Image Quality

Figures of Merit for Optical Systems

What does the optical system do? The figure of merit provides a number that tells how well the system functions

Optical imaging

Geometric image size

- RMS image diameter
- FWHM

– Fractional encircled energy

– MTF at particular spatial frequencies

– RMSWE (root mean square wavefront error)

– Beam divergence

– Boresight

Other

Coupling efficiency

Data rate

NETD

Two regimes for imaging systems

1. <u>Geometric limit</u>

   Use simple ray trace to determine image quality

   Rms spot size is most common FOM

   Valid for wavefront errors > 1 λ

2. <u>Near Diffraction limit</u>

   Must take the wave nature (interference and diffraction) into account.

   Valid for wavefront errors < $\lambda/4$

   Rms wavefront error is most common FOM

Aberrations - definitions

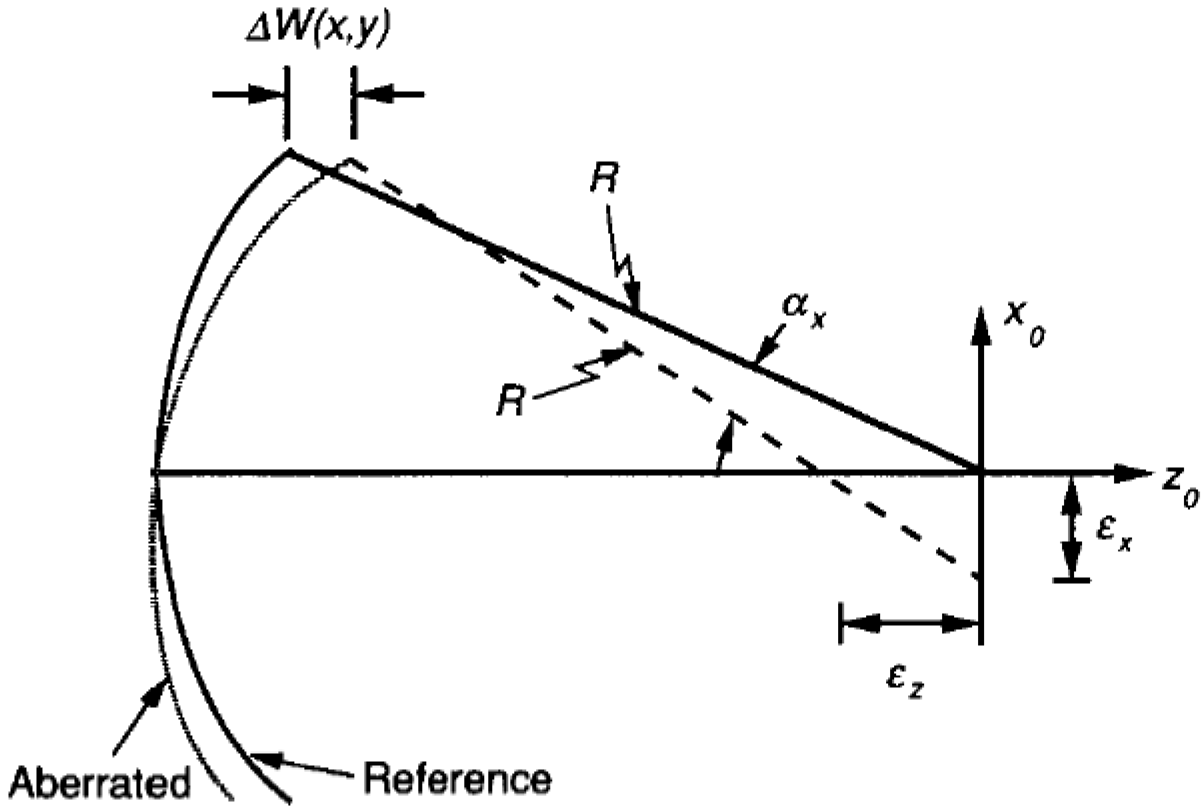
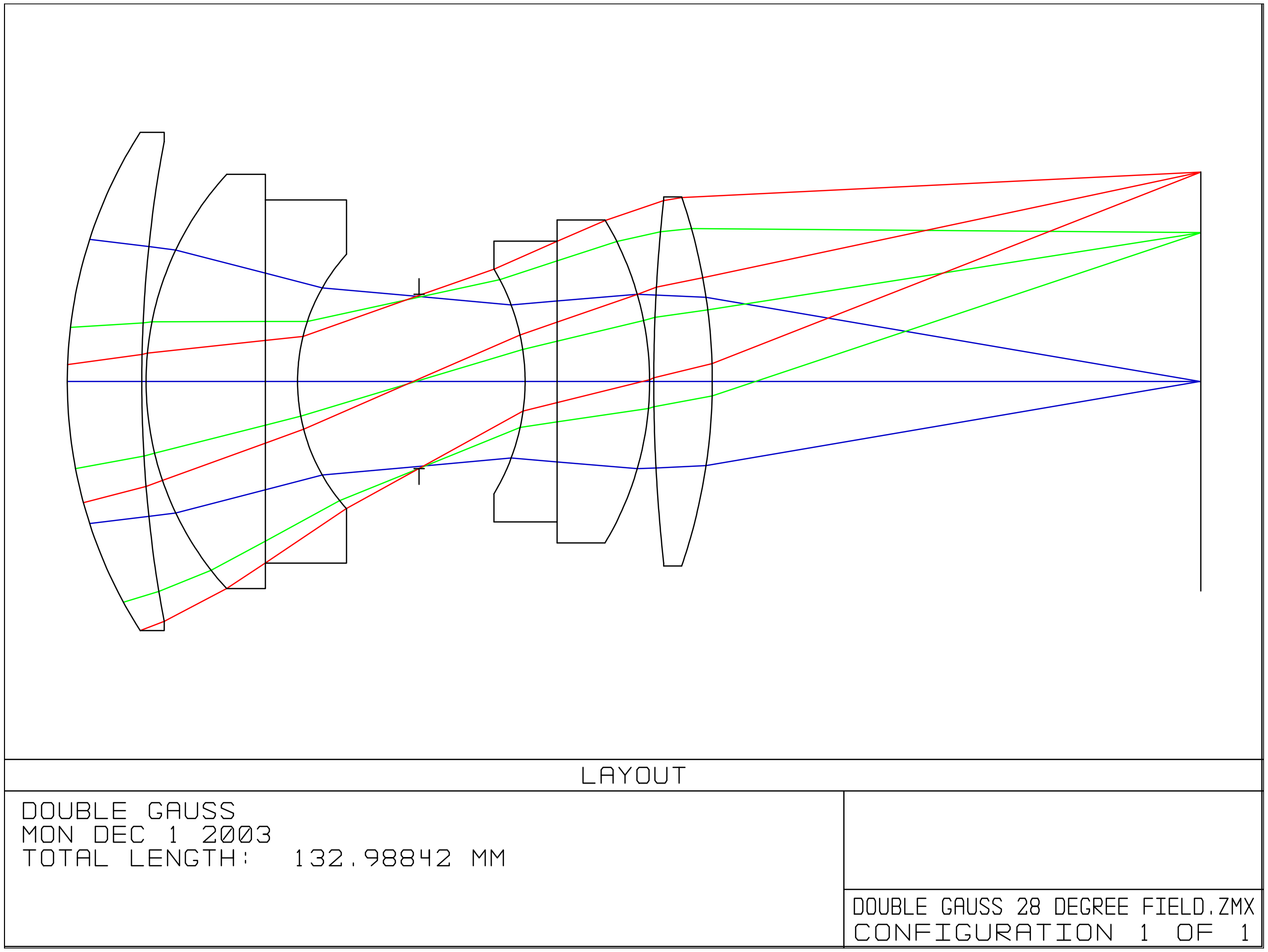
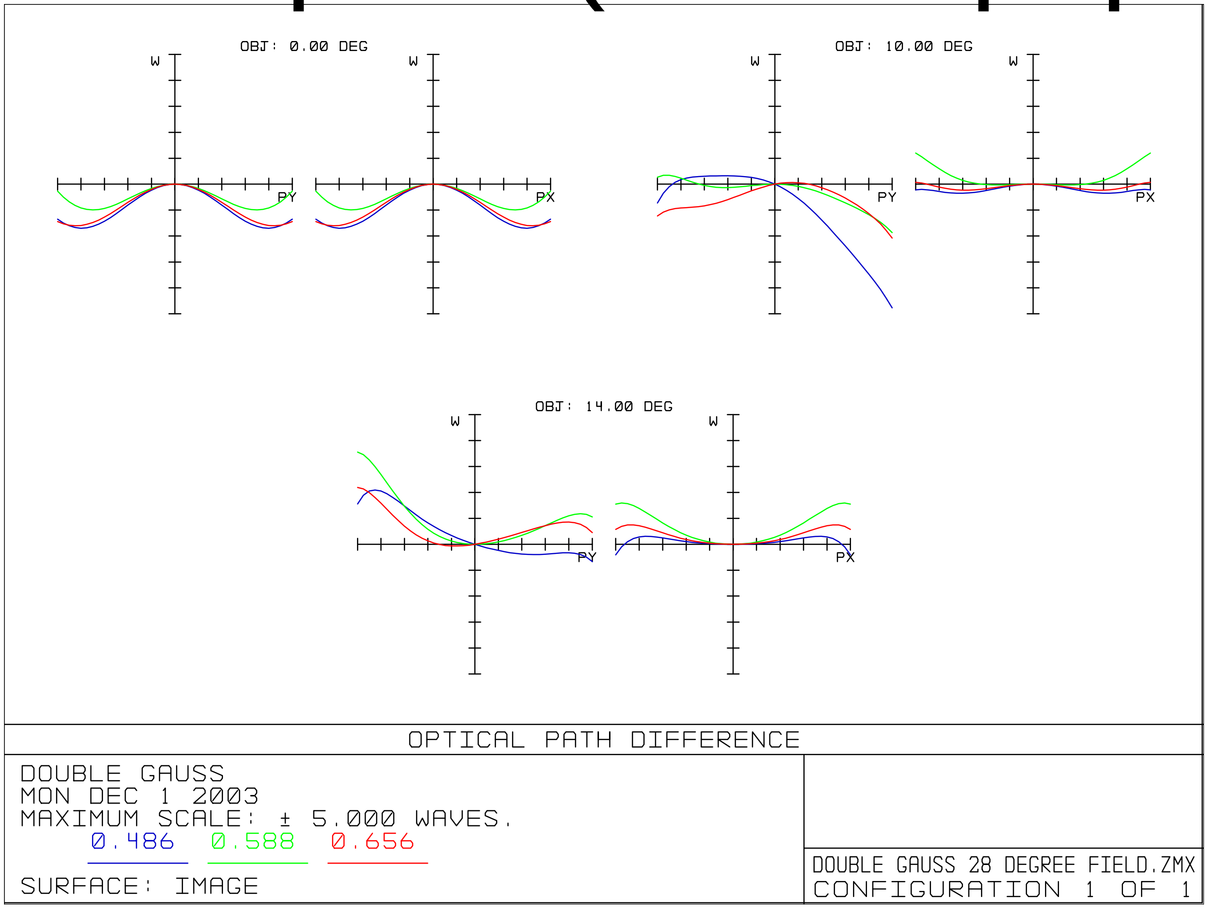


FIG. 15. Transverse and longitudinal aberrations. Δ W(x, y) = distance between reference wavefront and aberrated wavefront, ε_x = transverse or lateral aberration, ε_z = longitudinal aberration.

