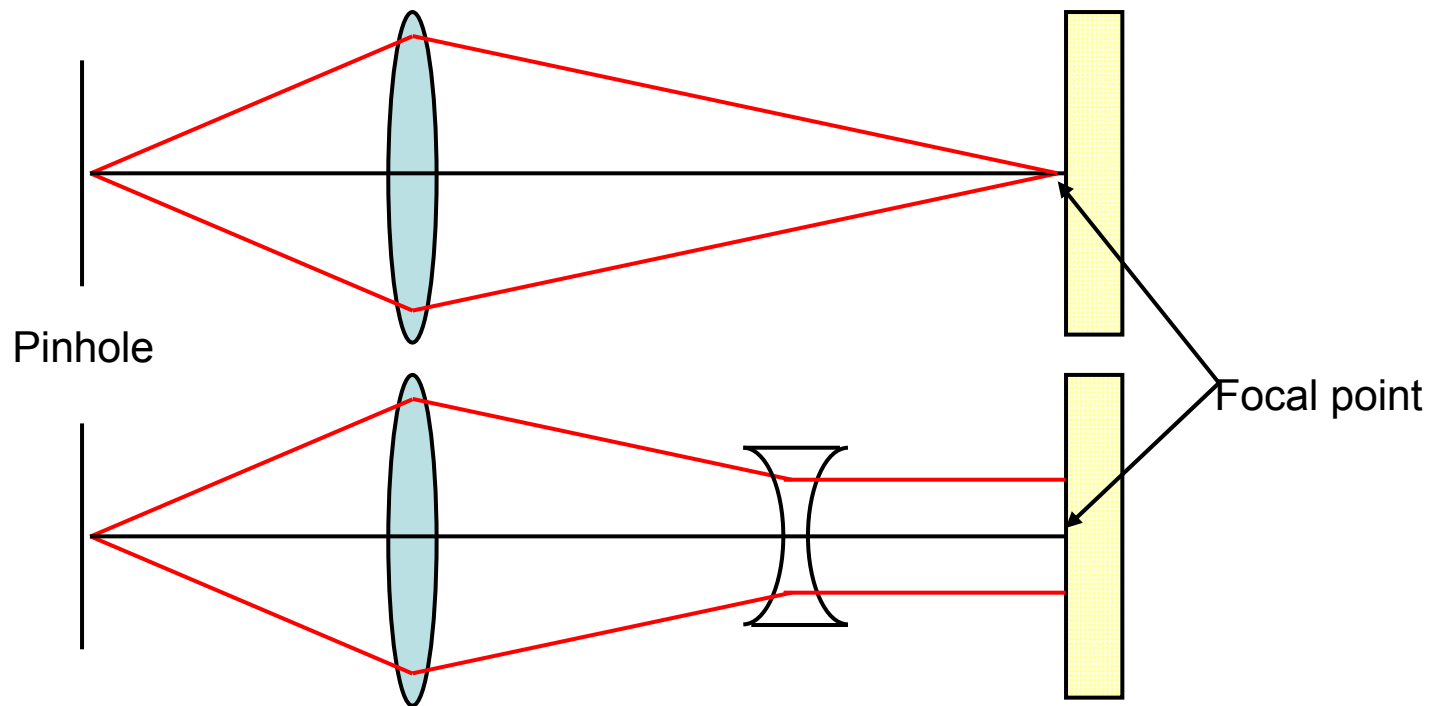
# Autocollimation

- Basic means of finding a focal point
- Project point source or object
- Reflect with a plane mirror, tilted slightly
- When image is in focus at same plane object is at focal point:
  - Pinhole image is minimum size and shifted slightly on pinhole disk
  - Sharp text from back illuminated newsprint



# Autocollimation (2)

- Negative lens requires an auxiliary lens to produce a virtual object to be collimated



# How far is infinity?

- 10 focal lengths  $\rightarrow$  image is 1/10 focal length away from focal point  $\rightarrow$  10% offset
  - More than adequate to tell the difference between a 200 mm and 300 mm focal length lens
- 100 focal lengths  $\rightarrow$  1% offset
  - Typical catalog singlets and doublets have a  $\pm 2\%$  tolerance on focal length
- Whether or not a 1% or 10% offset from focal point is significant depends upon the F/# of the system and wavelength, which determines the depth-of-focus

