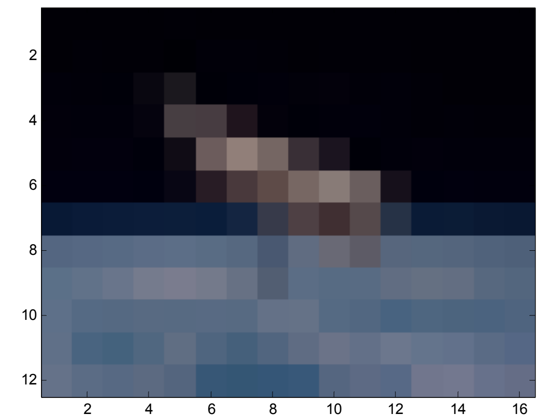
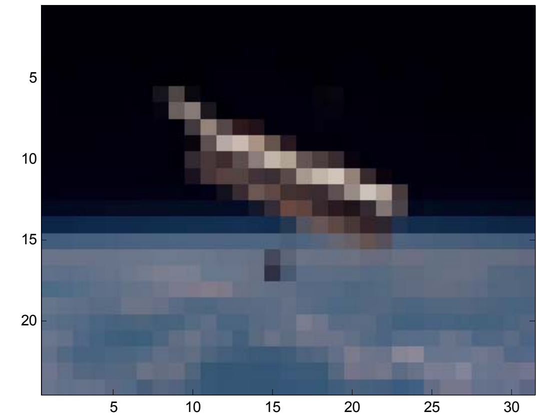
Opti 415/515

Introduction to aberrations and image quality metrics

Identification vs. resolution

- Detection of an object requires few samples
- Determining the orientation requires a bit more
- Recognition – jeep vs. truck requires still more
- Identification – what kind of jeep still more, but not many
- High quality images – require resolution that at least matches eye

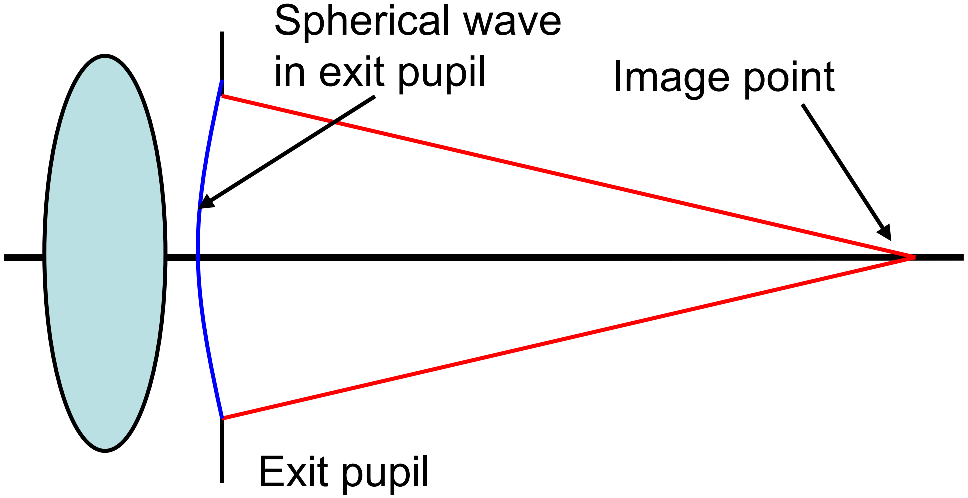




See for example, W. Wetherell, Applied Optics & Optical Egr. Shannon & Wyant, Vol VIII, P. 247

Images are a collection of points

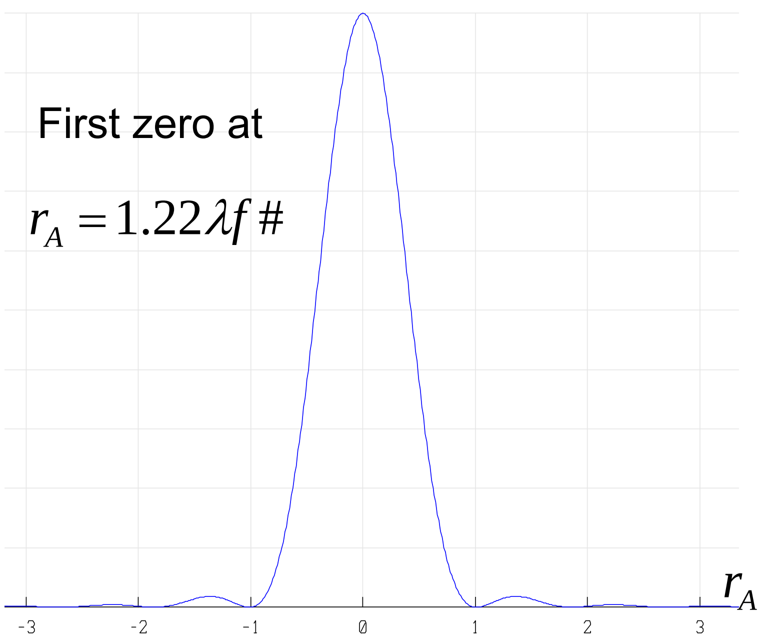
- An (incoherent) imaging system maps the object space FOV to image space point-by-point and the output image is the sum of all the points:
  - The system is linear
- Paraxial optics, in addition to ignoring imperfections in an optical system, ignores the wave nature of light
- Basic characteristic of a linear system is its impulse response.
- Impulse response of an optical system is the point spread function (PSF)
- For a well corrected (i.e. good) optical system the point spread function defines the ideal, but limiting performance of an optical system



- Fourier transform of a constant is a delta function
- Electronic signals are a function of time (1D)
- Optics function of space (2D)
- A constant wavefront is a plane wave through all space
- What is effect of aperture?