Example Lens layout



OPD plots (ΔW vs pupil)



$\Delta W = OPD >> 1\ \lambda$

This system is in the <u>geometric</u> limit

OPD (Optical Path Difference) is not a useful metric for this system

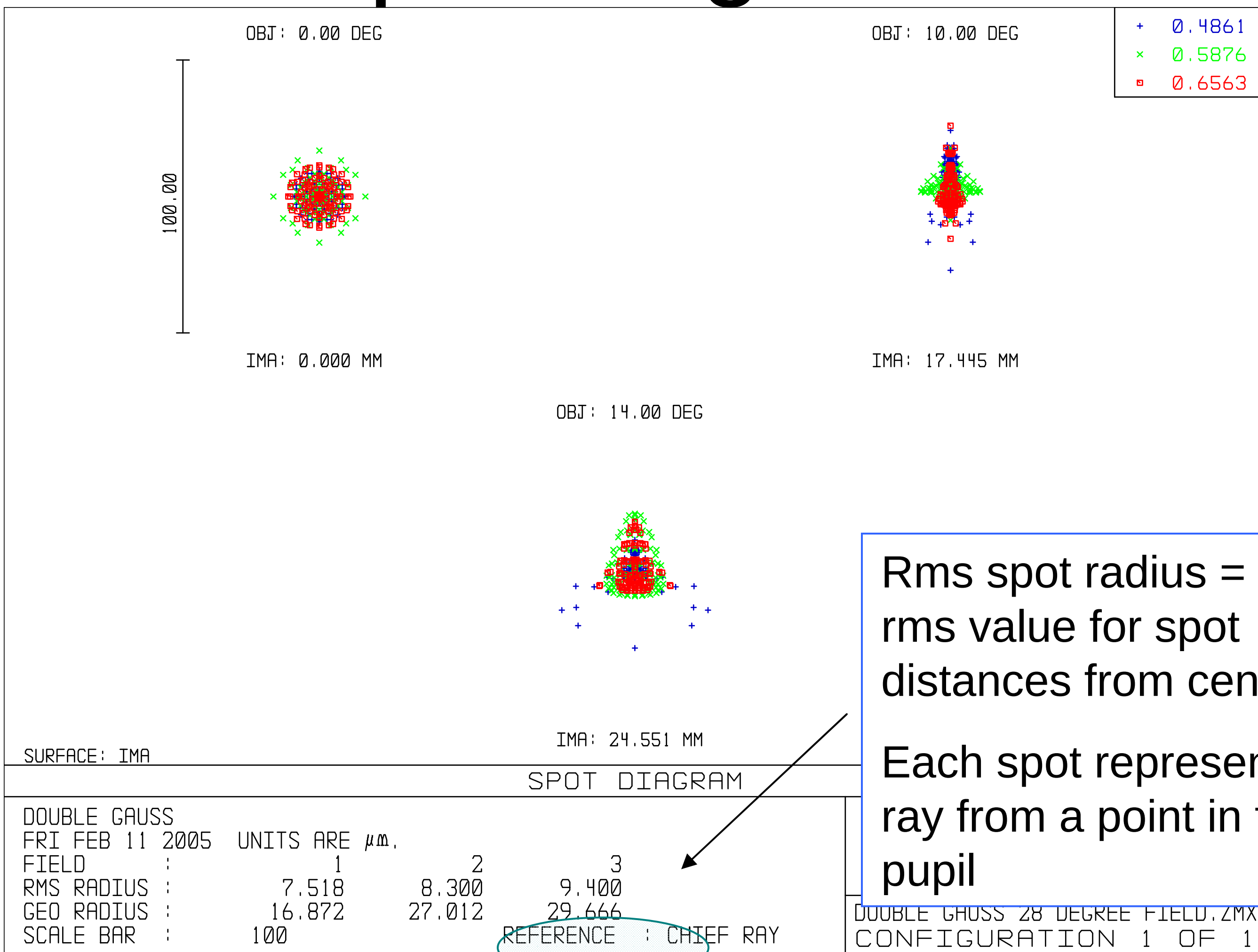
Image quality – point sharpness

- Look at the image of points
- In the geometric limit:
  - RMS diameter or radius (half-diameter)

    Easily calculated using raytrace programs by tracing a bunch of rays: only makes sense for geometric limit
  - FWHM
  - 80% encircled energy

    the circle that contains 80% of the spots

Spot diagrams



Rms spot radius = rms value for spot distances from centroid

Each spot represents a ray from a point in the pupil

In software, send a bunch of rays through the system and see where they intersect the image plane

Calculation of rms

- Mean value of x :

$$\bar{x} = \langle x \rangle = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{\int x\,dA}{\int dA}$$

- Mean value of x^2

$$\langle x^2 \rangle = \frac{\sum_{i=1}^{N} {x_i}^2}{N} = \frac{\int x^2\,dA}{\int dA}$$

- Root mean square deviation

$$rms = \sqrt{\langle x^2 - \bar{x}^2 \rangle} = \sqrt{\frac{\int x^2\,dA}{\int dA} - \left(\frac{\int x\,dA}{\int dA}\right)^2}$$

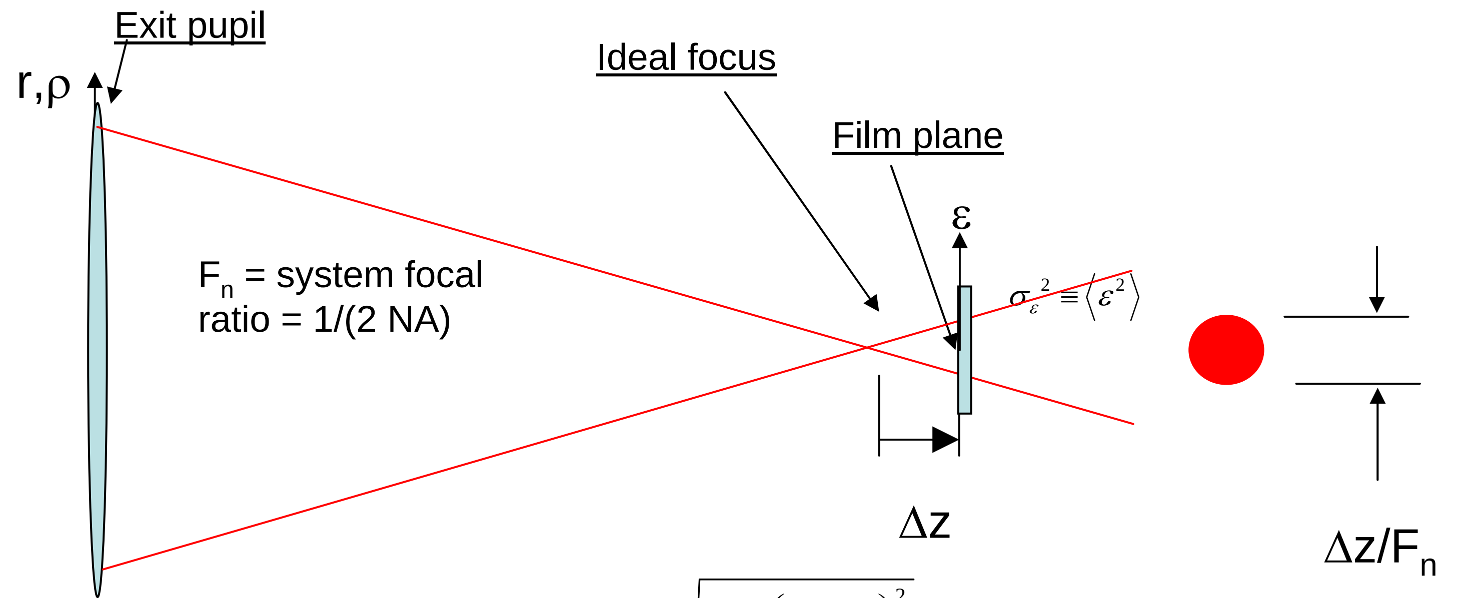
- If mean <x> = 0,

$$rms = \sqrt{\frac{\int x^2\,dA}{\int dA}}$$

PSF for defocus

ρ normalized

$\rho = r/r_{max}$



From geometry, $\varepsilon(\rho) = -\rho\ \Delta z/2F_n$

$$\varepsilon_{RMS} = \sqrt{\langle \varepsilon^2 \rangle} = \sqrt{\frac{\int dA \left(-\frac{\rho \Delta z}{2F_n} \right)^2}{\int dA}}$$

Where ε is position in image plane relative to center : $\langle\varepsilon\rangle = 0$

Calculate rms radius

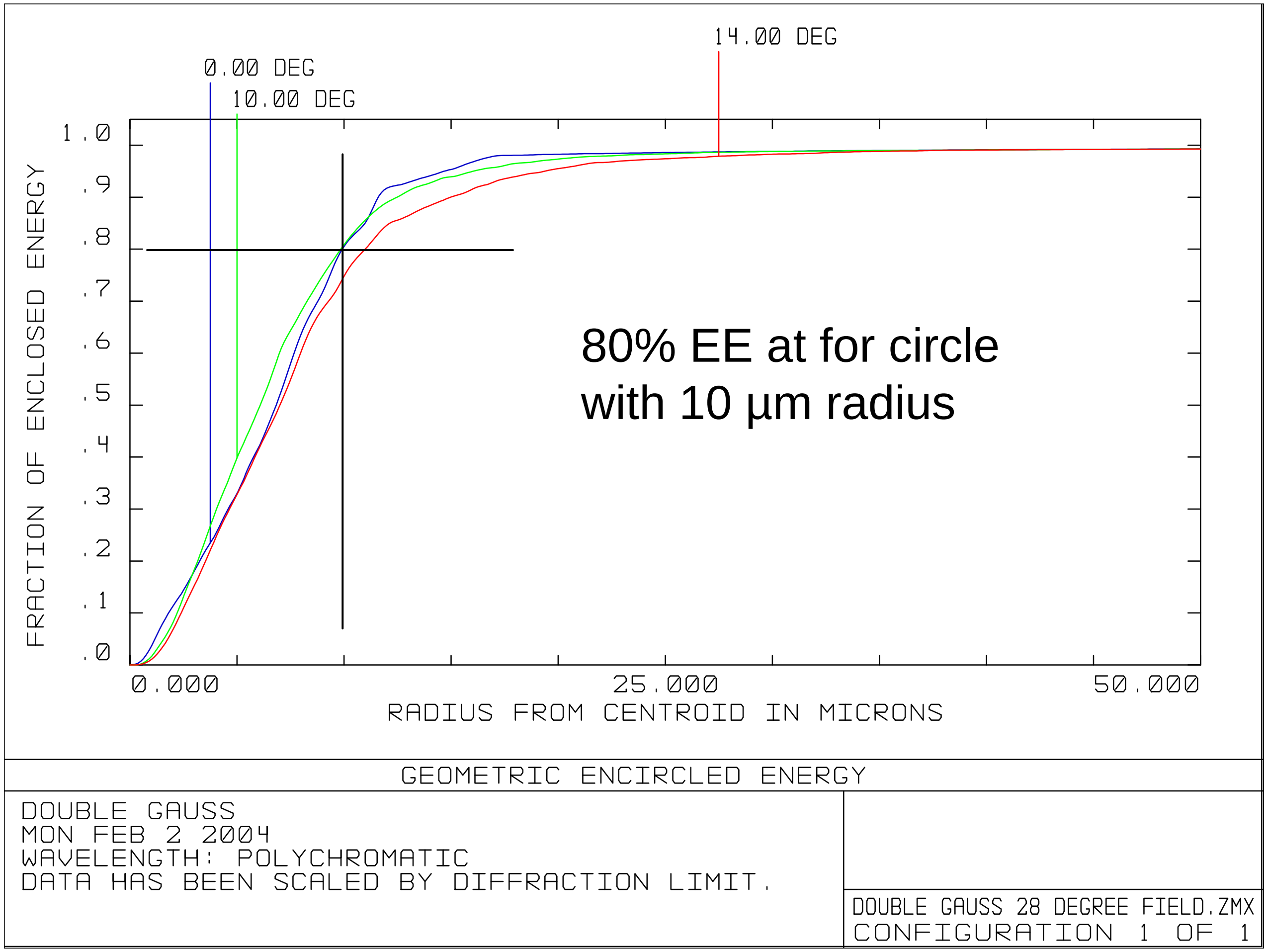
$$= \frac{\Delta z}{2F} \sqrt{\frac{\int 2\pi\rho\, d\rho \cdot \rho^2}{\pi}}$$

$$= \frac{\Delta z}{\sqrt{2}F_n} \sqrt{\left(\frac{1}{4}\rho^4 \Big|_0^1 \right)} = \frac{1}{2\sqrt{2}} \frac{\Delta z}{F_n}$$

$$= 0.353 \frac{\Delta z}{F_n}$$

$$\frac{rms\ radius}{diameter} \approx 0.35$$

Geometric encircled energy EE



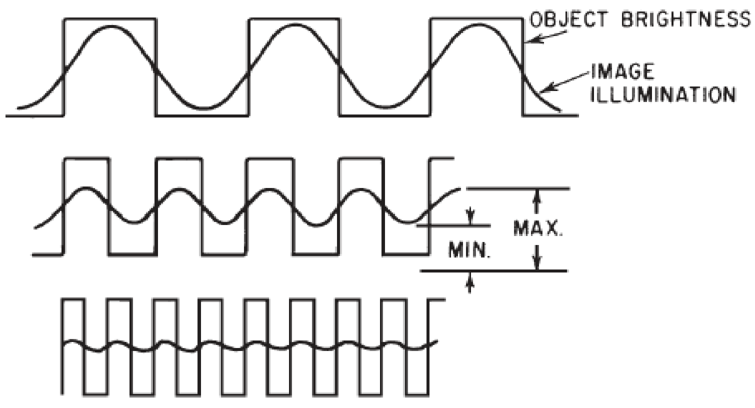
80% EE is often used as a threshold

Modulation Transfer Function

- Rather than using the blur size for image points – use the contrast reduction for high-frequency (small scale) features.
- MTF is plot of contrast (or modulation) vs. spatial frequency
- Has nice linear properties – system MTF = product of MTF for subsystems.
  - Have to be a little careful with sampled systems (detectors)