$$z' = -\frac{f^2}{z}$$

$$z = 10f \Rightarrow z' = -\frac{f}{10}$$

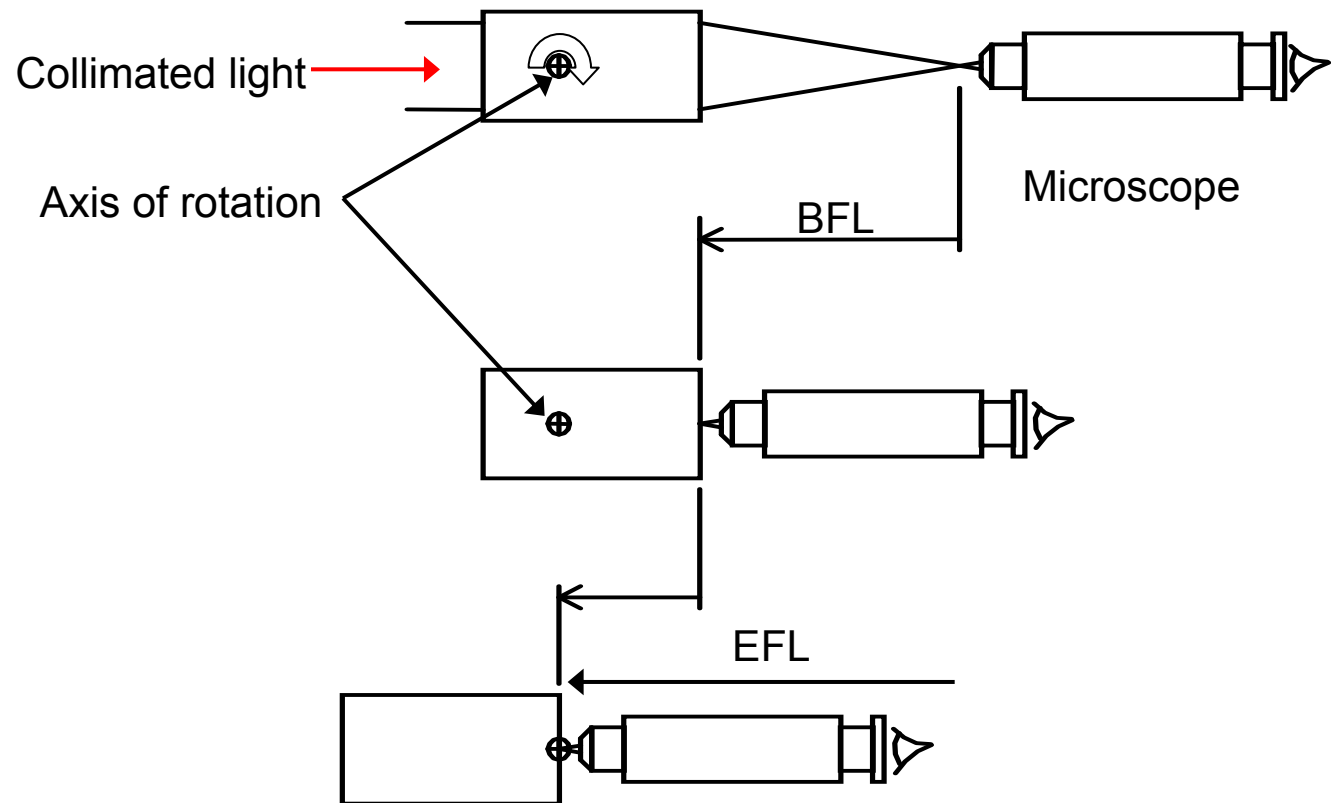
$$z = 100f \Rightarrow z' = -\frac{f}{100}$$

# Nodal slide use

1. Move lens and microscope to see a sharp image that does not move when the lens is rotated.

2. Move microscope to find BFL

3. Move lens and microscope to find distance from nodal point to lens vertex



# Measurement of cardinal points using a nodal slide

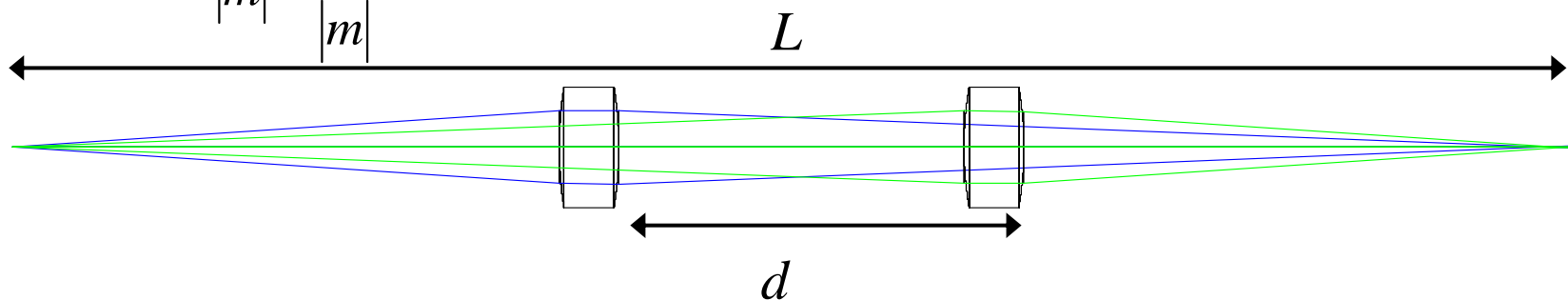
- Nodal slide works because angle of ray incident on first nodal point changes by rotation angle, and so does ray exiting second nodal point, if rotation is through second nodal point. In which case the image does not move.
- Find nodal point by illuminating the lens with collimated light and observing image motion as the lens is rotated. Translate the lens to minimize the image motion, maintaining focus by following with the microscope. When the image motion is minimized, the nodal point coincides with the axis of rotation.
- Find the back focal distance BFD directly by translating the microscope and imaging the rear surface of the lens. If the optic is clean, you will need to apply dust to the surface and focus on the dust. Measure the microscope motion.
  - Try using cross illumination, or better yet, a point source microscope to find surface.
- Find the distance from the rear surface of the lens to the nodal point. To do this, move the lens until the axis of rotation coincides with the rear surface. You can observe this by rotating the lens about this axis and observing the rear surface itself. This surface (or the dust on it) will not move when the rotation axis coincides with the surface. Measure the microscope motion. Can substitute another artifact in place of lens to find axis of rotation.
- Adjust lens axial location such that image of point source viewed by microscope does not move for small rotations of lens