Ideal image of a point source

- A well corrected imaging system produces a spherical wave in the exit pupil for each object point
  - An object point at infinity produces a plane wave
- In image plane, the spherical wave produces the Airy disk – a bright peak surrounded by rings
- Airy disk in units of Airy radii



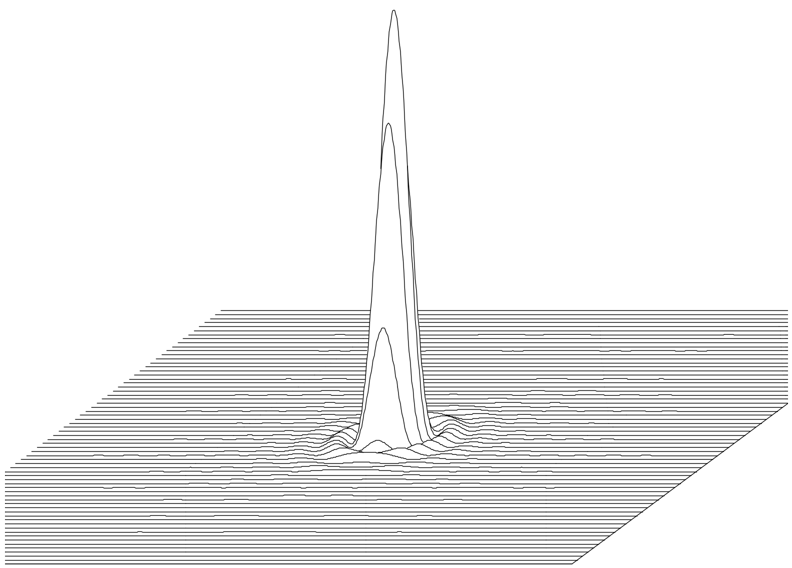
$$E(r) = \frac{E_A \pi^2 d^2}{16\lambda^2 (f\#)^2} \left[\frac{2 J_1(\pi r / \lambda f\#)}{\pi r / \lambda f\#} \right]^2$$

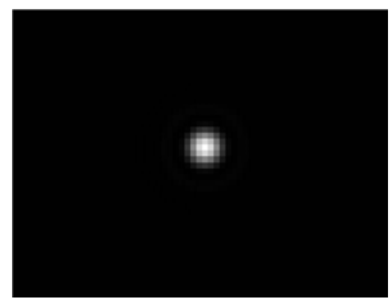
$$f\# = L/d$$

d	exit pupil diameter
λ	wavelength
E_A	aperture irradiance
$\pi d^2/4$	aperture area
r	radial dist. from peak to obs. point

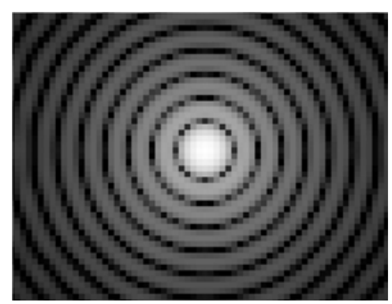
A few views of a perfect PSF

- Surface – rings are small
- Top image – rings are barely visible (in part because of display contrast
- Bottom image – log scale
- 84% of power is in first ring, 7.1% in 2^{nd}, 2.8% in 3rd, 1.5% in 4^{th}, 1.0% in 5^{th}, so on





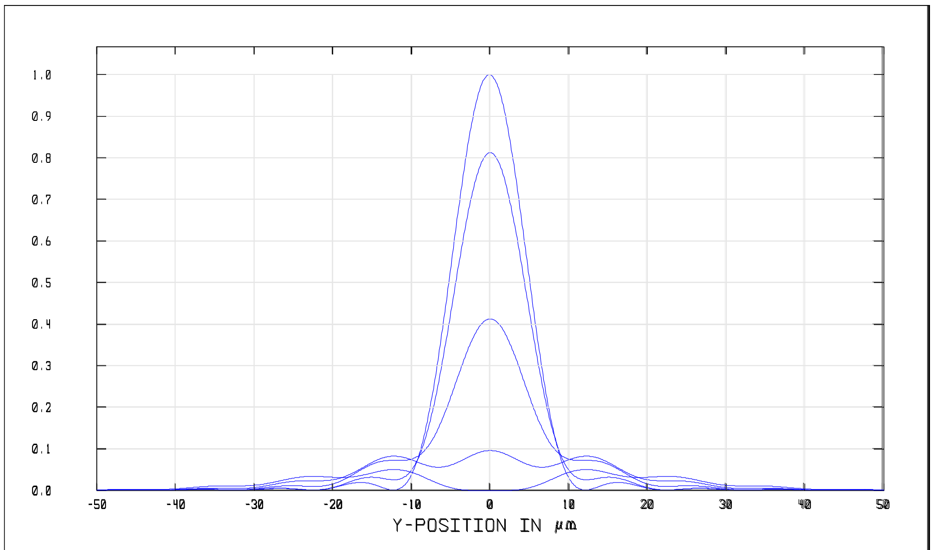
Linear scale



Log scale

Defocus

- System is “perfect”, F/10, 1 μm wavelength
- Defocus – shift the image plane – less energy in peak
- Wavefront in exit pupil is not a sphere centered about the image point –
- The difference between the reference sphere in the exit pupil centered in on the image point and the actual wavefront is an *aberration* – an imperfection in the wavefront



PSF CROSS SECTION

$$\varepsilon_z = -8 \Delta W_{defocus} \left(f\# \right)^2$$

- Defocus – shift the image plane – what happens?
- System is “perfect”, F/10, 1 μm wavelength
- Focal length is 100 mm
- Curves from top, correspond to focus shifts of 0, 0.2, 0.4, 0.6 and 0.8 mm