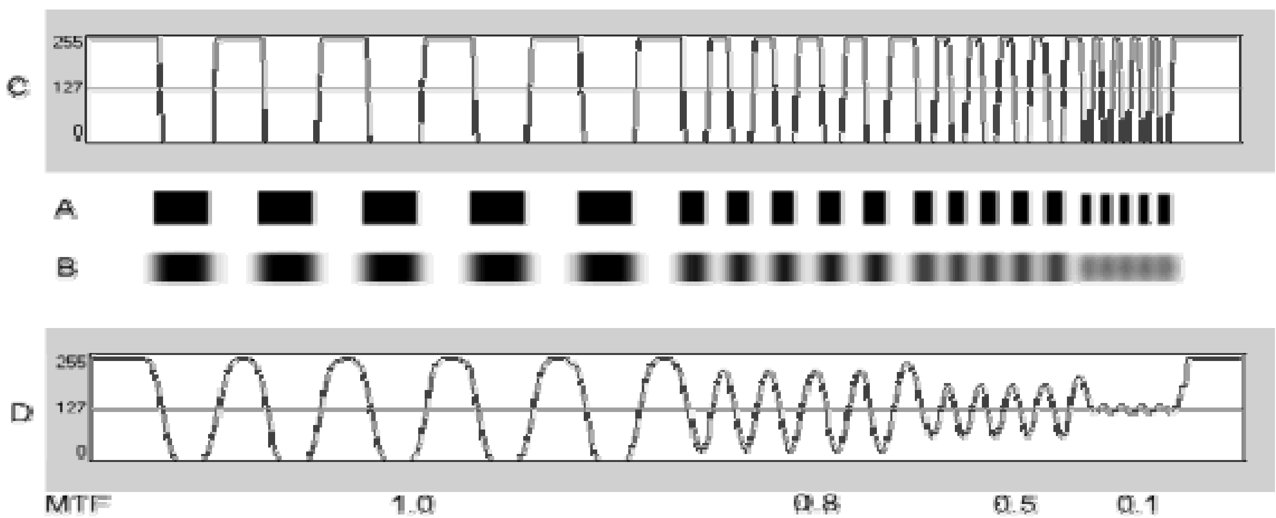
Definition of Modulation

(c)



(d)

$$\text{Modulation} = \frac{\text{max.} - \text{min.}}{\text{max.} + \text{min.}}$$



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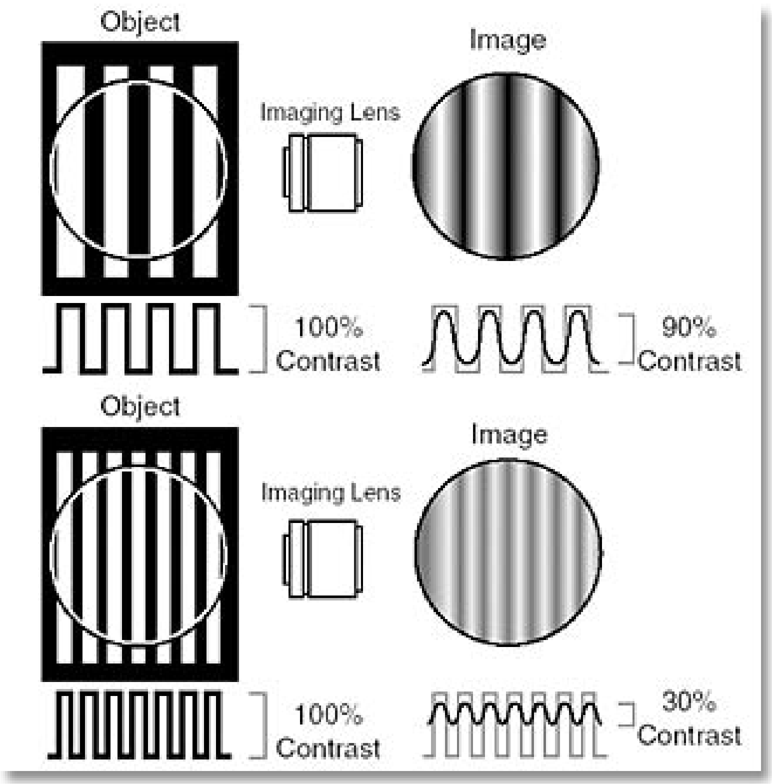
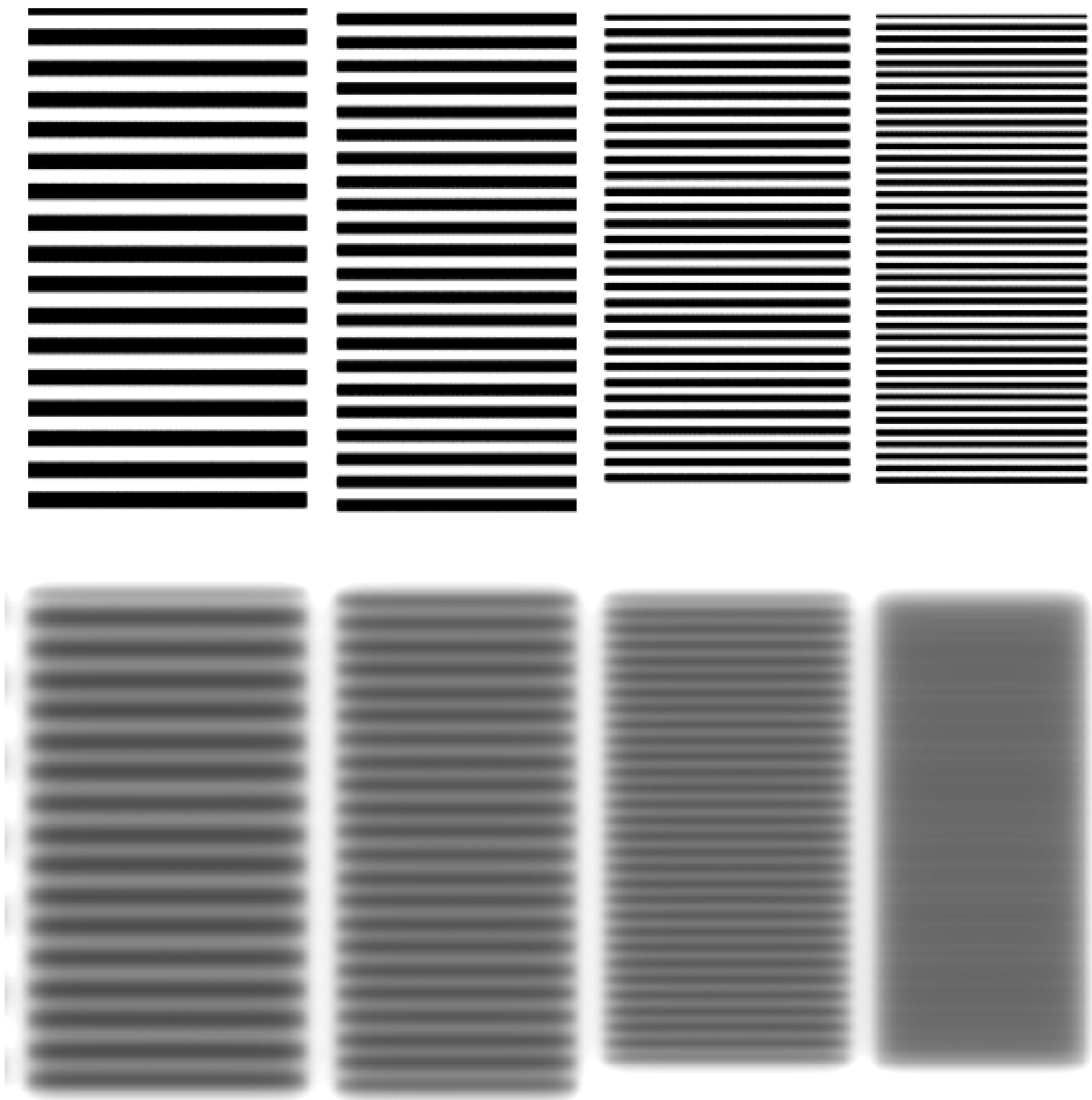
Contrast



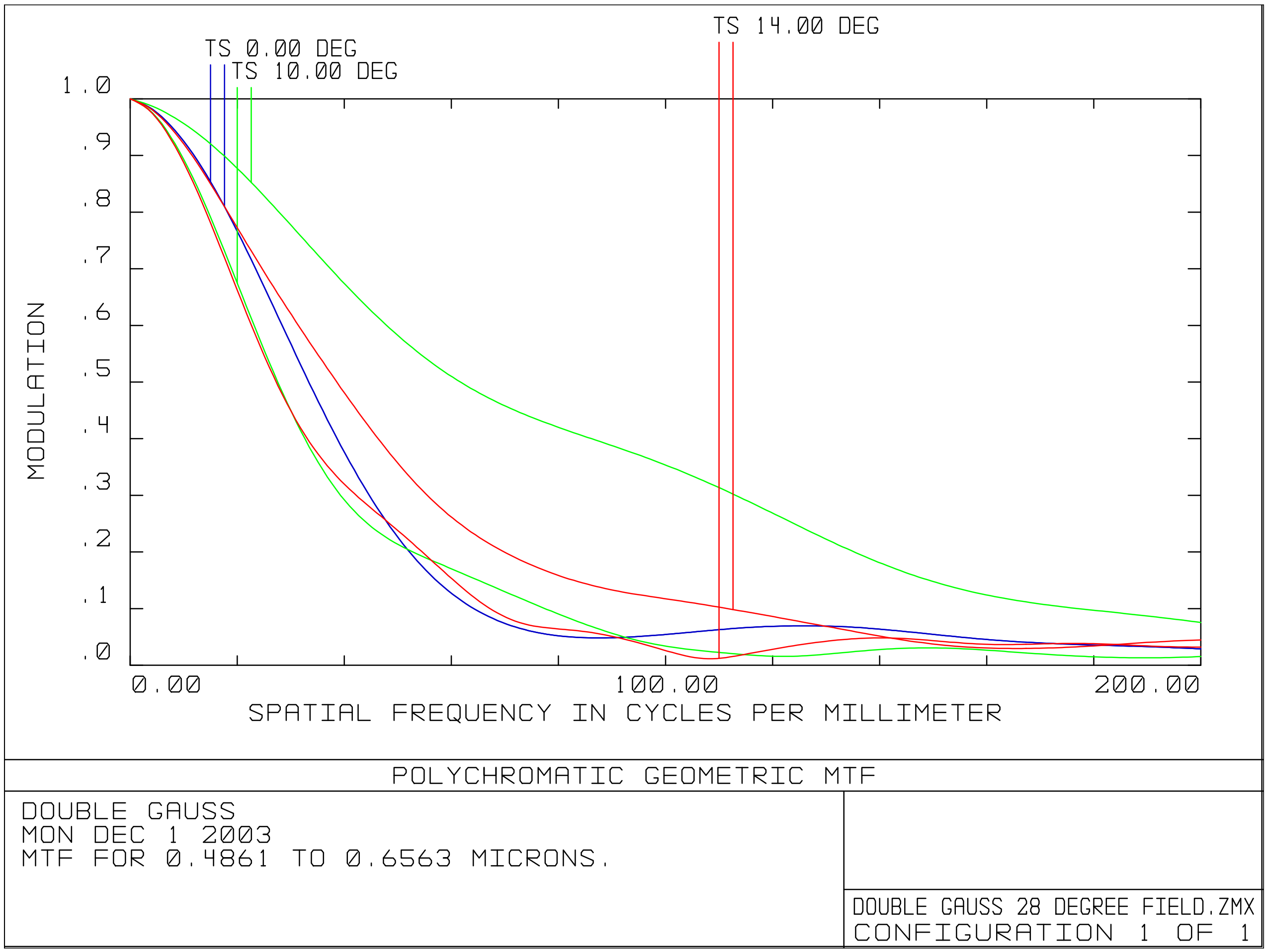
$$\text{Modulation} = \frac{\text{max.} - \text{min.}}{\text{max.} + \text{min.}}$$

$$Contrast = \frac{Amplitude}{Mean}$$

$$= \frac{\left(\dfrac{\max - \min}{2}\right)}{\left(\dfrac{\max + \min}{2}\right)}$$