# Reciprocal magnification

- An indirect measurement of focal length
- For a fixed track length (object to image distance) there are two positions for a lens to produce a real image of a real object
  - Minimum track length occurs at a magnification of -1
- Only need to measure magnification for one position and  $d$ 
  - Measure magnification at both positions to provide a check
  - Reticle for object – filar eyepiece for image
    - Filar eyepiece has a reticle and micrometer driven line for measurement
  - Grating for object, screen for image and camera imaging screen for measurement
- Equation is paraxial – good imaging is required or stop the lens down

$$f = \frac{d}{|m| - \frac{1}{|m|}}$$

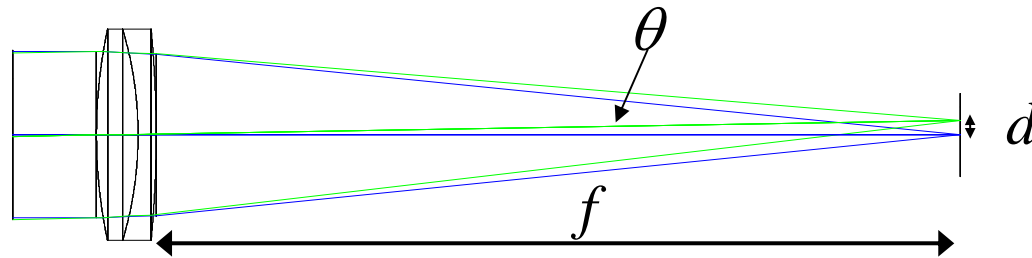
$$L \geq 4f + \overline{PP'}$$



# Transverse magnification

- Collimator → rotate lens with camera attached
  - Measure image displacement on camera as a function of known angle change
- Collimator to project two bars
  - Calibrated distance between bars and collimator focal length → no stage
- Sometimes it might be easier to do either of the above in reverse by using the lens under test as a projector and observing image with collimator
- Can also move an object at finite conjugates and observe with camera or microscope and follow object with stage
  - Extremely high accuracy is possible, but requires very high quality stages, environmental controls, ...

$$\tan(\theta) = \frac{d}{f}$$

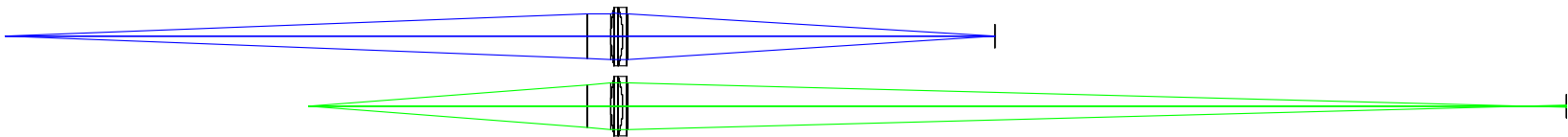


$$f = \frac{1.745\text{mm}}{\tan(1\text{deg})} = 99.97\text{mm}$$

Is magnification affected by F/#?

# Longitudinal magnification

- Axial shift of object results in axial shift of image in same direction
- Measure object shift and image shift  $\rightarrow$  2 values,
  - Newton's imaging equation has 3 values: focal length, and object and image positions
- Need to measure image shift for two object shifts – perhaps use optimizer to solve
- As the shift approaches zero,  $m_L = m^2$
- Other methods seem easier, but someday you might solve a problem using this idea
- Longitudinal magnification varies with object distance  $\rightarrow$  image of a 3D object is stretched asymmetrically

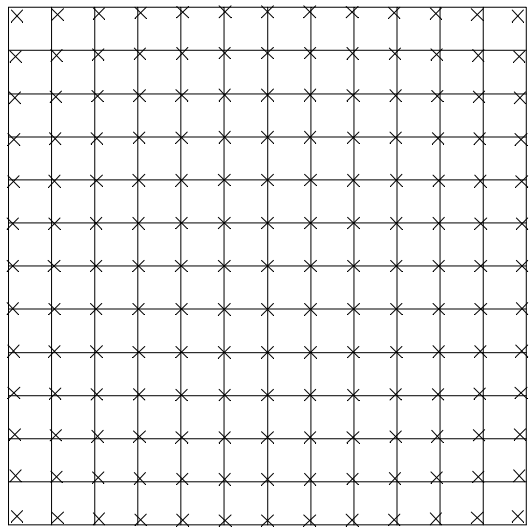


# Distortion

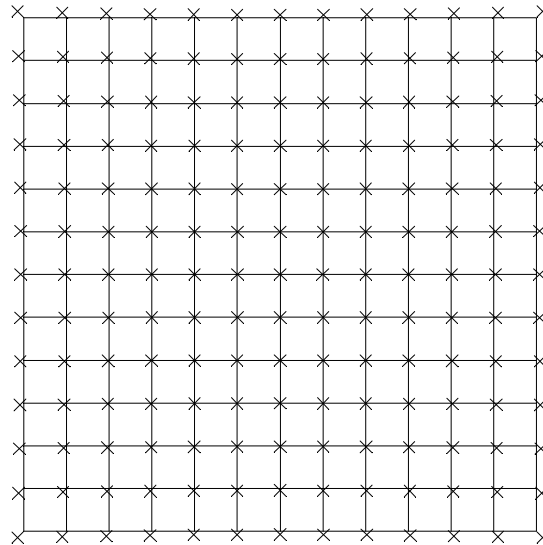
- Paraxial imaging → transverse magnification is a constant
  - Not generally true in real life
  - Difference is distortion
- Image (chief ray) location is not at expected location → centroid of image is displaced
- A more general concept is that an optical system *maps* an object surface to an image surface:
  - Usually map a plane to plane with constant magnification
  - Anamorphic system: different magnification used in vertical and horizontal directions – cameras and projectors for wide-screen movies
  - Scan lens – map angle to transverse position – a very specific type of distortion
- For our purposes distortion is an imperfect mapping of object space to image space due to non-constant (transverse) magnification
- Measurement of image or wavefront quality for a single object field point will not show distortion
- 3<sup>rd</sup> order aberration → image displacement is 3<sup>rd</sup> power of field, constant aperture because image position is defined by chief ray through center of aperture