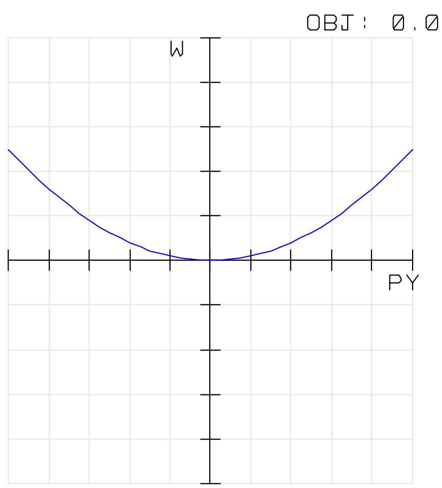
- Strehl ratio – ratio of actual peak irradiance to peak for unaberrated wavefront
- Maréchal – well corrected if Strehl ratio > 0.8

Diffraction depth-of-focus

- If assume only OPD error is defocus, then Rayleigh criterion (OPD error < λ/4) can be used to derive allowable depth-of-focus

$$\Delta W_{defocus} = \pm\frac{\lambda}{4} = \pm\frac{\varepsilon_z}{8(f\#)^2}$$

$$\varepsilon_z = \pm 2\lambda (f\#)^2$$



OPD curve ¼ wave error (½ wave full-scale – curve is a … parabola

Transverse aberrations

- OPD or wavefront error ΔW (difference between actual wavefront and the ideal reference sphere) is a general function of two pupil coordinates
- If ΔW is not a constant across the pupil, then the system is aberrated
- Transverse aberrations - a is unaberrated if it intersects the image plane at the desired image position. Displacement of the ray
  - in the image plane is a transverse aberration
  - along the optical axis is a longitudinal aberration
- The transverse aberration ε_x and longitudinal aberration ε_z can be expressed in real coordinates (left side), or normalized coordinates (max radial coordinate of 1, pupil radius *h*, index = 1).
- A spot diagram is a plot of ray intercept locations or transverse ray aberrations

$$\varepsilon_x = \frac{-\partial \Delta W(x,y)}{n \partial x}$$

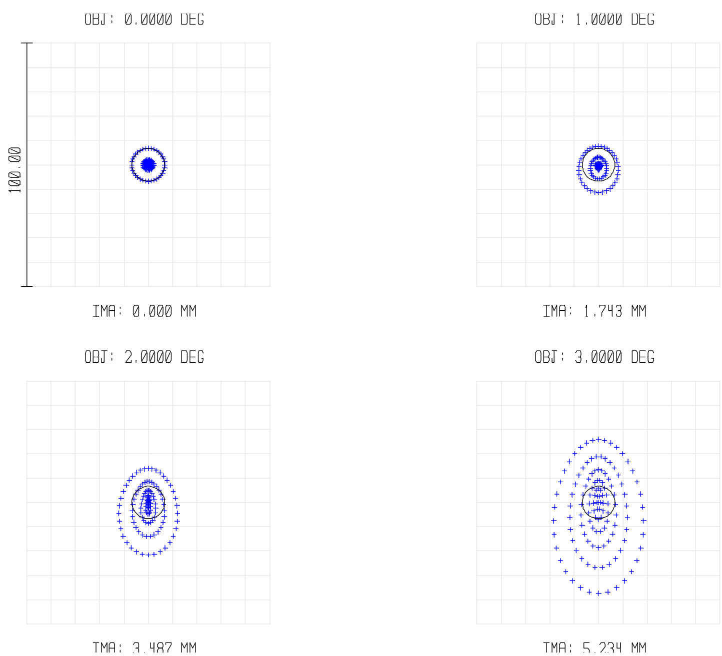
$$\varepsilon_x = \frac{-R}{h} \frac{\partial \Delta W}{\partial x}$$

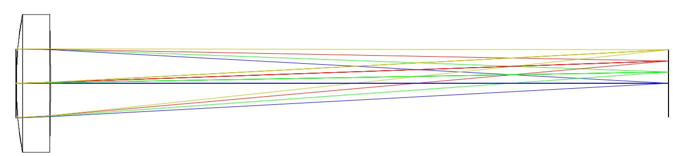
$$\varepsilon_z \approx \frac{R}{x} \varepsilon_x = -\frac{R^2}{x} \frac{\partial \Delta W(x,y)}{n \partial x}$$

$$\varepsilon_z \approx \frac{R}{x} \varepsilon_x = \frac{-R^2}{xh^2} \frac{\partial \Delta W}{\partial x}$$

A singlet

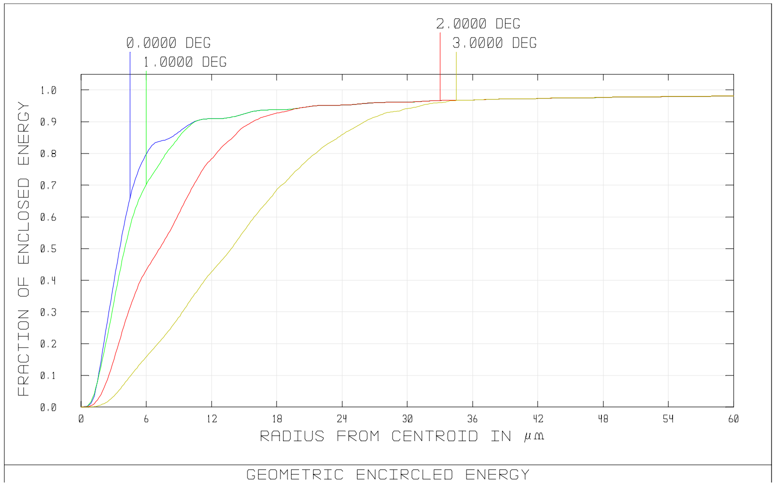
- Plano-convex single of N-BK7 with a 100 mm focal length at 550 nm.
- Object at infinity, angles of 0, 1, 2, 3 degrees
- Black circle in spot diagrams is the Airy disk radius
- Spot diagrams are a plot of ray intercept locations for rays distributed throughout the exit pupil
- Geometrically a perfect spot diagram with all ray intercepts at the same exact location is possible – diffraction does not let this happen in real life