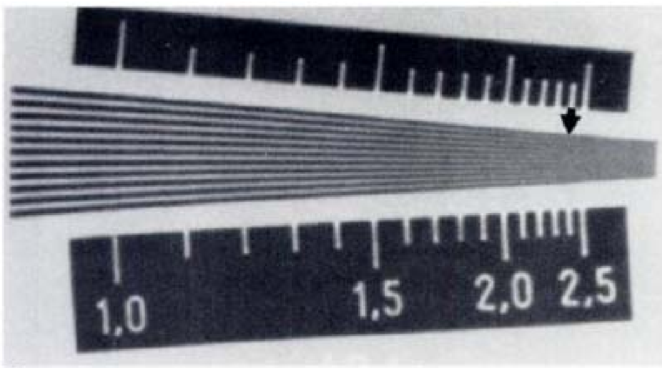


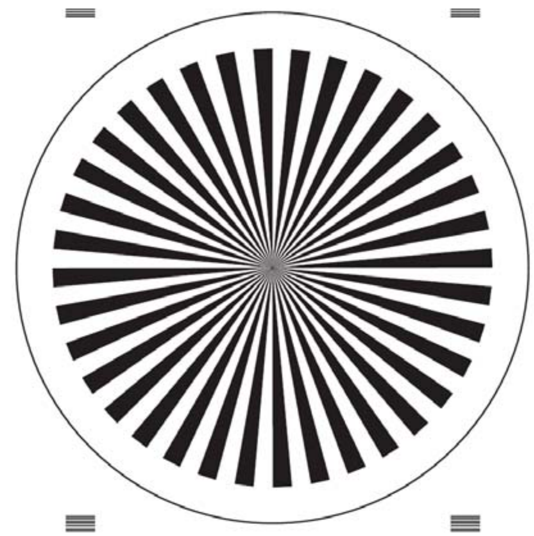
Modulation Transfer Function



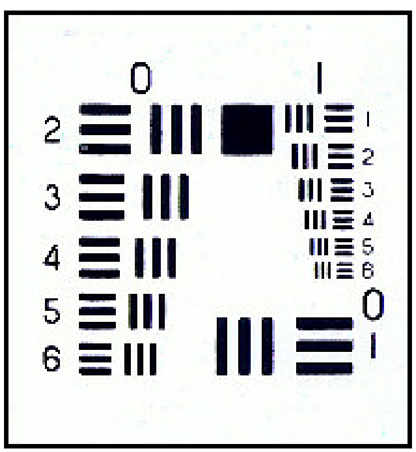
MTF targets

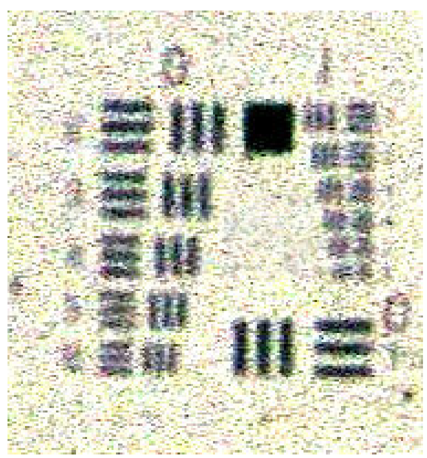
Siemens star





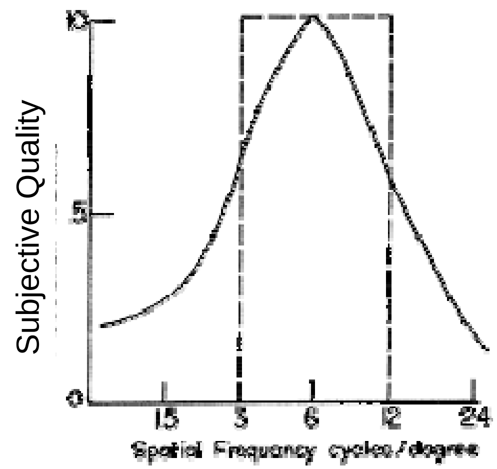
1951 USAF Target





Visual response

SQF: Integrate MTF over sensitive SFs



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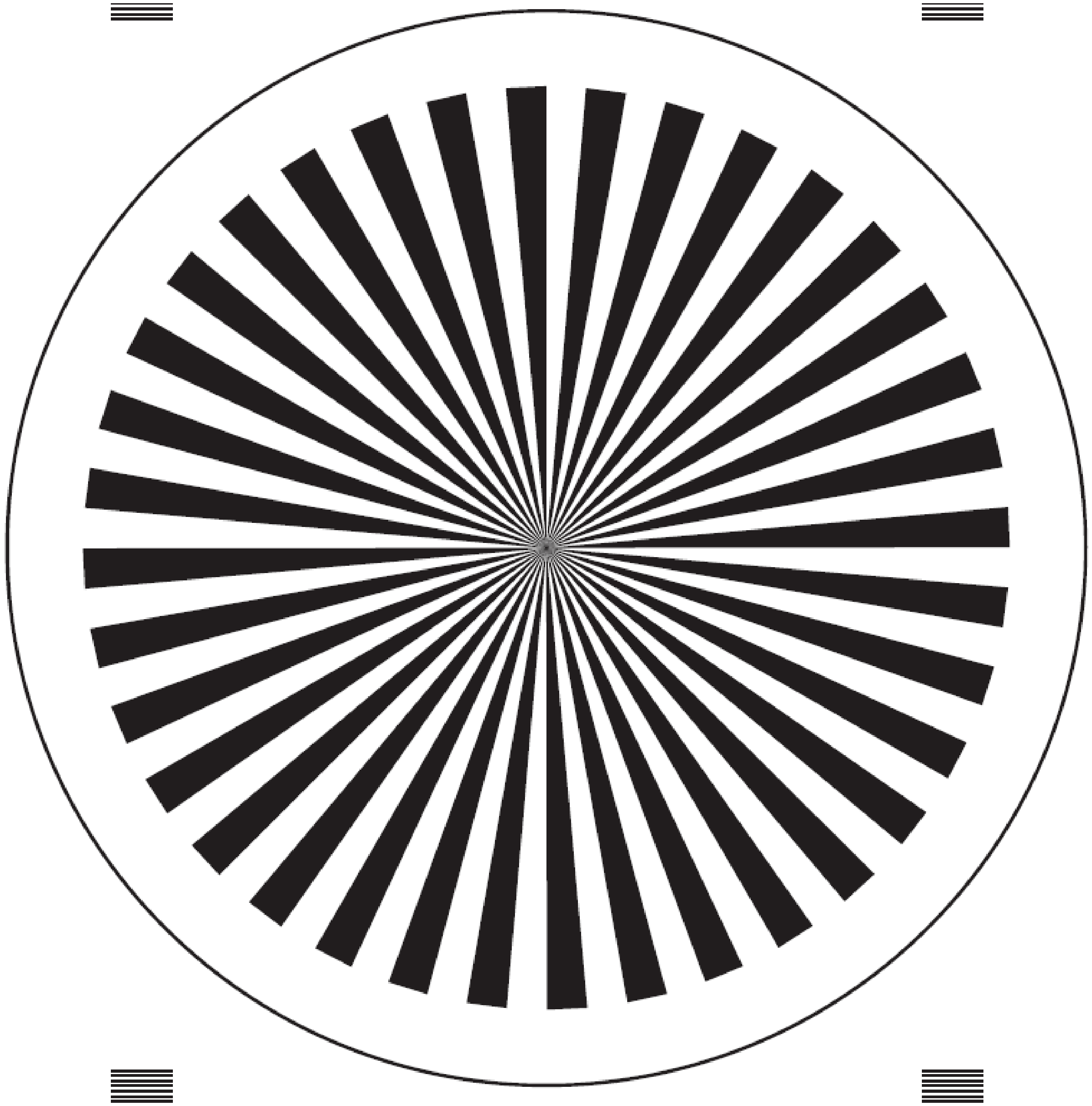
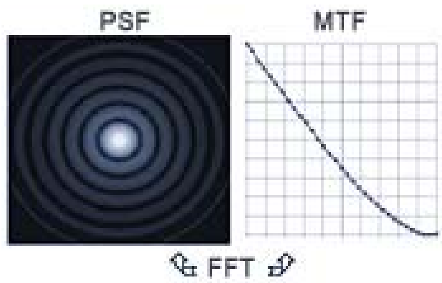
Phase inversion

defocus this image

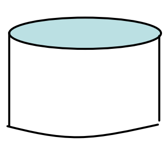


MTF, PSF

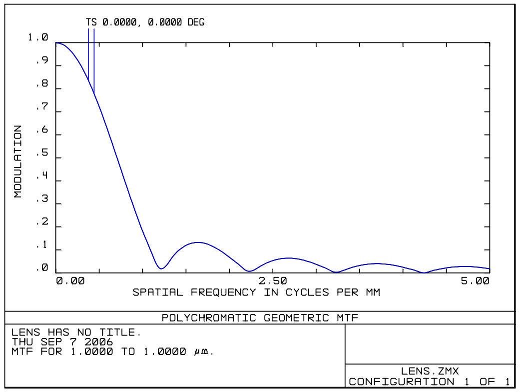
- MTF (really OTF) and PSF are Fourier transform pairs



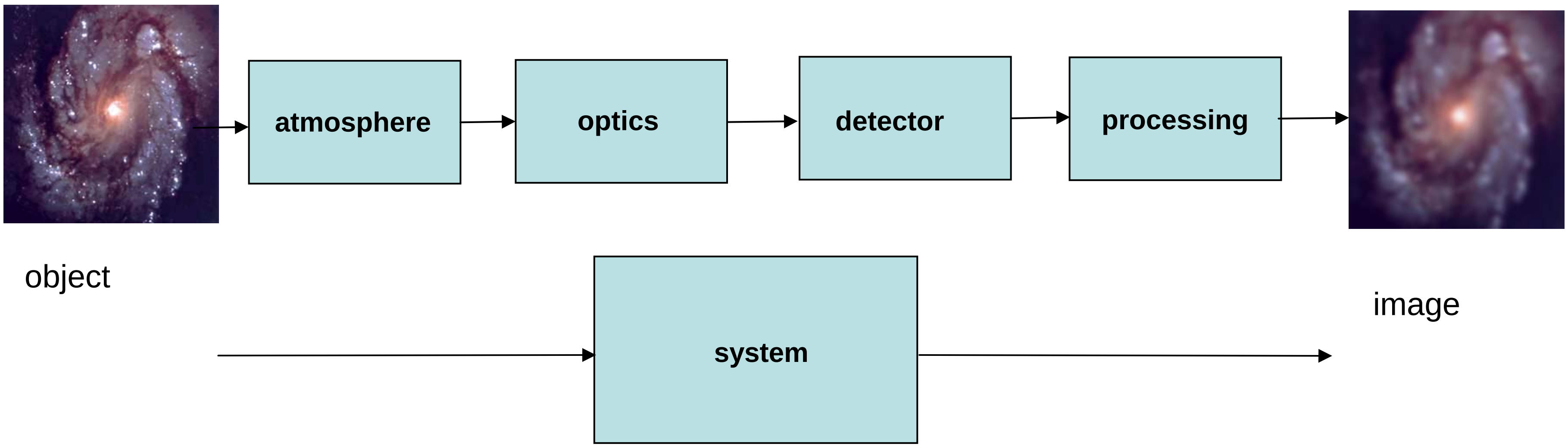
- What is MTF for large amount of defocus?



