

1.0 MEASUREMENT OF PARAXIAL PROPERTIES OF OPTICAL SYSTEMS

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If we wish to completely characterize the paraxial properties of a lens, it is necessary to measure the exact location of its cardinal points, that is, its nodal points, focal points, and principal points. For a lens in air the nodal points and principal points coincide. For a thin lens, the two principal points coincide at the center of the lens, so the only required measurement is the focal length, while for a thick lens two of the three quantities--focal length, two focal points, or two principal points--must be determined.

1.1 Thin Lenses

1.1.1 Measurements Based on Image Equation

The simplest measurements of the focal length of a thin lens are based on the image equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (1.1)$$

where p is the object distance from the lens (positive if the object is before the lens), q is the image distance from the lens (positive if the image is after the lens), and f is the focal length of the lens. If the lens to be tested has a positive power, a real image can be formed of a pinhole source, and the distances p and q can be measured directly. When the lens to be tested has a negative power, it should be combined with a positive auxiliary lens having sufficient power so that the combination forms a real image. The focal length can then be determined for the auxiliary lens alone and the combination of lenses. The resultant data can be used to determine the power, or focal length, of the negative lens, since (in the thin-lens approximation) the power of the combined lens system is simply the algebraic sum of the powers of the individual elements.

To obtain a rough measurement of the focal length of a positive lens, an image can be formed of a source located several focal lengths away from the lens, and the distance between the lens and the resultant image can be taken as the approximate focal length of the lens. The accuracy of this measurement depends, of course, on how far the object is from the lens. According to the Newtonian form of the image equation

$$zz' = f^2, \quad (1.2)$$

where z is the distance of the object from the first focal point, and z' is the distance to the image from the second focal point. If the object and image distances are measured in units of the focal length, then they are reciprocals of each other:

$$z' = \frac{1}{z}. \quad (1.3)$$

Thus, if the object is 10 "focal lengths" from the first focal point, the image will be located 1/10 of a "focal length" from the second focal point.

1.1.2 Autocollimation Technique

One of the simplest techniques for locating the focal point of a lens is the autocollimation technique illustrated in Fig. 1-1. Light from the source, often a laser, passes through a pinhole and then through the lens whose focal point is to be found. After passing through the test lens, the beam is reflected by a plane mirror that is tilted slightly so that the returning beam does not pass through the pinhole but forms a small spot to one side of it. The distance between the pinhole and the test lens is then adjusted until the size of this spot is a minimum. The pinhole then lies in the focal plane of the lens.

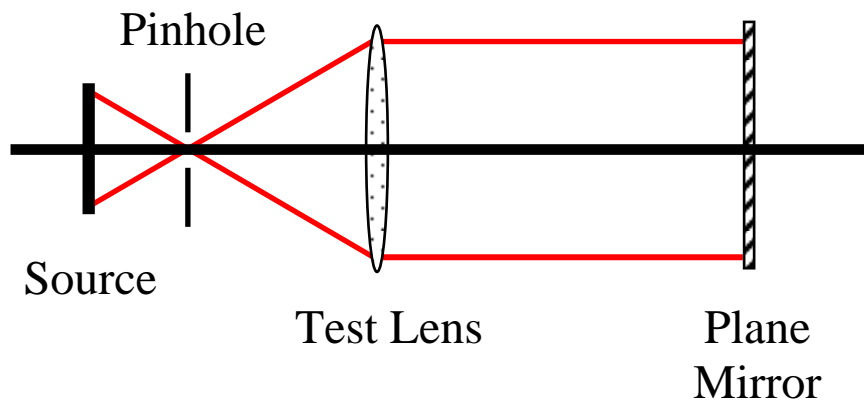


Fig. 1-1. Autocollimation for locating focal points.

The autocollimation techniques can be used to find the focal length of a negative lens if an auxiliary positive lens is added as shown in Figure 1-2.