# Distortion (2)

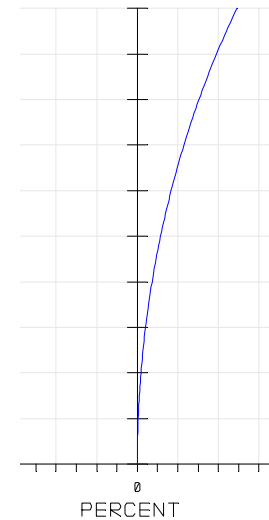
- Square grid is object \* paraxial magnification
- x intercepts are image points exhibiting distortion
- Distortion is inherent in design
- Distortion different than design results from fabrication or assembly errors, including an incorrectly positioned aperture stop



Barrel distortion



Pincushion distortion



Field vs. distortion

# Measurement of distortion

- Since distortion is non-constant magnification, measurement of distortion can be considered measurement magnification variation over the field.
- 3<sup>rd</sup> order aberration theory → distortion varies as cube of field position only, not always true
  - Equivalently, relative distortion is quadratic with field
- Nodal slide → measure image displacement as a function of lens rotation – convenient for an infinite conjugate system
- Measurement of 1% distortion for a 10 mm image height:
  - 100 μm at edge of FOV – fairly easy
  - 1 μm at 0.1 relative field – requires care to measure
- Required angle control for measurement can be demanding. Does maximum image height correspond to 50 degrees, or 1 degree?

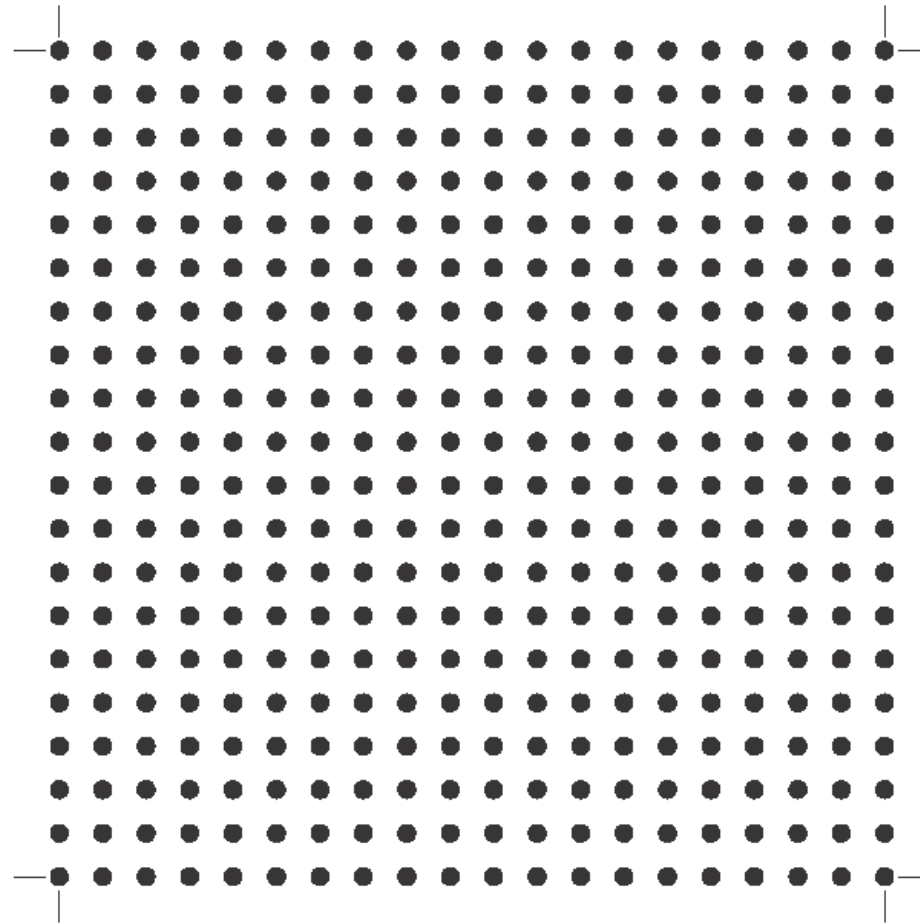
$$r' = r + ar^3 + br^5 + \dots$$

Normalized radial  
coordinates

$$D = \frac{r' - r}{r} = ar^2 + br^4 + \dots$$

Relative distortion

# Calibration grid



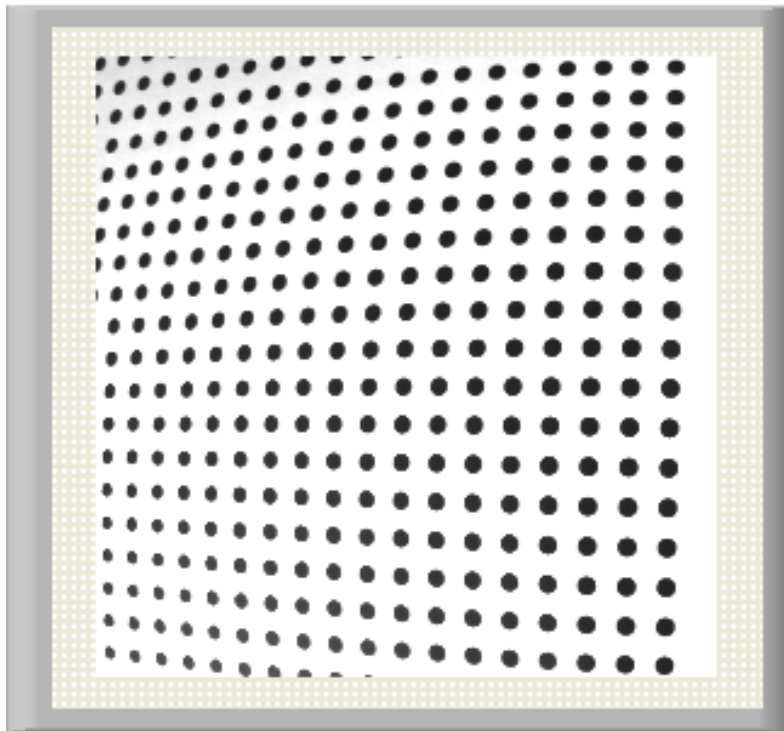
# Processing calibration grid

- Capture image of grid with camera
- It is fair to assume, for vast majority of applications, that pixels on camera are regularly spaced and the size specified in the data sheet
- Machine vision software to find spots and calculate centroid of each.
- Convert data into a transformation to remap
- Most machine vision toolkits support – even to the point of performing correction for projection