PSF



Combining multiple effects



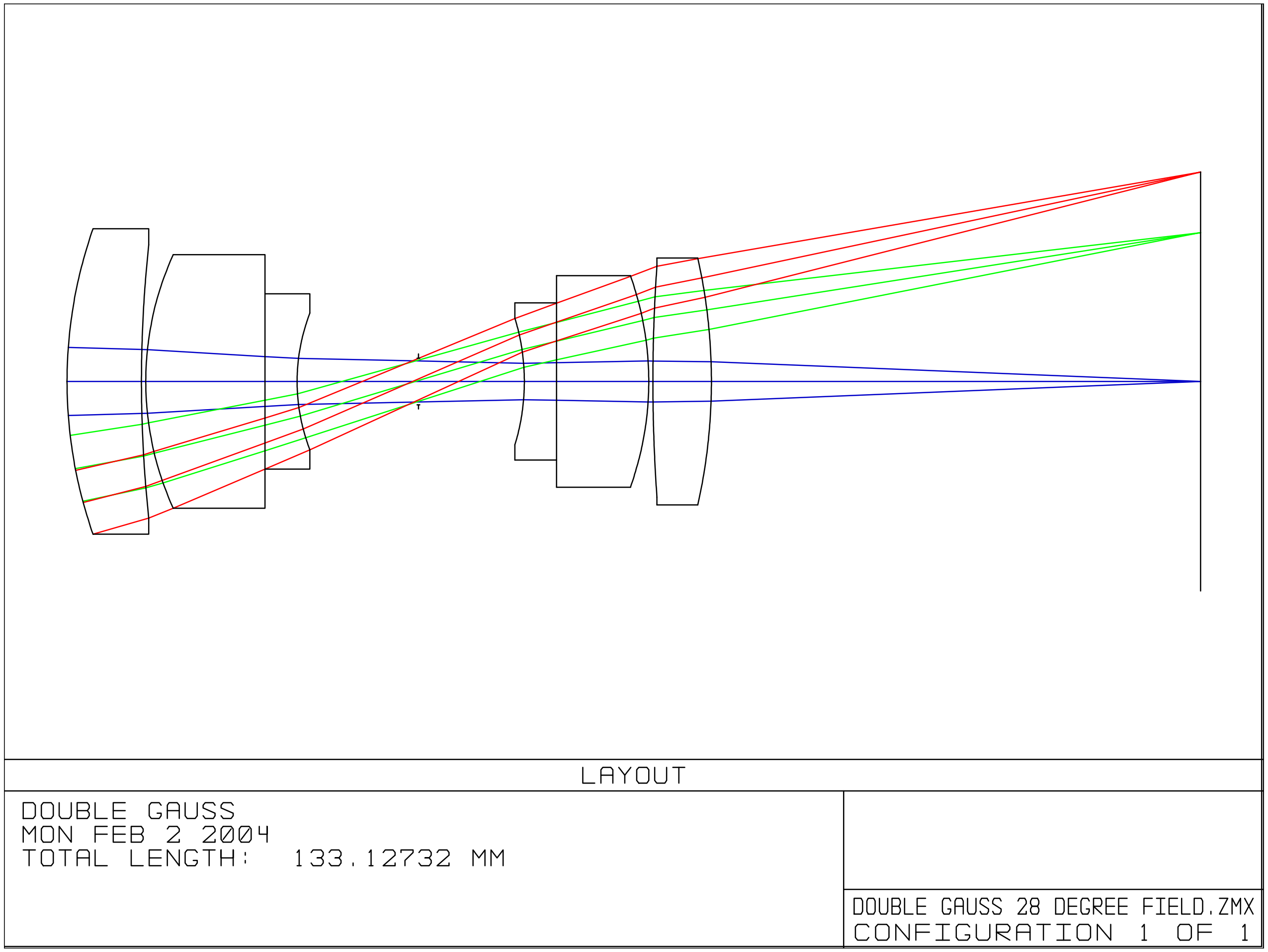
$$PSF_{system} = PSF_{atmosphere} ** PSF_{optics} ** PSF_{detector} ** PSF_{processing}$$

(** denotes a convolution)

$$MTF_{system} = MTF_{atmosphere} \times MTF_{optics} \times MTF_{detector} \times MTF_{processing}$$

Diffraction Limit

Stop the above system down



OPD < 1 wave

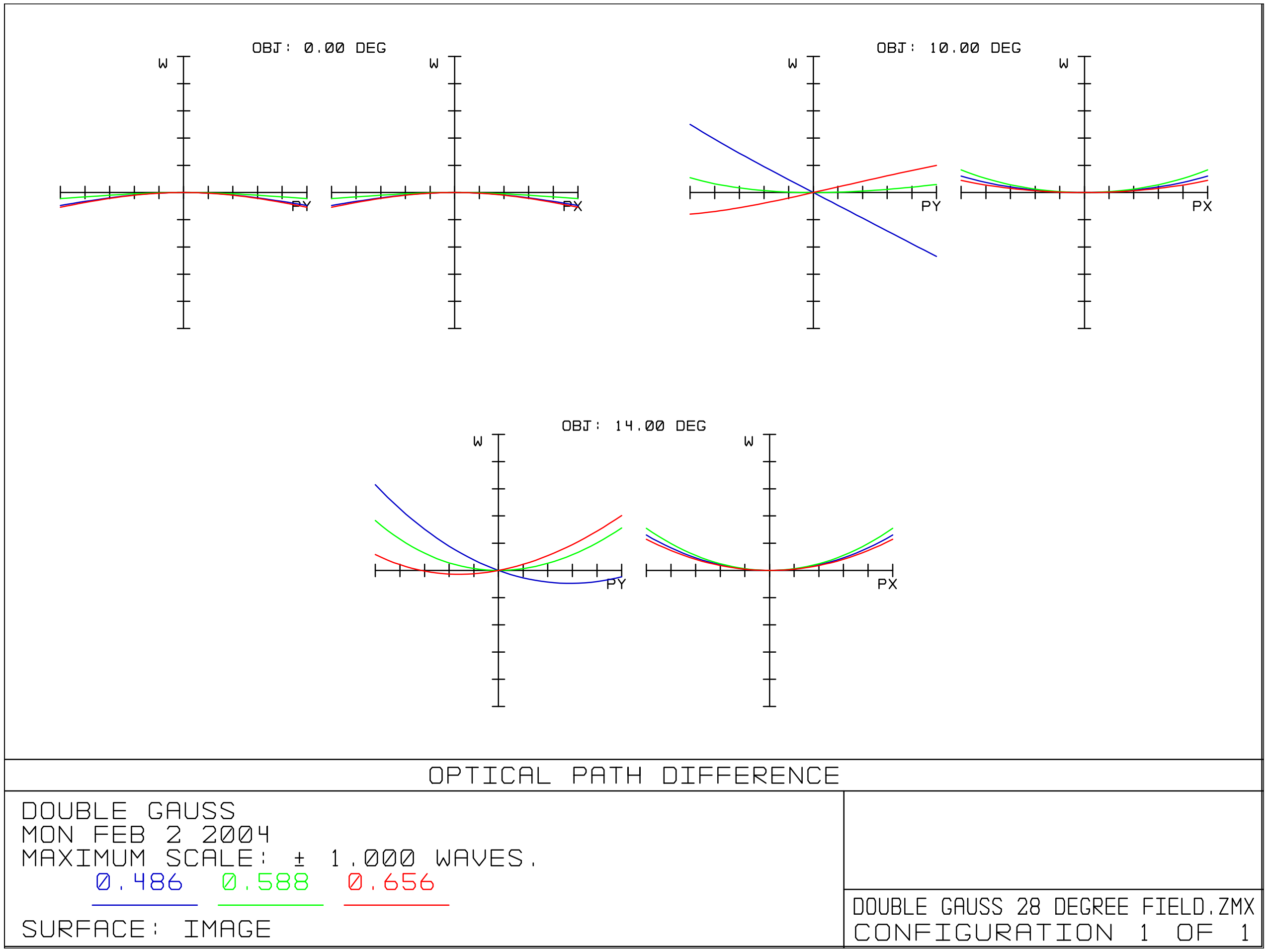
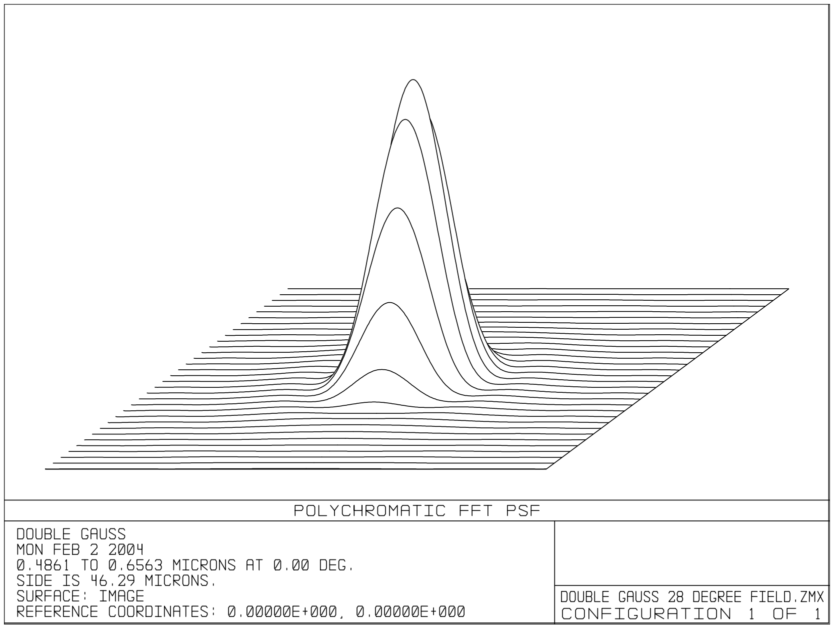
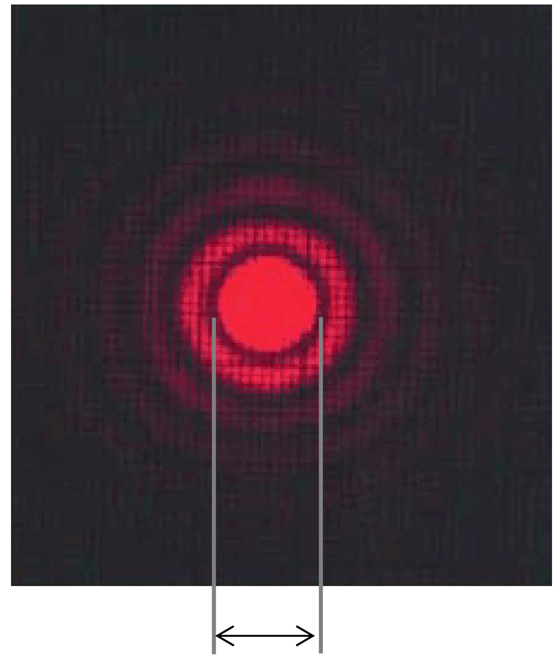

OBJ: 0.00 DEG
W
PY
PX
OBJ: 10.00 DEG
OBJ: 14.00 DEG
OPTICAL PATH DIFFERENCE
DOUBLE GAUSS
MON FEB 2 2004
MAXIMUM SCALE: ± 1.000 WAVES.
0.486 0.588 0.656
SURFACE: IMAGE
DOUBLE GAUSS 28 DEGREE FIELD.ZMX
CONFIGURATION 1 OF 1