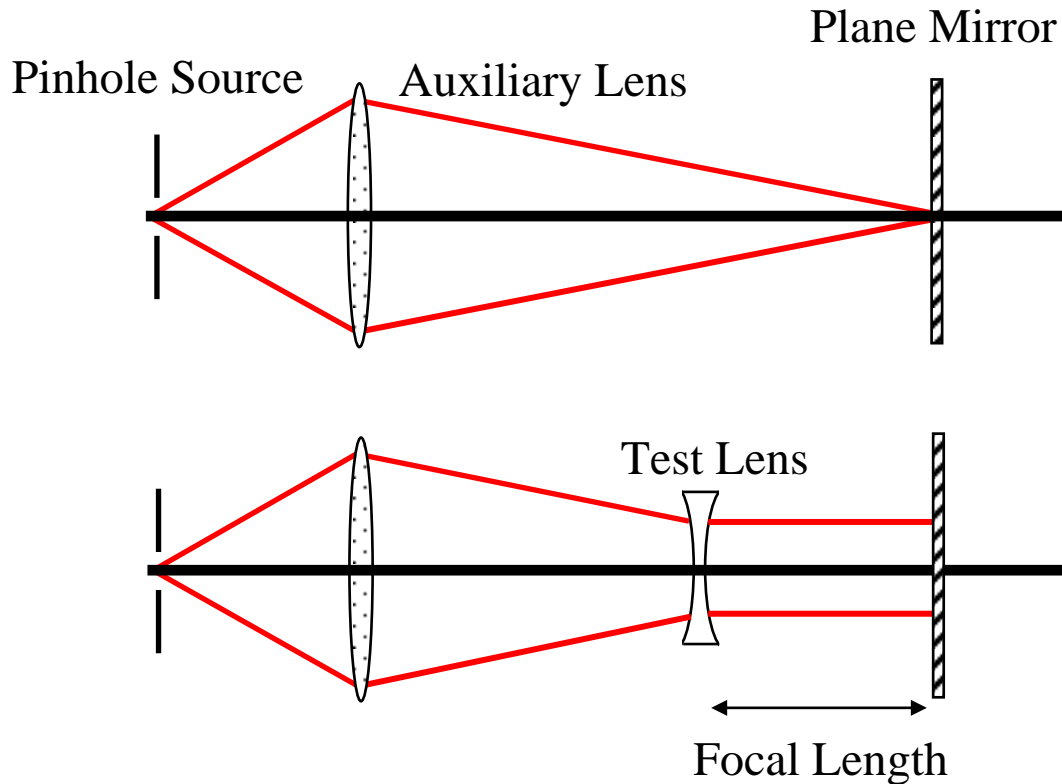


Fig. 1-2. Use of an auxiliary positive lens to find the focal length of a negative lens.

1.1.3 Geneva Gauge

A Geneva gauge, illustrated in Fig. 1-3, can be used to measure the focal length of a thin lens. It consists of three steel prongs, the outer two of which are fixed, and an inner prong that is free to move along its axis, and that is connected to an indicator gauge through a mechanical linkage.

In use, the gauge is pressed onto one surface of the lens to be tested, and the surface power is read directly from the dial. The procedure is then repeated for the other surface. The net power of the lens, in diopters (reciprocal meters), is the algebraic sum of the two readings. The focal length, in meters, is the reciprocal of the power.

The quantity actually measured by the Geneva gauge is the sag (sagitta) of the surface. The dial of the gauge is calibrated under the assumption that the refractive index of the glass is 1.523. The power of the surface is

$$\phi = \frac{n-1}{R} = \frac{0.523}{R}.$$

(1.4)

Using this equation, the actual radius of curvature for a surface can be determined from measurements using a Geneva gauge.

A Geneva gauge can be used to determine the focal length of a lens having a refractive index other than 1.523, if the actual index n_{lens} is known. The true focal length of the lens is found from the equation

$$f_{\text{true}} = \frac{0.523}{n_{\text{lens}} - 1} f_{\text{measured}} \quad (1.5)$$

Generally, measurements made with a Geneva gauge are accurate to +0.25 diopter. Before use, a Geneva gauge should be pressed against a piece of window glass (or other flat surface) to see if it reads zero power. It is important, when using a Geneva gauge, to make sure that it is perpendicular to the surface under test.

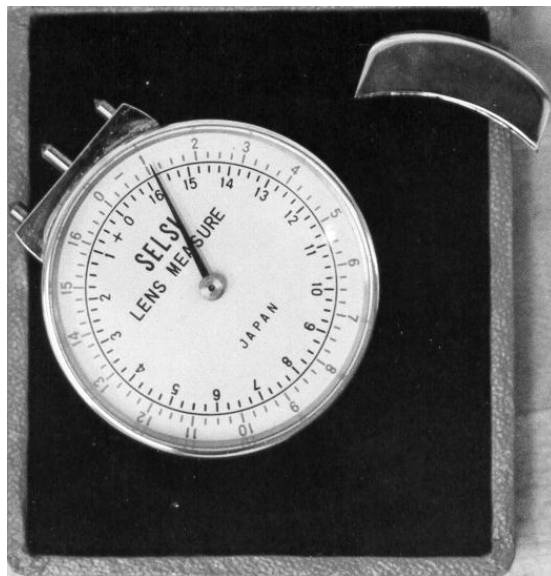


Fig. 1-3. Geneva Gauge.