# LabVIEW example

- Nonlinear calibration example – calibration grid exhibiting both distortion and perspective (camera) projection
- Real object image of coins

Calibration Template



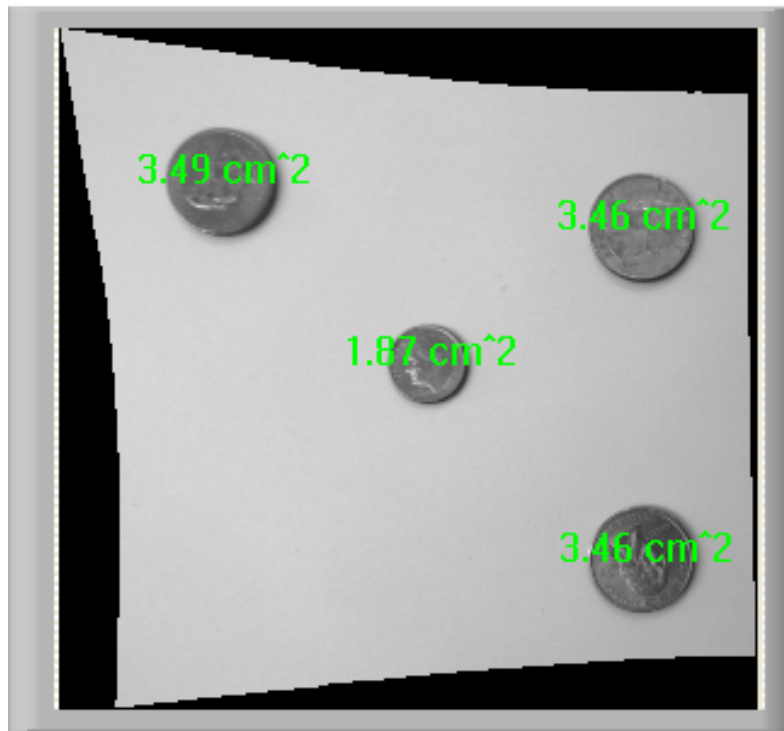
Coins Image



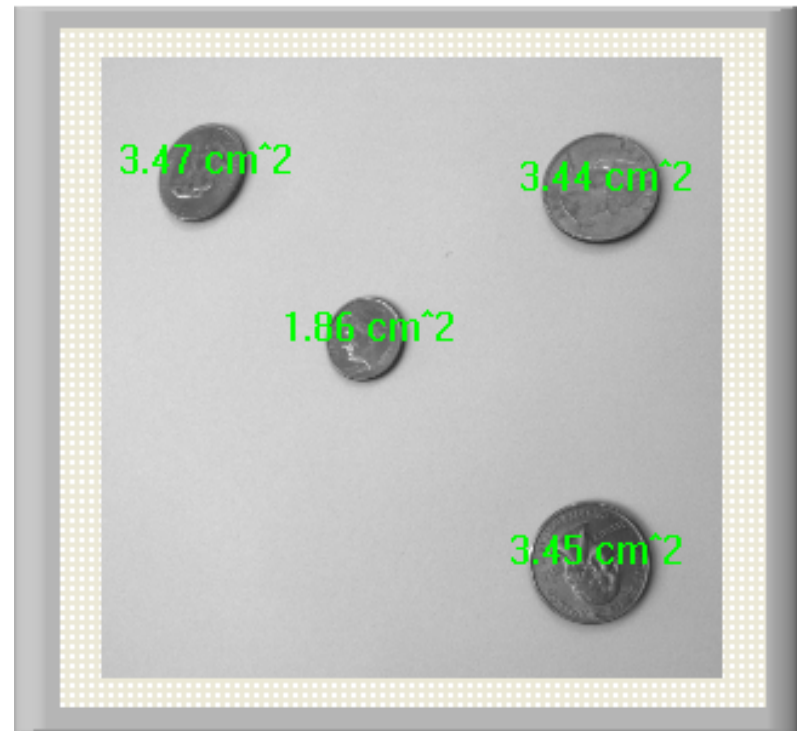
# LabVIEW example (2)

- Calibration grid is used to learn the mapping, which is used to produce a corrected image and measurements of object area before and after image correction
- The measured area of the quarters range from 3.44 cm<sup>2</sup> to 3.49 cm<sup>2</sup>