Image size comes from diffraction





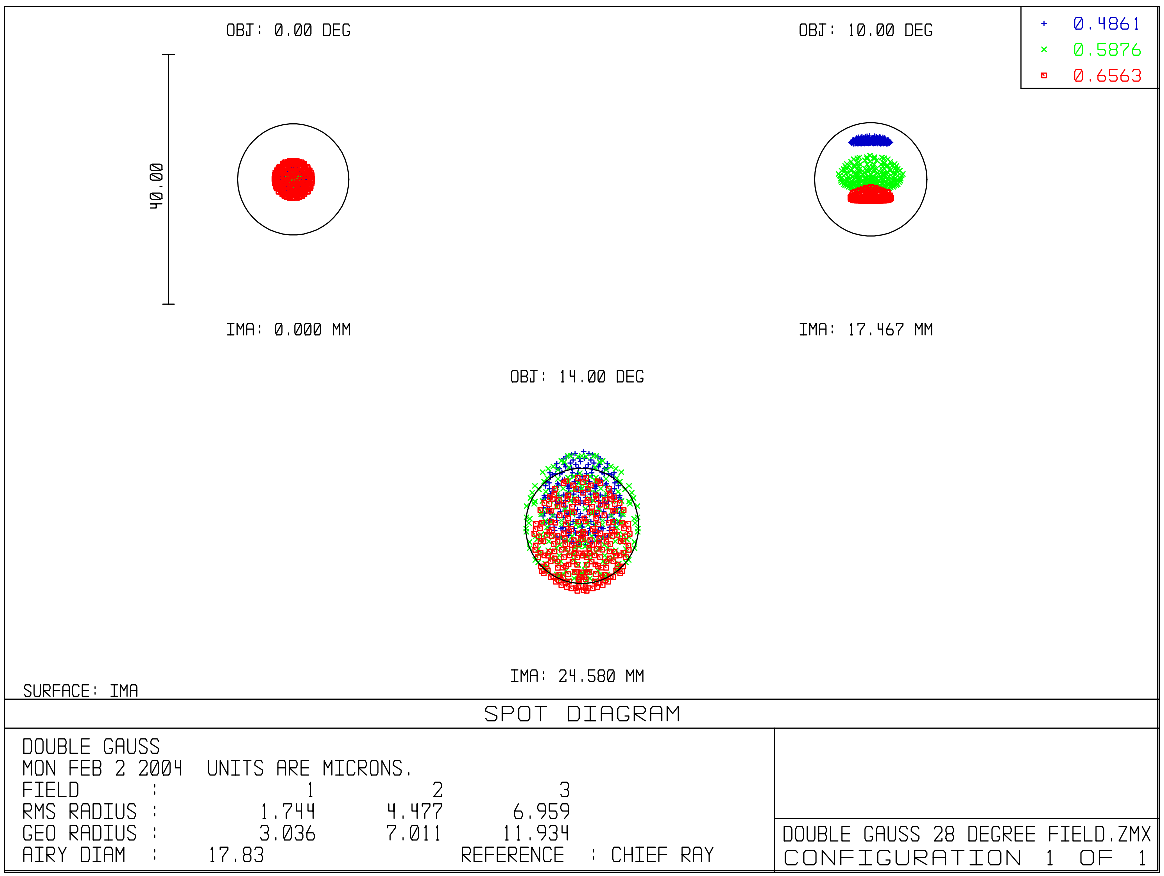
$2.44\lambda F_n$

FWHM = $1.02\ \lambda\ F_n$

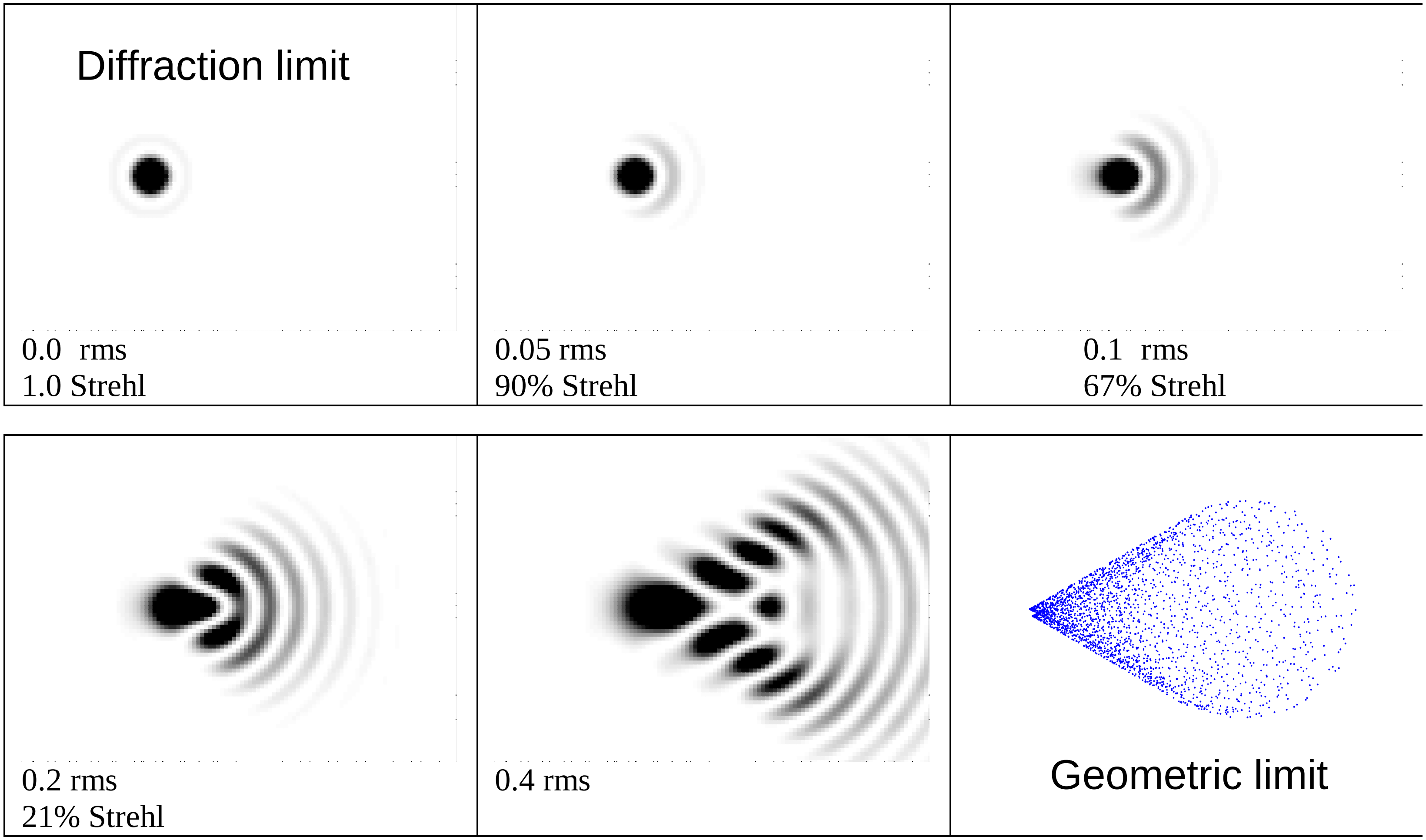
In angle space, $\theta_{FWHM} = 1.02\dfrac{\lambda}{D}$

Spot diagrams

- Now these are meaningless
  (compare with Airy diameter)



Transition from Diffraction limit to Geometric Limit



Strehl Ratio

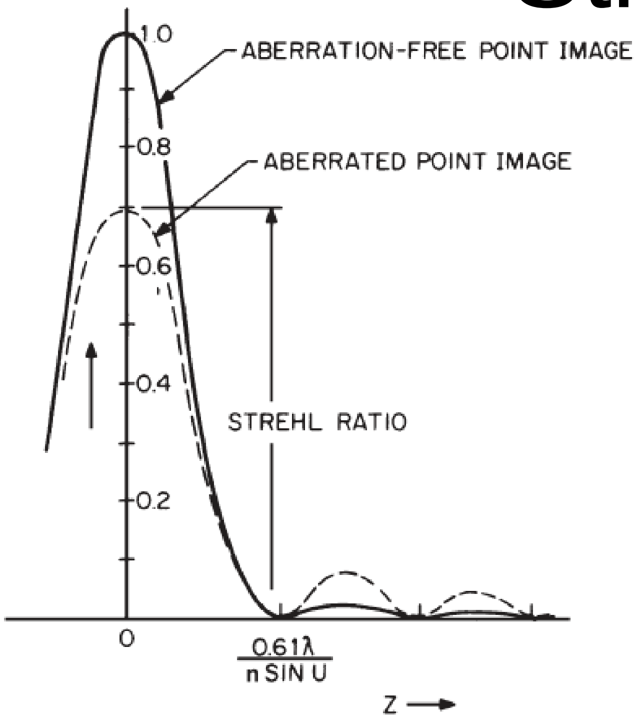


Figure 11.5

$$SR \cong e^{-\sigma^2} \cong 1-\sigma^2$$

Where σ is RMS wavefront error in ***radians***

$$SR \cong e^{-(2\pi W_{rms}/\lambda)^2} \cong 1-(2\pi W_{rms}/\lambda)^2$$

Where W_{rms} is RMS wavefront error in µm (assuming λ in µm)

Relation of Image Quality Measures to OPD

P-V OPD	RMS OPD	Strehl ratio	% energy in Airy disk	% energy in Rings
0.0	0.0	1.00	84	16
0.25RL = λ/16	0.018λ	0.99	83	17
0.5RL = λ/8	0.036λ	0.95	80	20
1.0RL = λ/4	0.07λ	0.80	68	32
2.0RL = λ/2	0.14λ	0.4*	40	60
3.0RL = 0.75λ	0.21λ	0.1*	20	80
4.0RL=λ	0.29λ	0.0*	10	90

*The smaller values of the Strehl ratio do not correlate well with image quality.

Rule of thumb for diffraction limit:

λ/4 P-V wavefront (0.07 λ rms)

80% SR

Diffraction calculation

Treat small wavefront errors using Fourier

For wavefront ripples with spatial period Λ, rms σ radians

Light is diffracted from central core

How much? σ^2

Where does it go? $\Delta\theta = \pm\lambda/\Lambda$

Wavefront
0.07λ rms, 10 cycles/diam
Λ = D/10

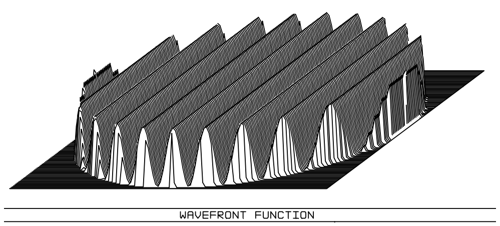
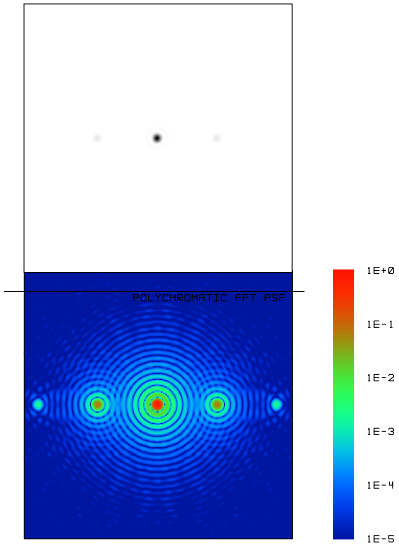


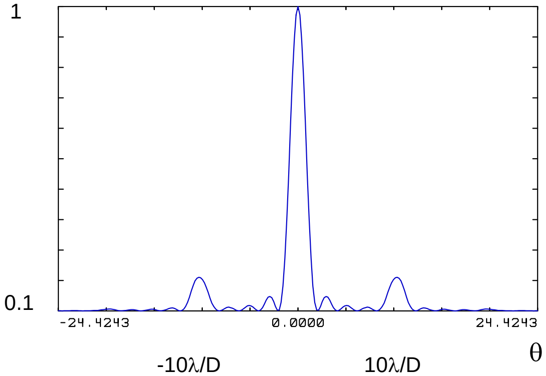
Image
Satellite peaks at $\theta = \pm 10\lambda/D$



(LOG SCALE)