1.1.4 Neutralization Test

Another method for measuring the focal length of a thin lens is the so-called neutralization test, in which the unknown lens is placed in contact with a lens that has a power equal in magnitude, but opposite in sign, to that of the unknown. In this case (in the thin lens approximation), the powers of the two lenses cancel so the total system has zero power.

In use, the unknown lens and the known lens are placed in contact, and a distant scene is viewed through the resultant combination. The lenses should be held as close to the eye

as feasible, to minimize the effects of mismatched power during the first trials. The total system power is determined by observing the motion of the scene as the observer moves his head from side to side. If the scene moves in the same direction as his head motion, the total system power is positive; if the scene moves in the opposite direction to his head motion, the total system power is negative. The focal length of the unknown lens is equal to the focal length of a known lens of opposite sign, which results in no apparent motion of the scene when the observer moves his head from side to side. During the latter stages of the neutralization test, somewhat increased sensitivity can be obtained by moving the lenses farther from the eye.

1.1.5 Focometer

A particularly handy instrument for measuring the power of a thin lens is the focometer, sometimes called the vertex focometer or vertometer. To use a focometer, the unknown lens is placed in a holder on the instrument, and a drum on the side of the instrument is rotated until a target pattern (typically a cross) can be seen in sharp focus through the eyepiece. The power, in diopters, of the unknown lens can then be read directly on the drum.

The optical system of a focometer is shown in Fig. 1-4. The instrument consists of a target that can be moved back and forth along the optical axis, a collimating lens, a telescope objective and an eyepiece and reticle. A mount is arranged so that the lens to be tested is located at the second focal point of the collimating lens.

The focometer is adjusted so that the telescope and the eyepiece are focused on infinity. When no test lens is present, the target will be in focus when it is located at the first focal point of the collimating lens. When a lens to be tested is inserted in the instrument, it will be necessary to move the target to restore the focus. Since the unknown lens is at the second focal point of the collimating lens, the Newtonian image equation for this lens becomes

$$z(f_{unknown}) = f_o^2 \quad (1.6)$$

The power $\phi_{unknown}$ is linearly proportional to the distance z that the target must be moved to restore the focus:

$$\phi_{unknown} = \left(\frac{1}{f_o} \right)^2 z. \quad (1.7)$$

Because of this linearity, the focometer is a particularly simple and reliable instrument. Typically, the drum is marked in units of 0.25 diopter, and measurements can easily be interpolated to a fraction of this. The lens holder on a focometer is designed so the back surface of the unknown lens is located at the second focal point of the collimating lens. The quantity actually measured by the focometer is the back, or vertex, focal length of the unknown lens; hence the names vertex focometer and vertometer for this instrument.

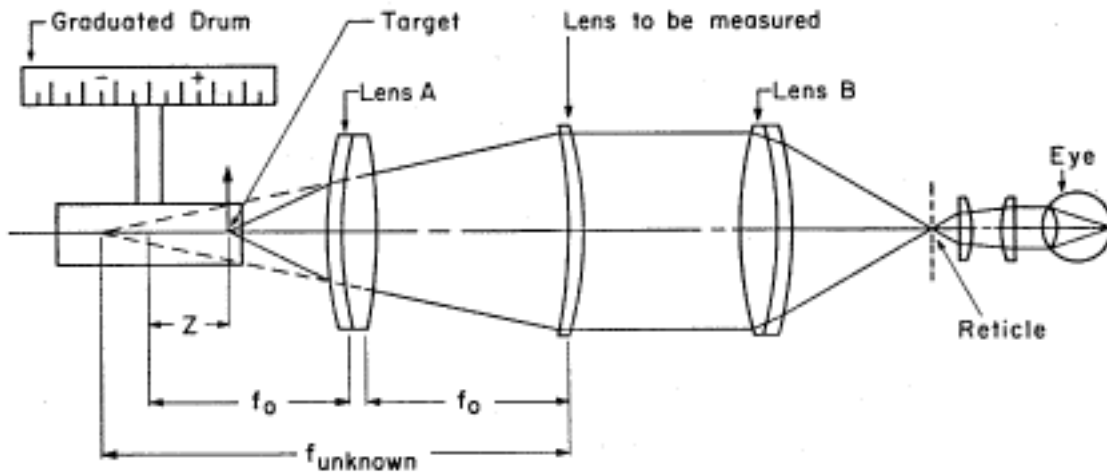


Fig. 1-4. Optical System of the Focometer.