Corrected Image



Calibrated Measurements



# Direct distortion measurement

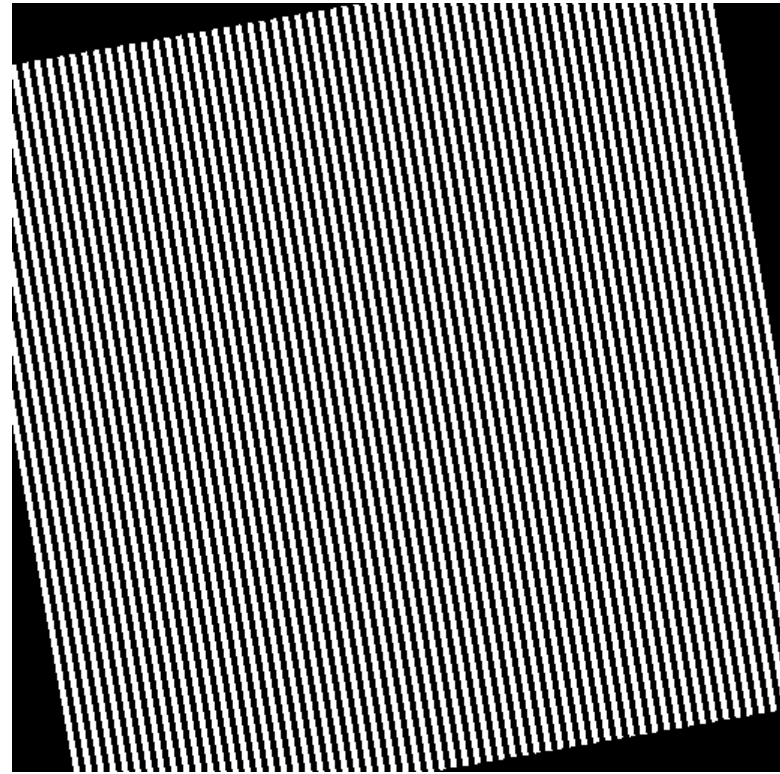
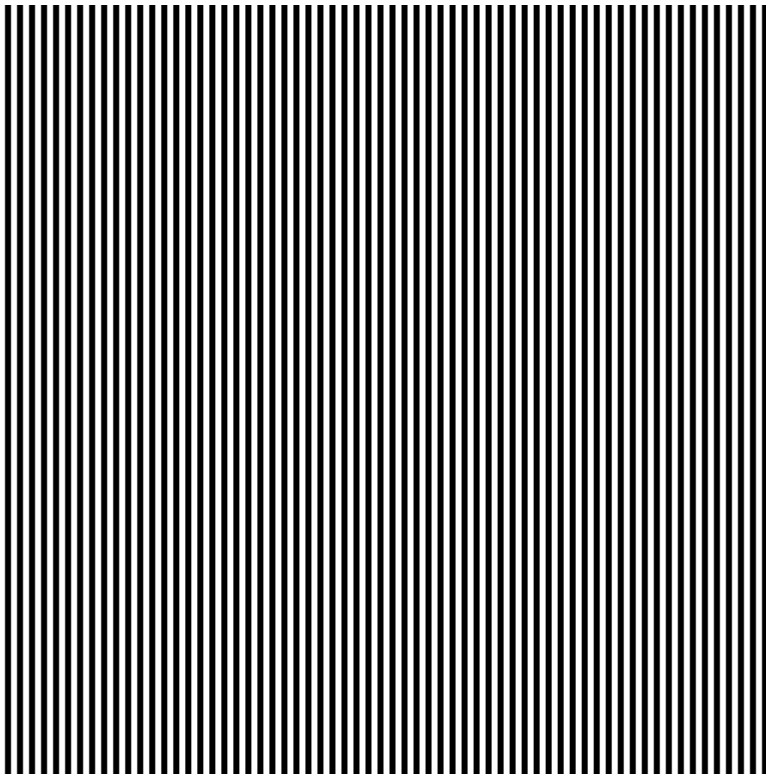
- Measurement of distortion is essentially a measurement of a linear dimension (along a radial line)
  - Microscope on a stage used to find centers of spots in image of a calibration grid
- Measuring linear dimensions with a dynamic range of:
  - $10^3$  is generally easy (look at a ruler, or certainly a meter stick)
  - $10^4$  takes some care (e.g. a good, calibrated caliper)
  - $10^5$  costs more – 1  $\mu\text{m}$  out of 100 mm
- Typical image sizes are perhaps 10 mm x 10 mm – certification of less than 0.1% distortion likely requires measurements of distances with better than 10  $\mu\text{m}$  accuracy (dynamic range of  $10^3$ )
- A system requirement of  $< 0.5 \mu\text{m}$  distortion over 50 mm x 50 mm field requires a measurement uncertainty of better than 1 part in  $10^6$ 
  - Note, aluminum TCE  $\sim 24 \cdot 10^{-6}/\text{C} \rightarrow 1\text{C}$  change over 50 mm is  $> 1 \mu\text{m}$
  - Possible, but a general purpose machine to do this is *expensive*

# Moiré techniques

- Moiré interference – multiplication of two irradiance patterns (or addition with a nonlinear detector)
  - Cross terms result in a secondary pattern that behaves like a fringe pattern
  - Synthetic wavelength related to period of grating, not wavelength of light
- For very demanding applications gratings can be written with an e-beam writer (used for making masks for lithography) - use one as an object and one as an image