Slice through image
80% Strehl

Energy/satellite peak = 10% of central lobe



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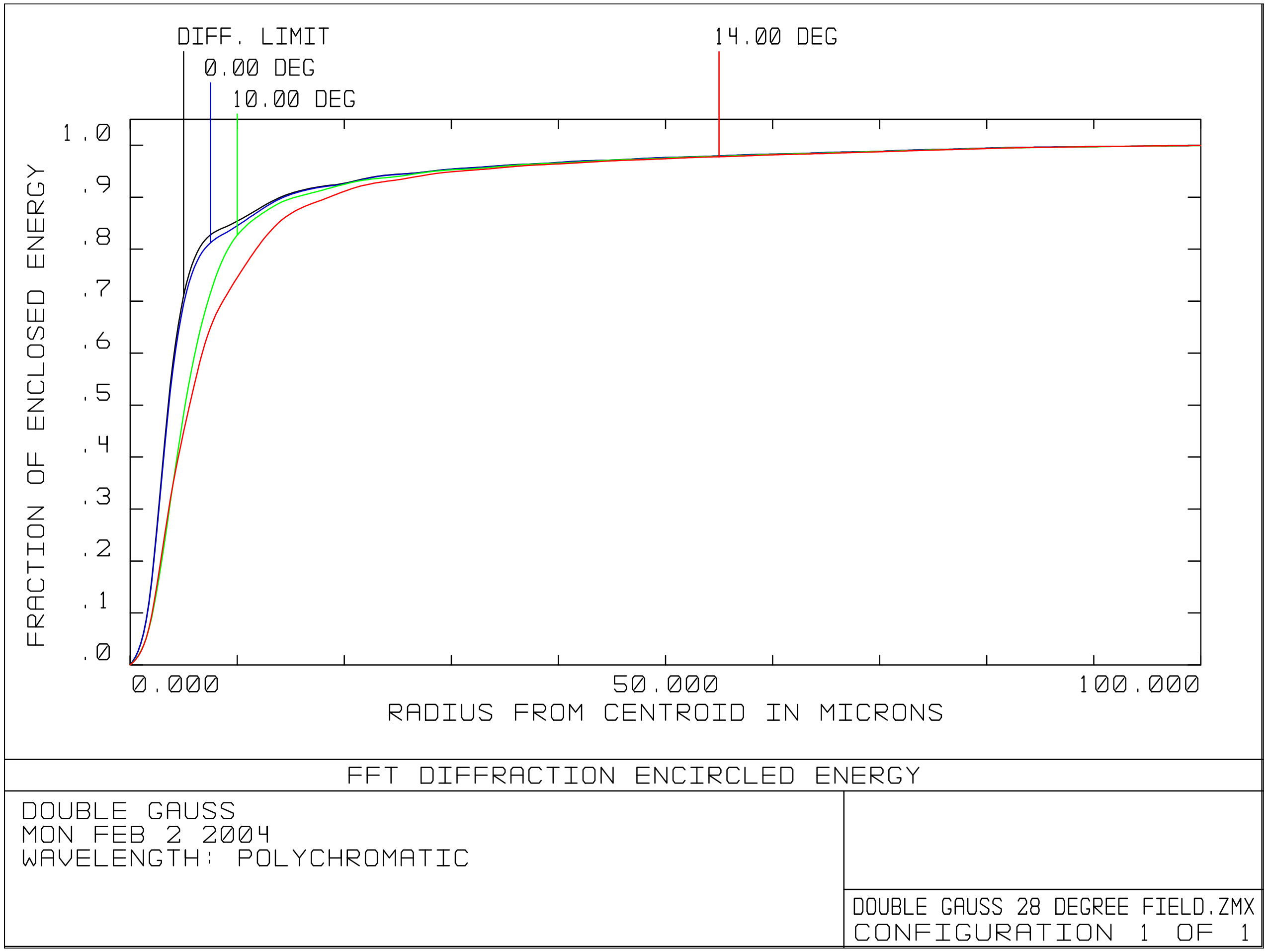
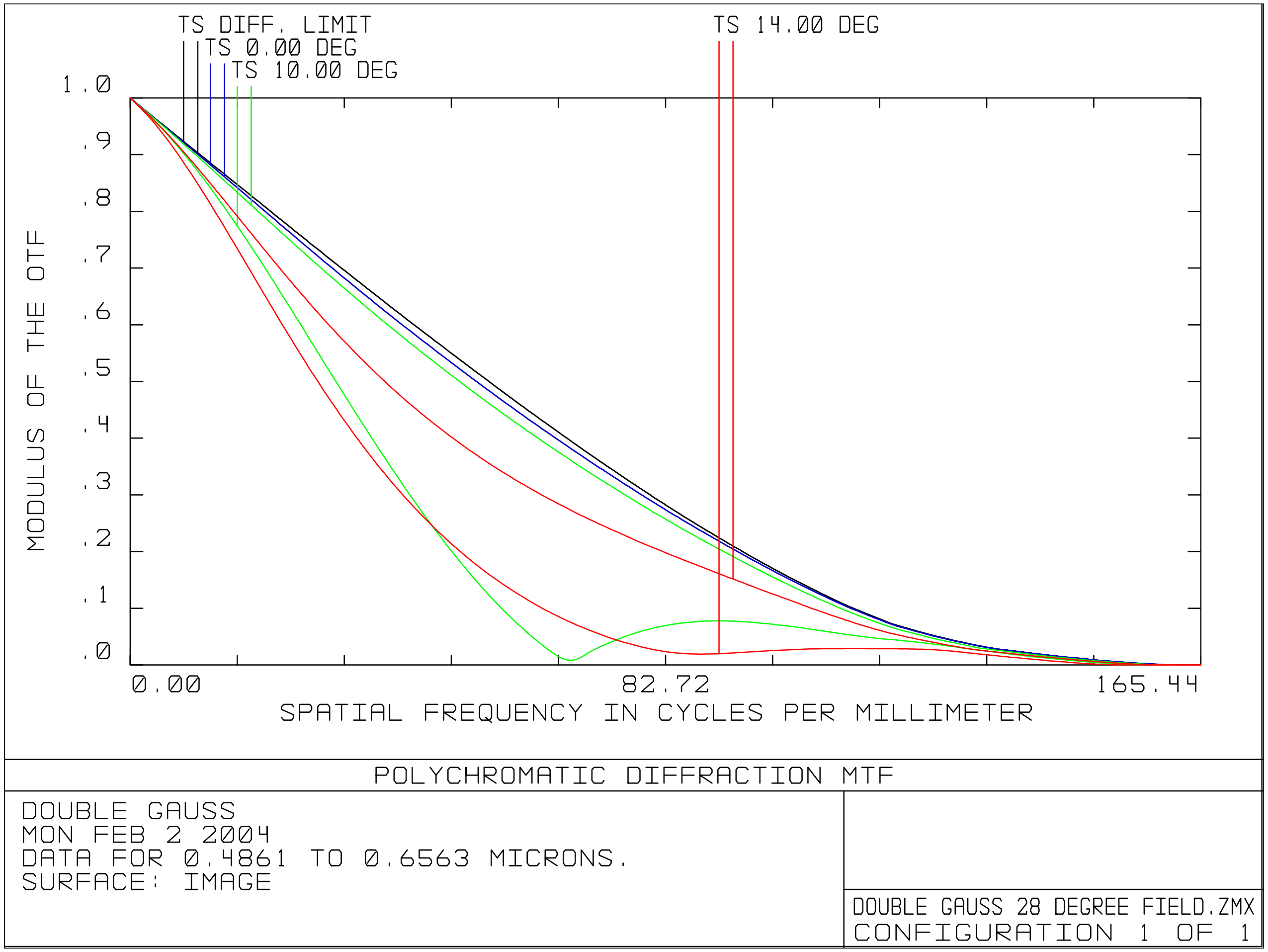
Encircled Energy



MTF


TS DIFF. LIMIT
TS 0.00 DEG
TS 10.00 DEG
TS 14.00 DEG
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
MODULUS OF THE OTF
0.00
82.72
165.44
SPATIAL FREQUENCY IN CYCLES PER MILLIMETER
POLYCHROMATIC DIFFRACTION MTF
DOUBLE GAUSS
MON FEB 2 2004
DATA FOR 0.4861 TO 0.6563 MICRONS.
SURFACE: IMAGE
DOUBLE GAUSS 28 DEGREE FIELD.ZMX
CONFIGURATION 1 OF 1

Diffraction Limited MTF

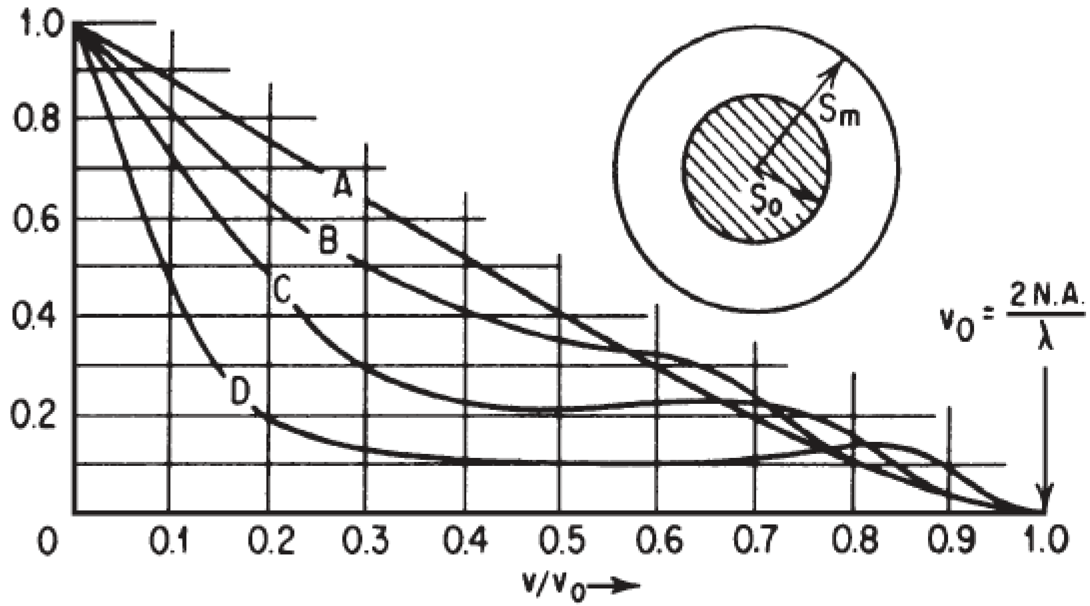


Figure 11.19 The effect of a central obscuration on the modulation transfer function of an aberration-free system.

(a) $s_0/s_m = 0.0$

(b) $s_0/s_m = 0.25$

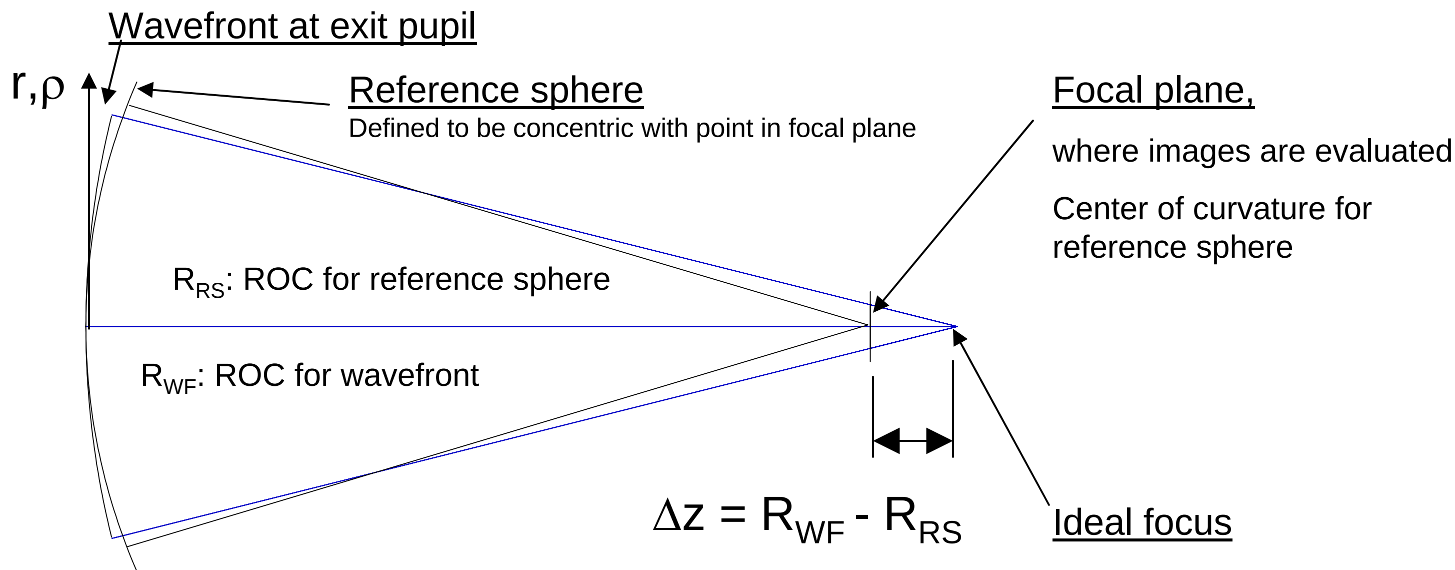
(c) $s_0/s_m = 0.5$

(d) $s_0/s_m = 0.75$

Can be calculated as the autocorrelation of the pupil function!

Wavefront error from defocus

- Shift in focus is exactly the same as the wavefront with the wrong radius



- Wavefront $$W(r) = \frac{r^2}{2R}$$
- Differentiate $$\Delta W = -\frac{r^2}{2R^2}\Delta R = -\left(\frac{D^2}{8R^2}\Delta R\right)\rho^2$$
- But the change in wavefront radius ΔR is equivalent with a change of focus -Δz

$$\Delta W(\rho) = \left(\frac{D^2}{8R^2}\Delta z\right)\rho^2 = \left(\frac{\Delta z}{8F_n^{\,2}}\right)\rho^2$$

Relate focus to RMSWE

A shift in focus is equivalent to wavefront error $W = a_2 \rho^2$

where $$a_2 = \frac{\Delta z}{8 F_n^2}$$

Define diffraction limit for $a_2 = \lambda/4$ ⟹ $$\Delta z = 2\lambda F_n^2$$

Convert to rms: $$W_{rms} = 0.289 a_2 = .0361 \frac{\Delta z}{F_n^2}$$

Convert to radians $$\sigma = \frac{2\pi W_{rms}}{\lambda} = 0.227 \frac{\Delta z}{\lambda F_n^2}$$

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RMSWE for focus

A shift in focus is equivalent to the wavefront error of power

$$W(\rho,\theta) = a_2 \rho^2$$

Calculate rms value for x as:

$$\sigma_x{}^2 \equiv \left\langle x^2 \right\rangle - \left\langle x \right\rangle^2$$

So get RMSWE as

$$W_{rms} = \sqrt{\frac{\iint \left[W(\rho,\theta)\right]^2 dA}{\iint dA} - \left(\frac{\iint W(\rho,\theta) dA}{\iint dA}\right)^2}$$

Substitute the power function for *W* and work out the integrals

$$W_{rms} = 0.289 a_2$$

So if $a_2 = 1\ \lambda$, we would say there is 1 wave P-V power in the wavefront.