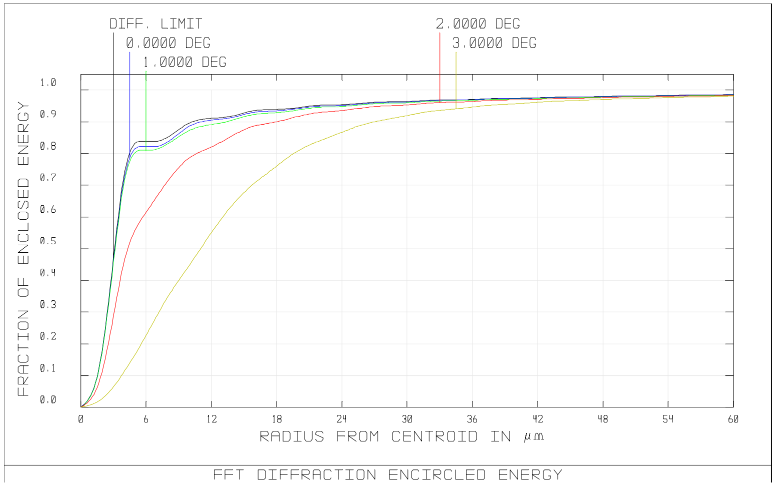




Encircled energy plots

- Geometric calculation is done by tracing a set of rays and then counting the fraction within the specified distance
- Diffraction does an FFT on an OPD map
- If spot diagram is smaller than the Airy disk the geometric calculation will overestimate the performance
- If spot is much larger than the Airy disk, which means there are large wavefront slopes or transverse ray aberrations, the diffraction calculation may be in error due to aliasing of the FFT.





Specifying system performance

- Strehl ratio is a very reasonable specification for a system and can be used as performance criterion for a tolerance analysis,
  - but can you measure Strehl ratio directly?
  - In general, it is not easy to measure the very peak irradiance of a spot with another optical system having calibrated radiometry
- Encircled energy – fraction of total energy within a circle, or ensquared within a box, centered on image point
  - Can specify in design programs
  - Can measure for many systems if have adequate image sampling or may require microscope to magnify image
- Effective specification for a system used with an area detector. For example, if pixels are 7.4 μm on a side then a performance specification of:
  - "80% of energy shall fall within a square 7.4 μm x 7.4 μm"
- Could qualify a system by measuring the ratio of energy on a pixel to the total energy on a 3x3 or 5x5 pixel area
  - Sometimes referred to as EOD – energy on detector
  - Should scan spot in sub-pixel increments since location of PSF peak might be near a pixel edge

Wavefront expansion

- Some will write W and others ΔW for wavefront. Another case of different assumptions. In general we are concerned with the difference between the ideal spherical wavefront and the actual wavefront
- Eqn. is in same form as the Field Guide to Geometrical Optics, pg 73
- *H* – field dependence
- *ρ* – pupil dependence
- Focus is an aberration, tilt is too
  - Generally these are a rigid body alignment in 3DOF
- Spherical aberration – hard / soft sides of focus – spacing errors – wrong conjugates
- Coma – tilts / decenters
- Astigmatism – tilts / decenters
- Field curvature – can't distinguish from focus at a single field point
- Distortion – can't distinguish from tilt with a single of field point

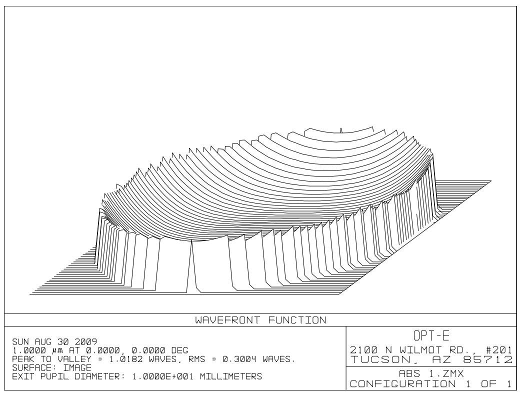
$\Delta W = W_{020}\rho^2 +$	focus
$W_{111}H\rho\cos(\theta)+$	tilt
$W_{040}\rho^4 +$	spherical aberration
$W_{131}H\rho^3\cos(\theta)+$	coma
$W_{222}H^2\rho^2\cos^2(\theta)+$	astigmatism
$W_{220}H^2\rho^2 +$	field curvature
$W_{311}H^3\rho\cos(\theta)+$	distortion

Piston (constant), tilt and focus are not considered Seidel aberrations, but they are perturbations of the wavefront corresponding to 1st order properties.

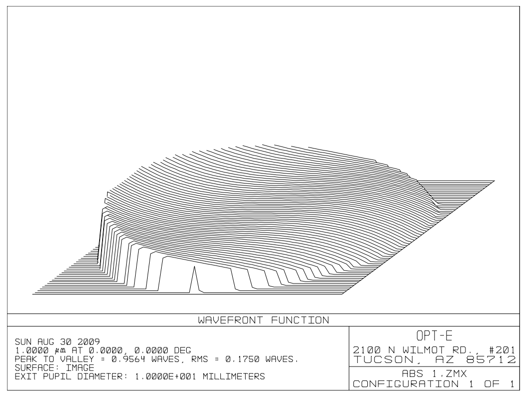
For relationship to Seidel's see Wyant Ch3.1, table II.