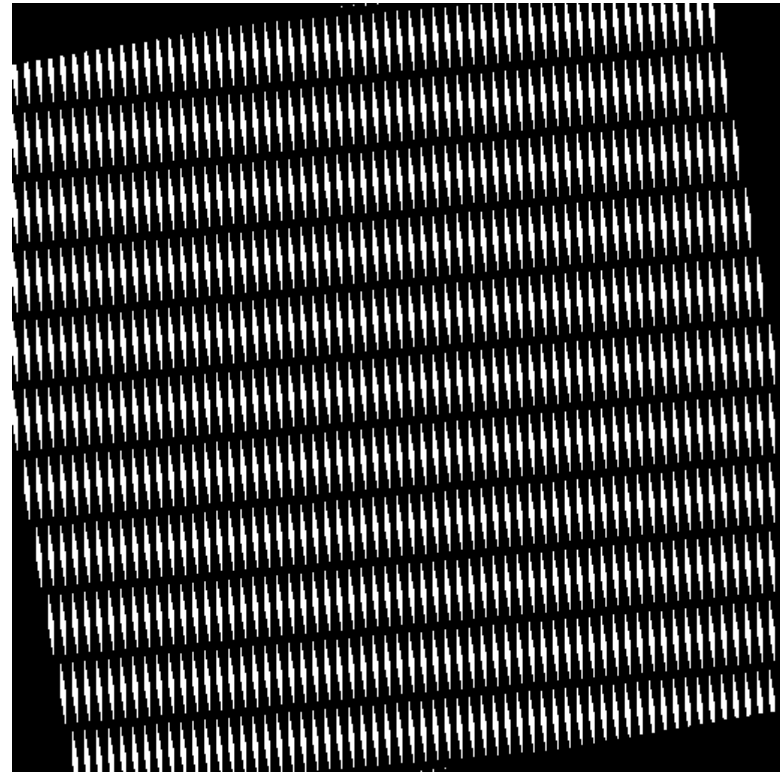
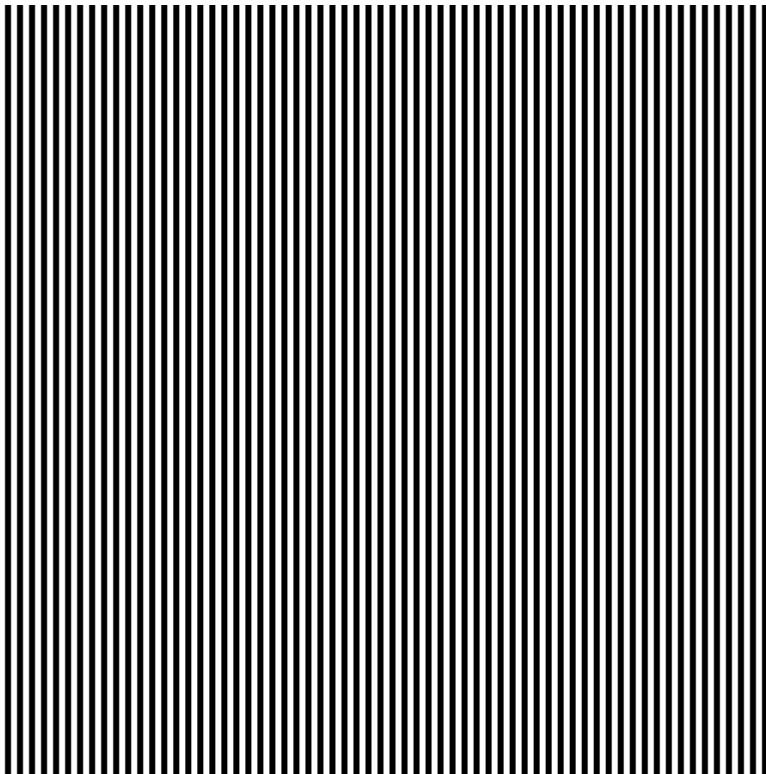
# Moiré (2)

- Reference “wavefront” or pattern on left
- Test on right
- Simulations performed using Matlab logical arrays and logical and to perform “multiplication”



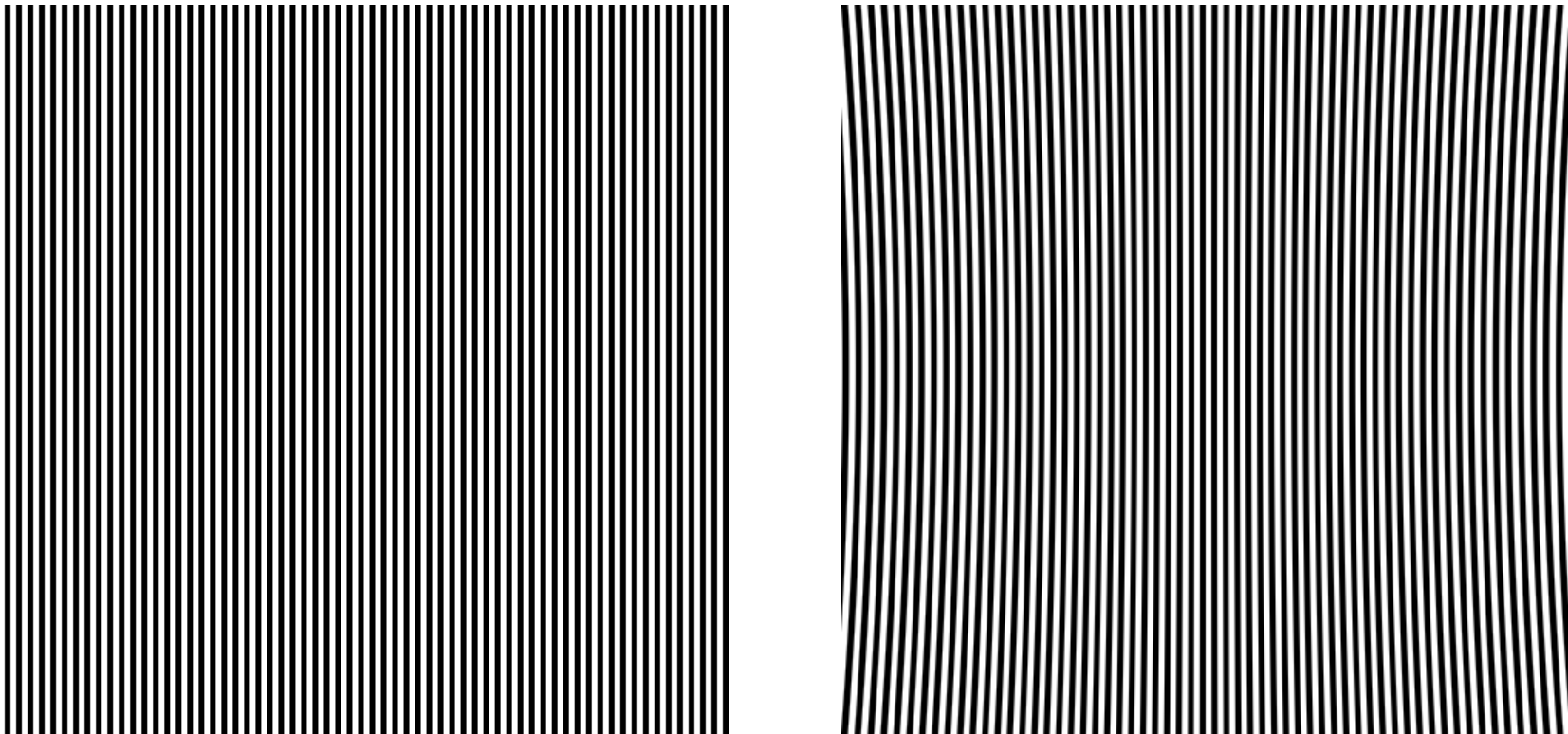
# Moiré (3)