1.2 Thick Lenses

1.2.1 Focal Collimator

The focal length and focal points, and hence the principal points, of a thick lens can be measured using an instrument called a focal collimator. A focal collimator consists of a reticle at the focal point of an achromatic collimating lens, and its use in measuring the focal length of a lens is illustrated in Fig. 1-5. The focal collimator is illuminated by an extended source, and the lens to be tested is placed in the emergent beam. A filar eyepiece inspects the image formed at the focal plane of the test lens. The focal length, f , of the test lens is given by

$$f = A' \left(\frac{F_o}{A} \right) \quad (1-8)$$

where A' is the measured size of the image, A is the size of the reticle, and F_o is the focal length of the collimator objective. Note that the focal collimator may be used to measure negative focal lengths as well as positive; one simply uses a microscope objective with a working distance longer than the negative back focus of the lens under test. In setting up the focal collimator, it is necessary to determine the collimator constant F_o/A to as high a

degree of accuracy as possible. The lens must be turned around to determine the second focal point position. The principal points are of course located a focal length distance from the focal points.

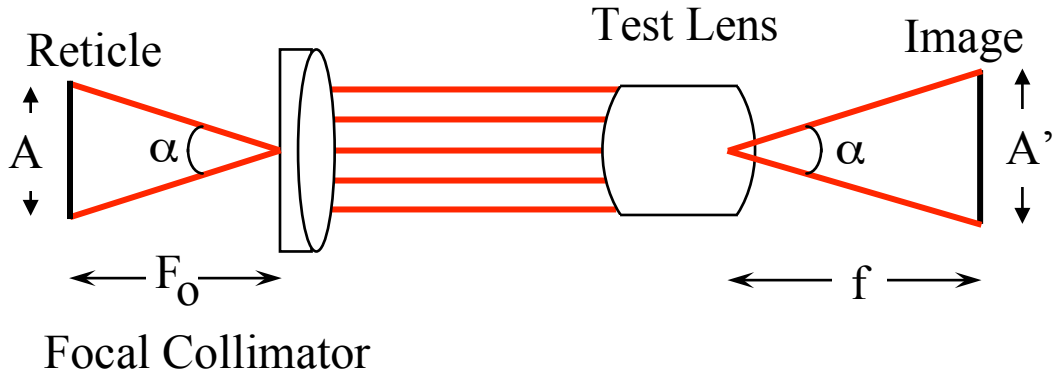


Fig. 1-5. The focal collimator.

1.2.2 Reciprocal Magnification

The cardinal points of a thick lens can also be measured using the reciprocal magnification method, which utilizes the fact that for given positions of object and image, there are two possible positions of the lens, as shown in Fig. 1-6. When the lens is in position 1, the object of height h forms an image of height h'' ; when the lens is in position 2, the image height is h' . As shown in the Appendix, if the distance between lens positions 1 and 2 is d , the focal length of the lens is given by

$$f = \frac{d}{m - \frac{1}{m}}, \quad (1.9)$$

where m is the magnitude of the magnification in position 1. In practice, the magnification is most easily measured by using a trans-illuminated millimeter scale as an object, and a filar eyepiece to inspect the image

Also as shown in the Appendix, the reciprocal magnification method gives the location of the principal planes since if p is the distance between the object and the first principal plane for position 1, and q is the corresponding distance between the image and the second principal plane,

$$p = \frac{d}{m-1} \quad \text{and} \quad q = \frac{d}{1 - \frac{1}{m}}. \quad (1-10)$$

The focal points are then a focal distance from the principal points.

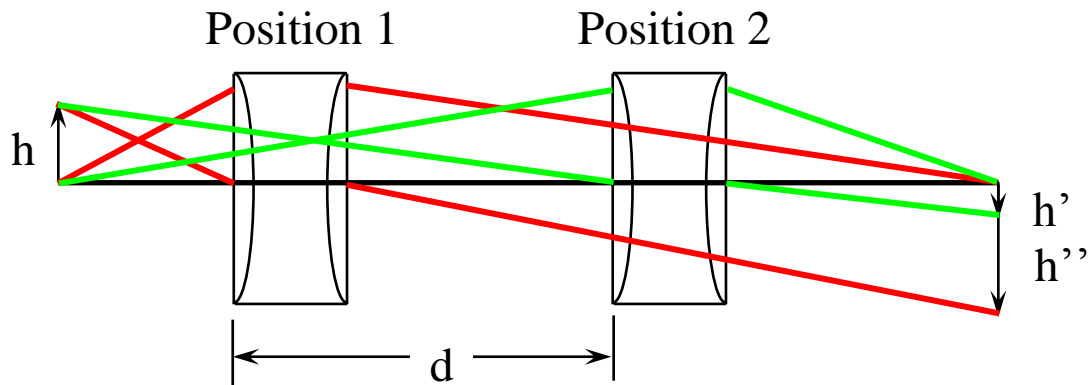


Fig. 1-6. Reciprocal magnification test.