Some wavefronts

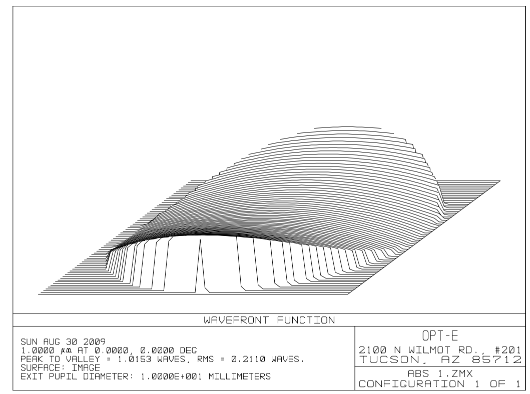
- Coma and astigmatism have orientation, and so does trefoil



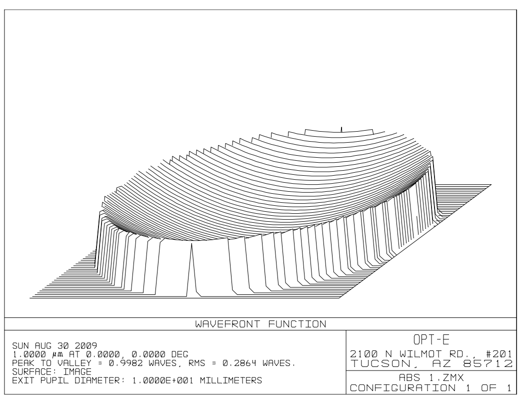
Spherical



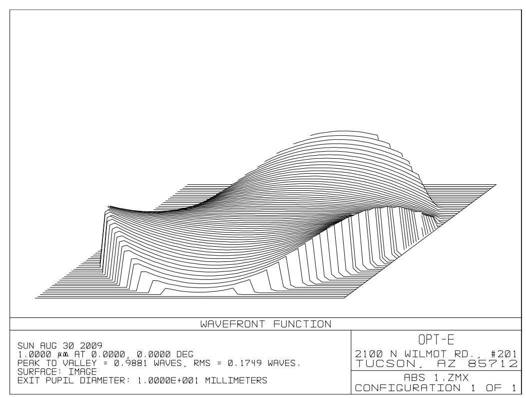
Coma



Astigmatism



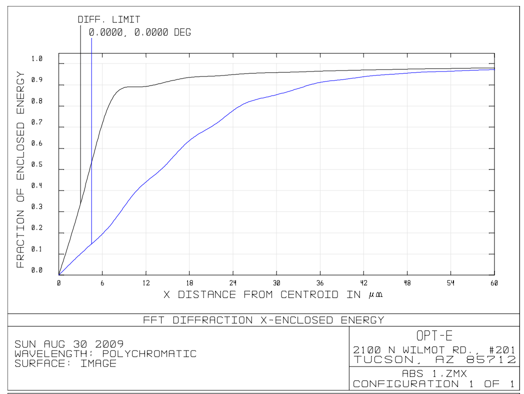
Focus

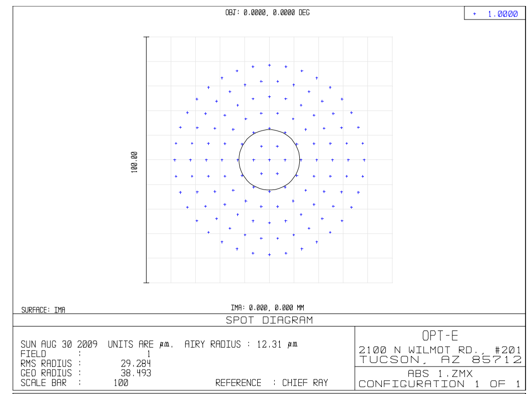


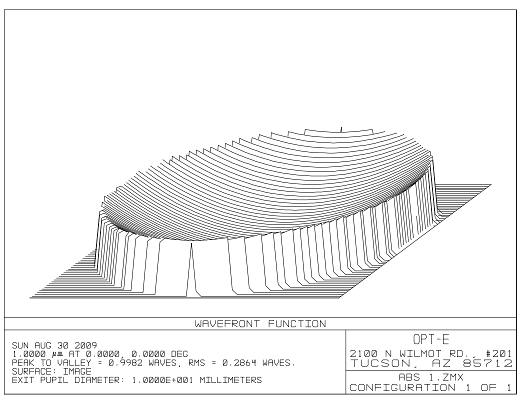
Trefoil

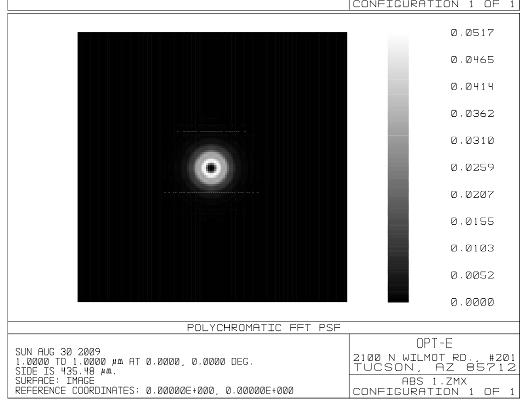
Focus

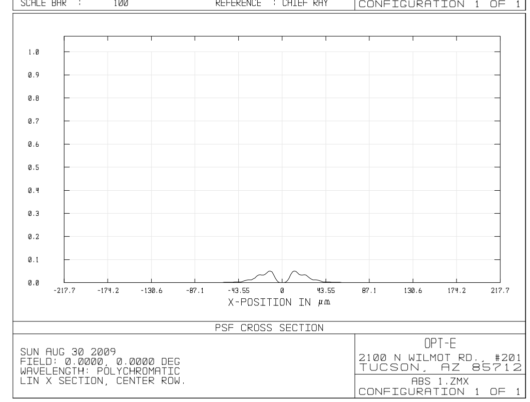
- 1 wave of defocus
- Strehl ratio is obviously ZERO since on-axis illumination is zero