- Reference on left
- Combination of reference and tilt on right



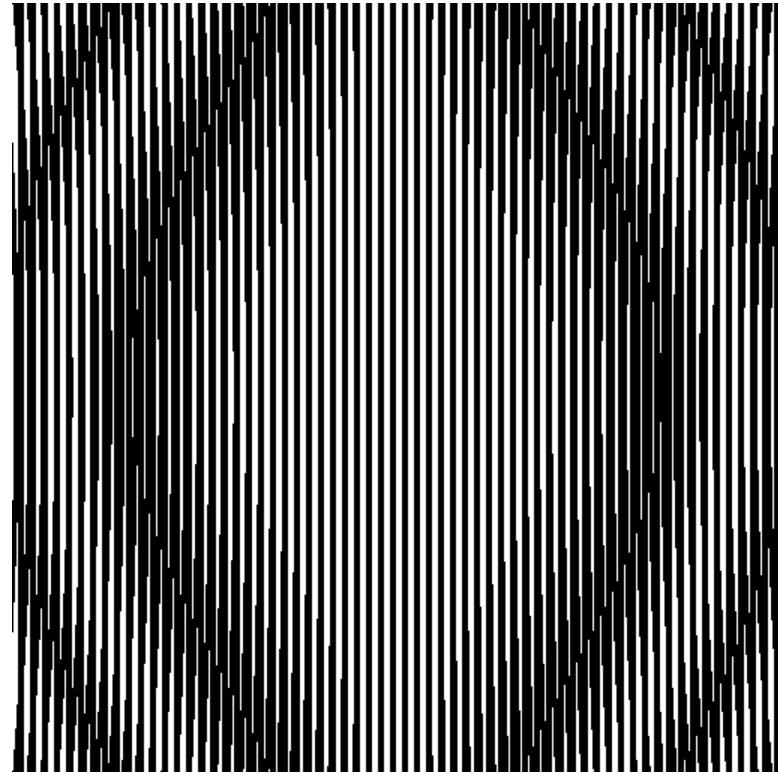
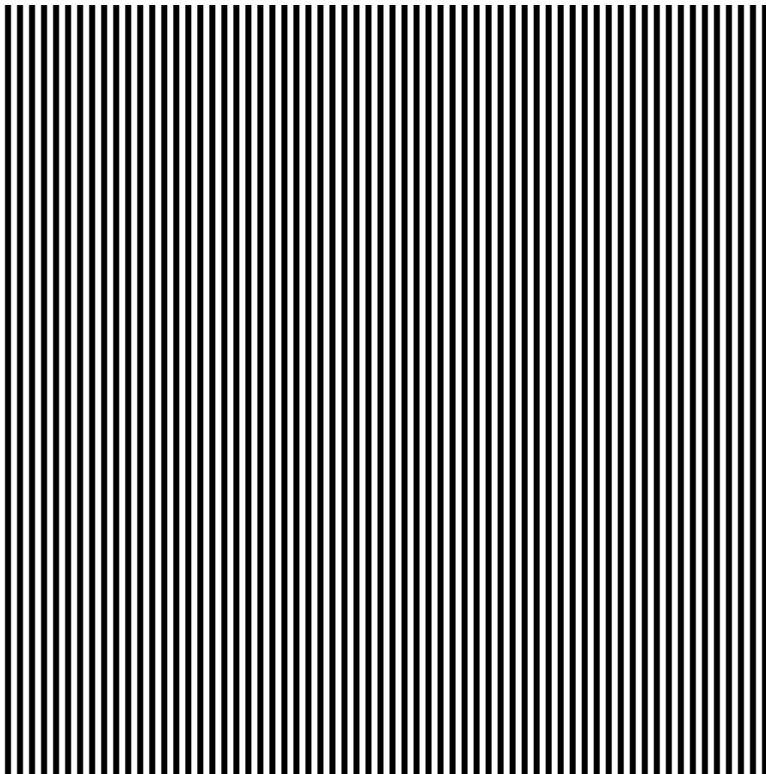
# Moiré (4)

- Reference on right
- 10% distortion on right (used Matlab griddata to remap a distorted coordinate system)



# Moiré (5)

- Reference on left
- Reference and distorted map on right



# More companies with useful products

- National Instruments: LabVIEW™ and their extensive toolkits for motion, vision and more
- The Mathworks: Matlab™ and their toolkits
- Machine Vision software: Cognex, Matrox, etc.
- Stages to build that ridiculous machine: Newport, Aerotech, Zygo or Agilent for DMI, etc.
  
- And so many other companies worth looking at