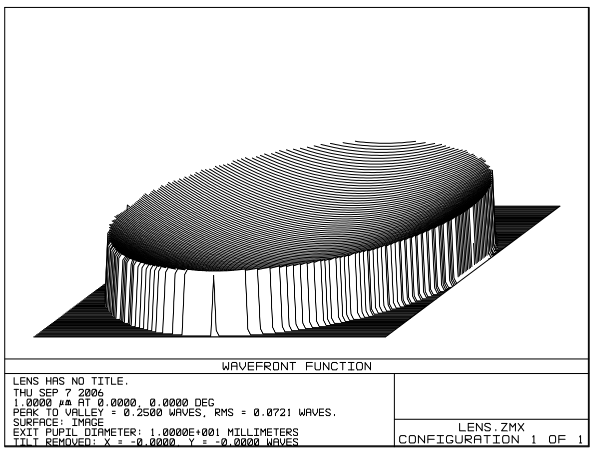
equivalently, there is 0.289 waves RMS

Diffraction limit for focus

- For focal shift of $2\lambda F_n^2$

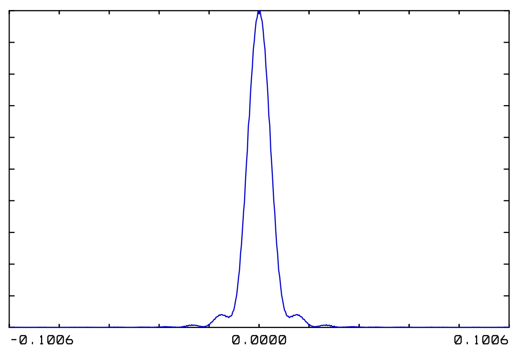
Wavefront error

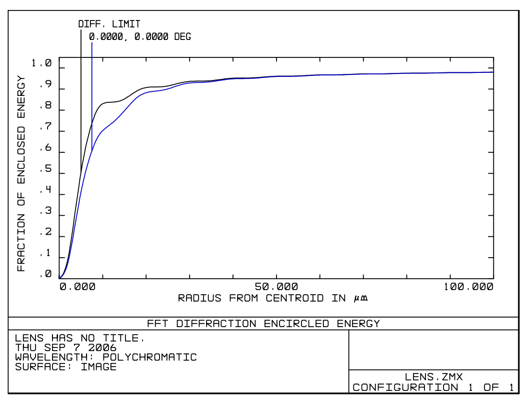
$\lambda/4$ P-V, 0.07 λ rms



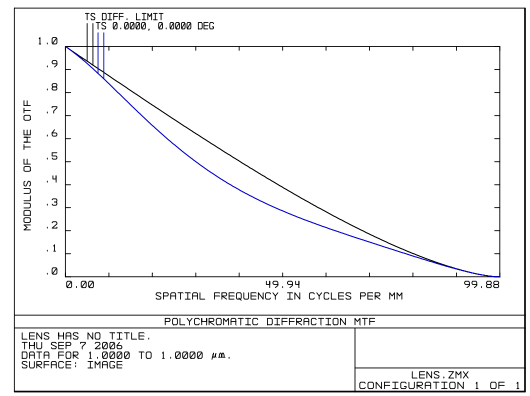
PSF

80% SR

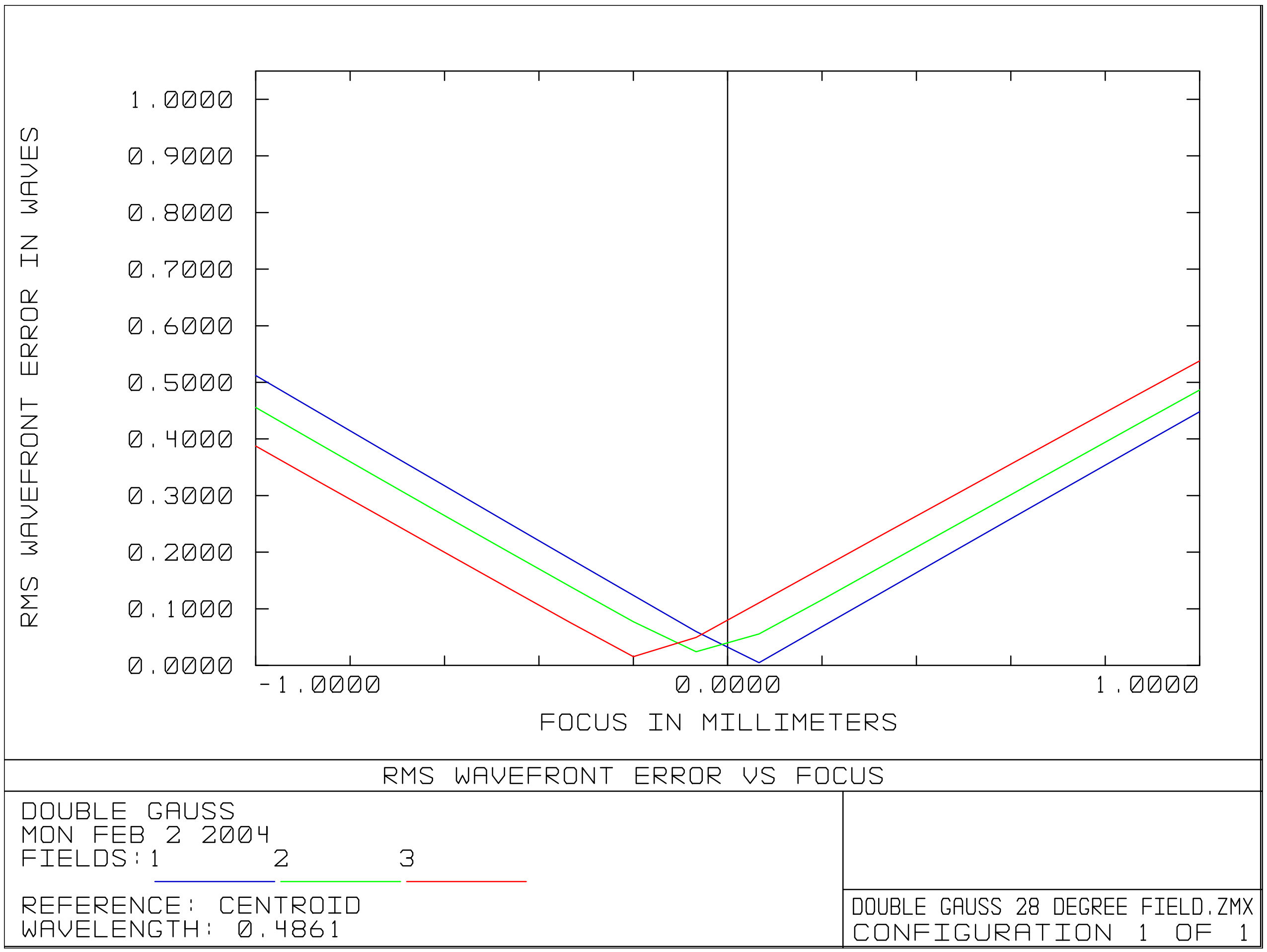




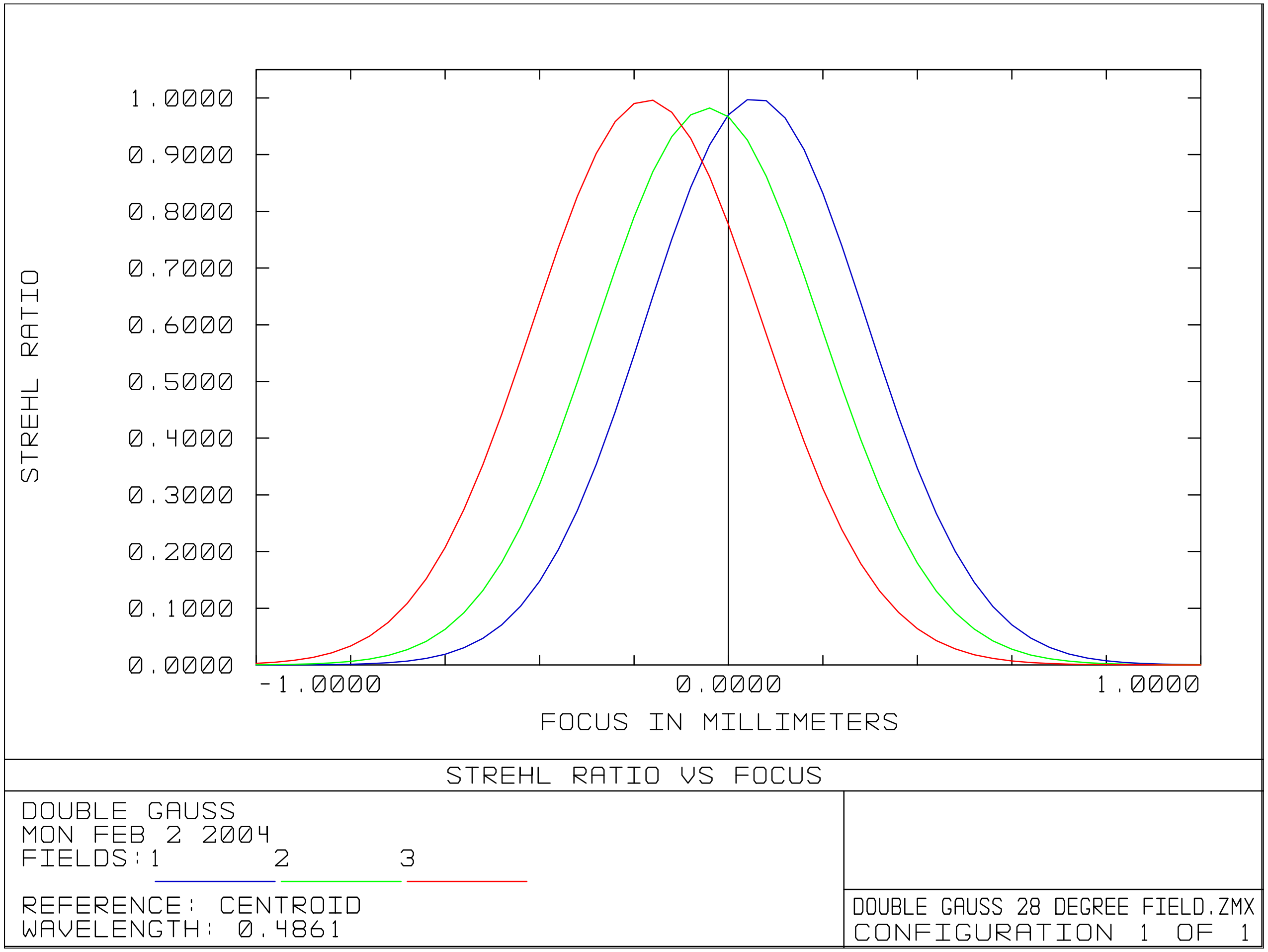
MTF



Wavefront error effect of focus



Effect of focus



<u>Figures of Merit for optical systems</u>

	Description	When to use it	*How to combine terms*
MTF Modulation transfer function	Gives the image contrast as a function of spatial frequency f	For imaging systems looking at extended objects	$MTF_{total}(f) = MTF_1(f) \times MTF_2(f) \times \ldots$
RMS Wavefront Error	Gives magnitude of wavefront errors, relative to ideal	Diffraction limited systems looking for resolution of small objects (rms OPD < $\lambda/6$)	$rms_{total} = \sqrt{rms_1^2 + rms_2^2 + \ldots}$
Image size	Usually this is given as the rms image diameter	Systems looking for resolution of small objects for the case when not diffraction limited (rms OPD > $\lambda/6$)	$rms_{total} = \sqrt{rms_1^2 + rms_2^2 + \ldots}$
Strehl ratio	Defined as the central intensity of the PSF relative to a perfect system	Diffraction limited systems looking for resolution of small objects (rms OPD < $\lambda/6$)	$SR_{total} = SR_1 \times SR_2 \times \ldots$
Boresight	Relative angular alignment between optical systems	When different systems must be pointed to the same thing	$\alpha_{total} = \sqrt{\alpha_1^2 + \alpha_2^2 + \ldots}$
Image motion	Motion of the image. Jitter – in Hz Stability – in secs, days, …	Jitter – increases image size Stability – affects calibration	$\alpha_{total} = \sqrt{\alpha_1^2 + \alpha_2^2 + \ldots}$