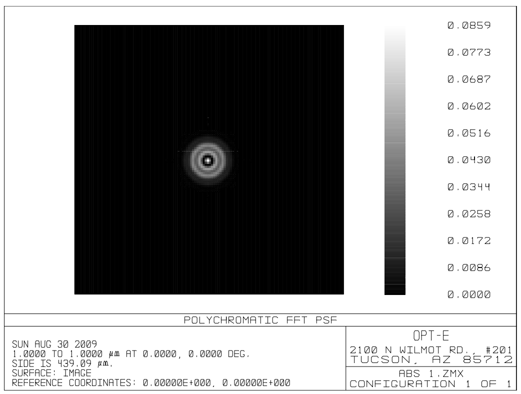


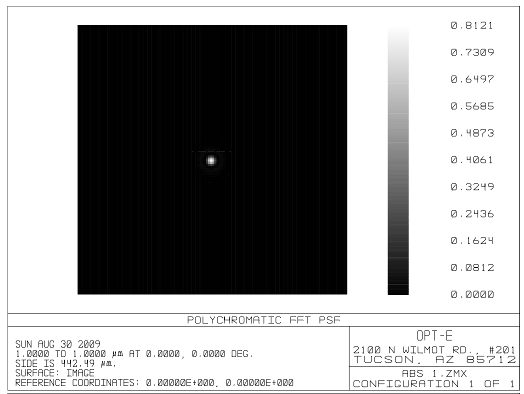


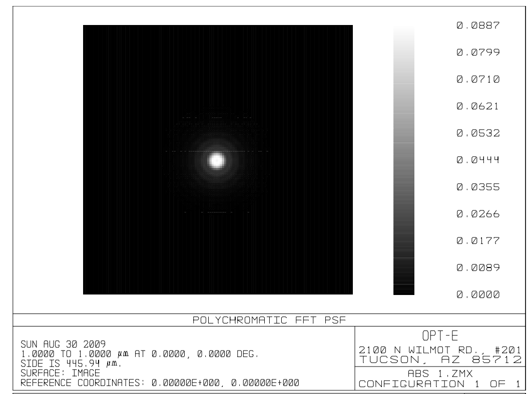


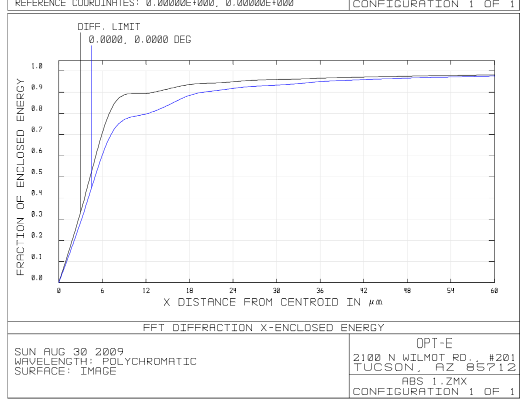
Spherical through focus

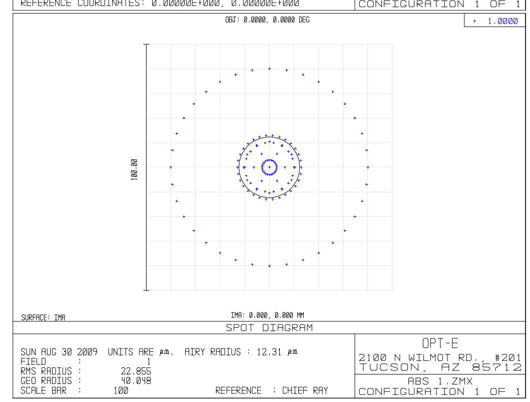
- Top row - 0.25 waves PV spherical, -1,0,1 wave defocus (0.8, 0, -0.8 mm shift)
- Strehl ratio of 0.81 at best focus. 0.073 waves rms to chief ray / centroid





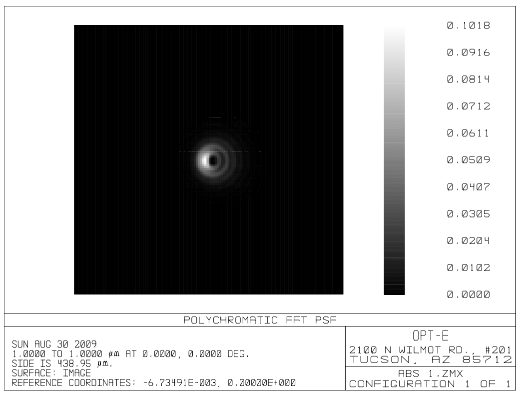


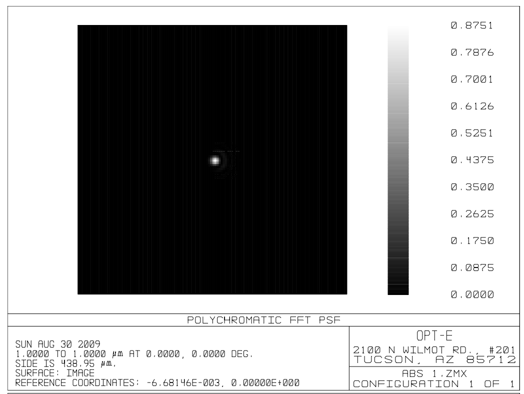


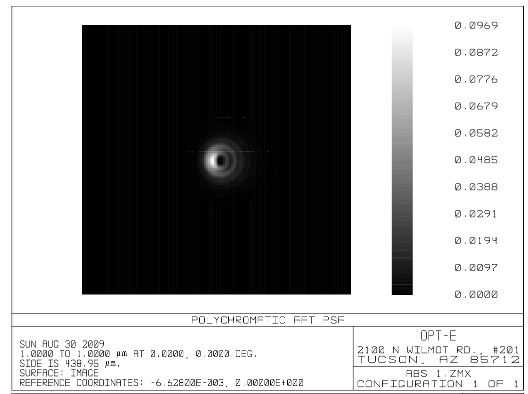


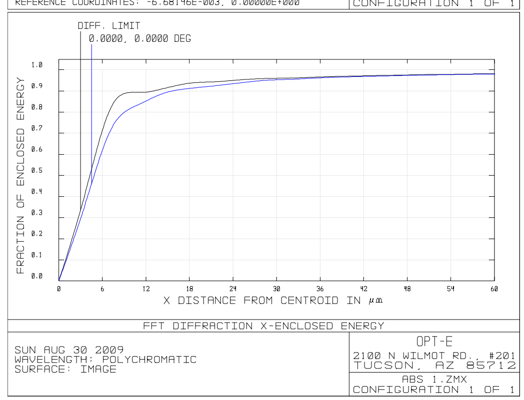
Coma through focus

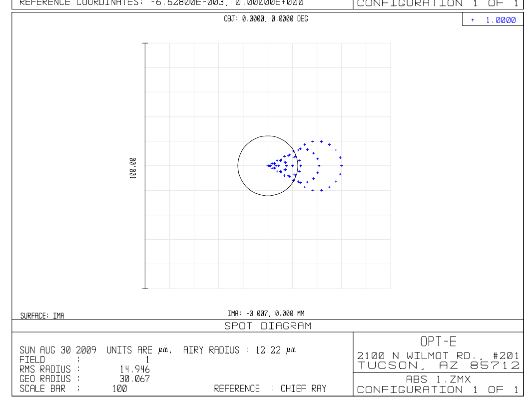
- Top row - 0.34 waves PV coma (to centroid), -1,0,1 wave defocus (0.8, 0, -0.8 mm shift)
- Strehl ratio of 0.87 at best focus. 0.17 waves rms to chief ray, 0.06 to centroid





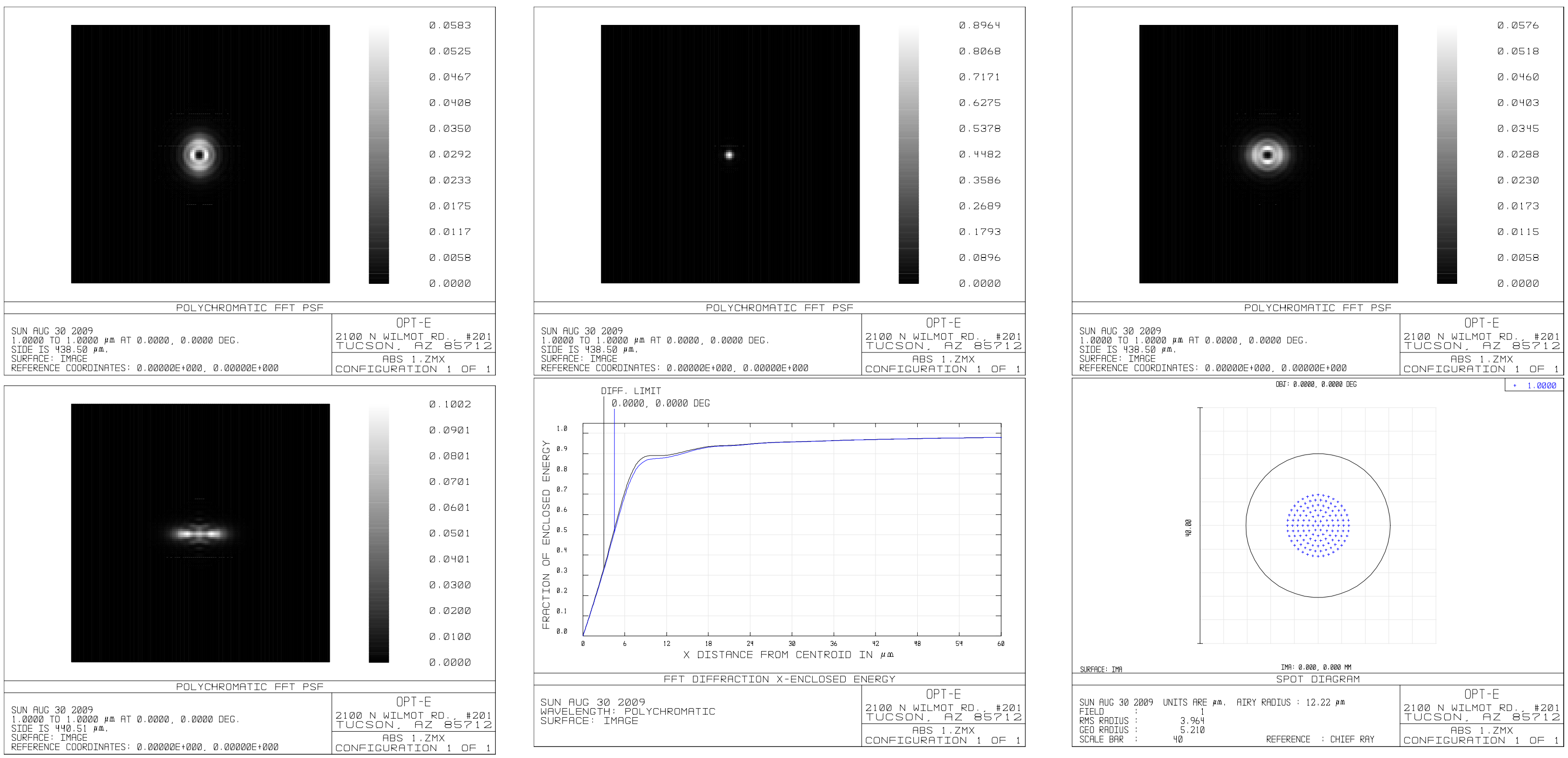






Astigmatism through focus

- Top row – 0.25 waves PV astigmatism, -1,0,1 wave defocus (0.8, 0, -0.8 mm shift)
- Strehl ratio of 0.89 at best focus. 0.05 waves to chief ray / centroid



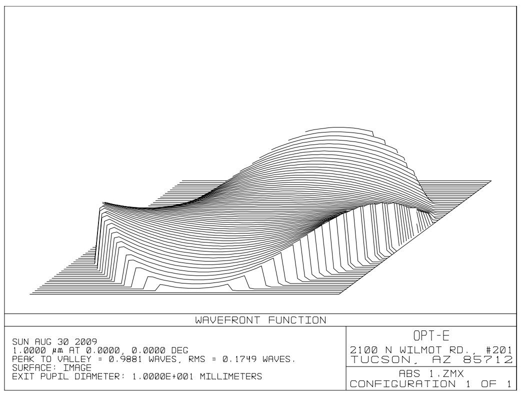
1 wave astigmatism

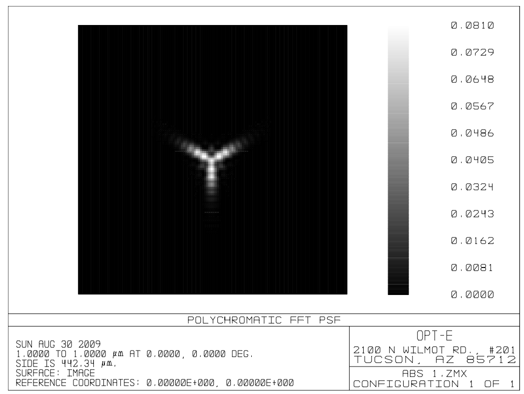
Star test

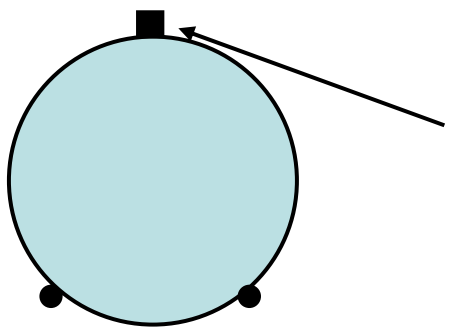
- Examine point image produced by a system using a distant object, such as a star
- Previous slides are simulated star test images – it is very easy to see ¼ PV OPD errors, with care detection of 1/10 wave is possible
- In lab, a collimator with an aperture as large or larger than the system under test is needed. A microscope with an eyepiece, or better, a camera is used to collect imagery
- Need to go through focus
  - To distinguish spherical from focus
  - To detect astigmatism – best focus is circle of least confusion
- For small amounts of aberration ±1 wave of defocus is a practically useful amount of defocus – make sure you have an idea how much translation of the microscope equates to about 1 wave of defocus
- Notice that coma, like all odd aberrations, displaces the image centroid
- Even in the lab turbulence can be a real problem

Trefoil

- What could cause the three period aberration trefoil?



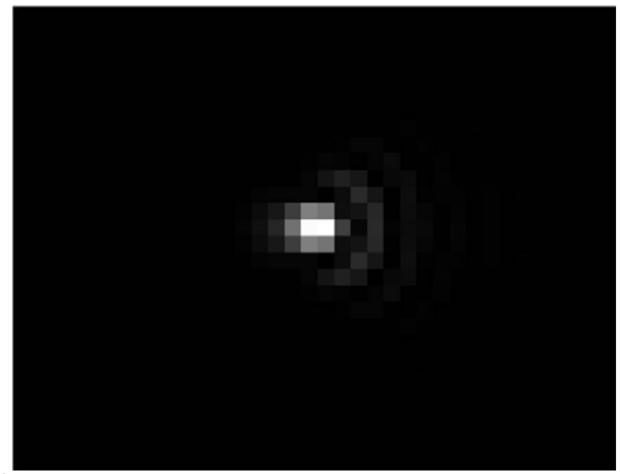


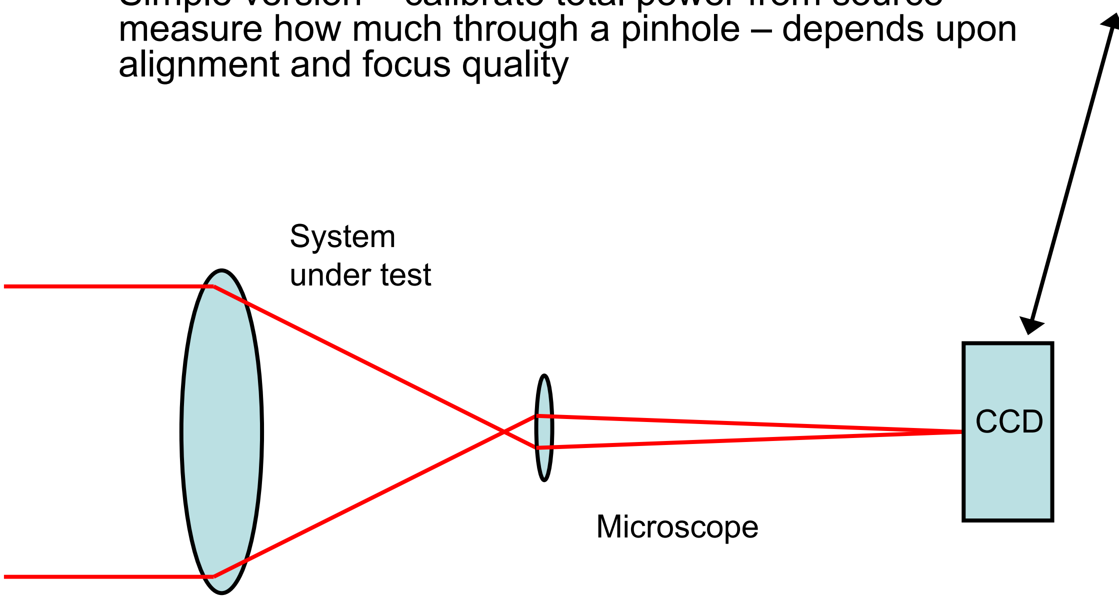


Just push a little too hard on the mirror retaining screw

Measurement of encircled energy

- Camera – Find peak – calculate ratio peak to sum over 3x3 or 5x5 pixels centered on peak – EOD – energy on detector
- Move microscope so that spot is symmetric to center PSF
  - only works if spot is symmetric
  - May need to repeat calculation for a series of positions in x&y of microscope
- High quality measurements require detector calibration, background subtraction, …
- Beam profiling – 1D version shown
- Simple version – calibrate total power from source – measure how much through a pinhole – depends upon alignment and focus quality





Beam profiler to measure size of focused spot