Another way to determine the principal points from the reciprocal magnification procedure is to combine it with the autocollimation procedure described earlier. Let the object be a millimeter scale. As shown in Fig. 1-7, for fixed positions of the scale and plane mirror, there are three positions of the lens in which the scale will be imaged back in its own plane. The location of the focal point with respect to the lens is first established using the autocollimation procedure. The lens is then moved to position (b) in Fig. 1-7, and the magnification of the scale in the plane of the mirror is measured (it is convenient to cover the mirror with a sheet of graph paper for this measurement). The lens is then moved to position (c), and the distance d is measured. The focal length of the lens is calculated using Eq. (1.9). Since the focal point is separated from the principal point by the focal length, the location of the principal point is then established. By reversing the lens, the location of the other focal point and principal point can be found.

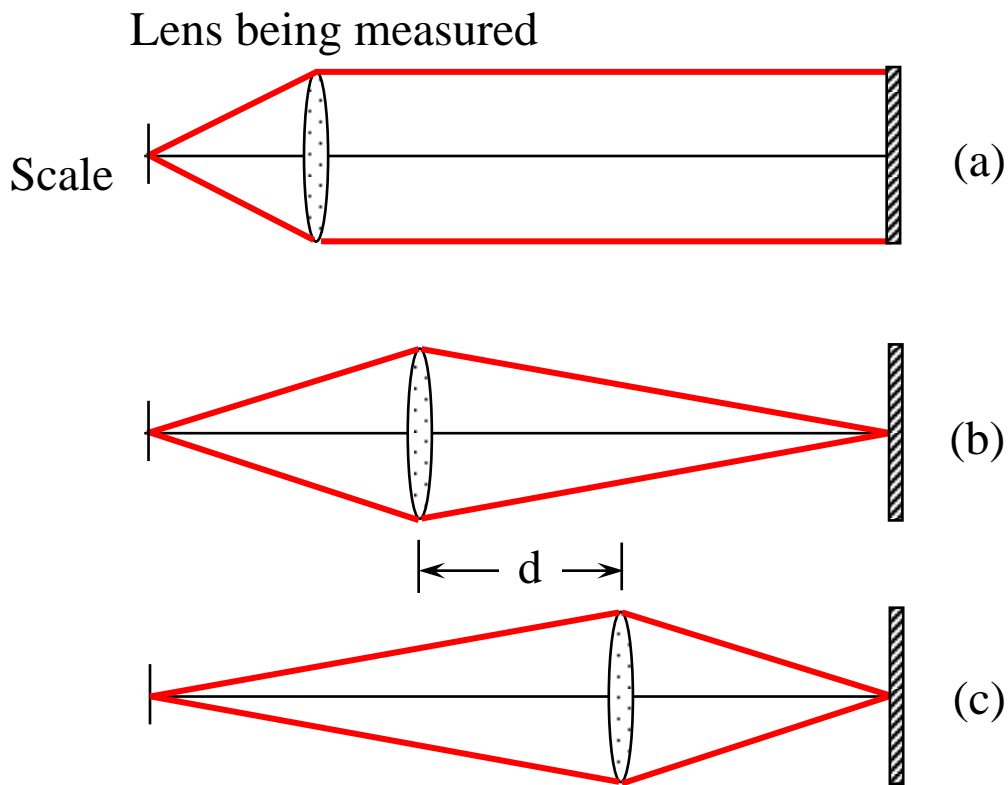


Fig. 1-7. Locating the principal points of a lens by use of reciprocal magnification test and auto-collimation procedure.

1.2.3 Nodal-Slide Lens Bench

Probably the easiest way to measure the positions of the cardinal points of a lens accurately is to use a nodal-slide lens bench, which consists of a pivoted lens holder equipped with a slide that allows the lens to be shifted axially with respect to the pivotal axis. Thus, by moving the lens forward or backward, the lens can be made to rotate about any desired point. Now note that if the lens is pivoted about its second nodal point, as indicated in Fig. 1-8, the ray emerging from this point (which by definition emerges from the system parallel to its incoming direction) will coincide with the bench axis (through the nodal point). Thus there will be no lateral motion of the image when the lens is rotated about the second nodal point. Once the nodal point has been located in this manner, the lens is then realigned with the collimator axis and the location of the focal point is determined. Since the nodal points and principal points are coincident when a lens is in air, the distance from the nodal point to the focal point is the equivalent focal length.

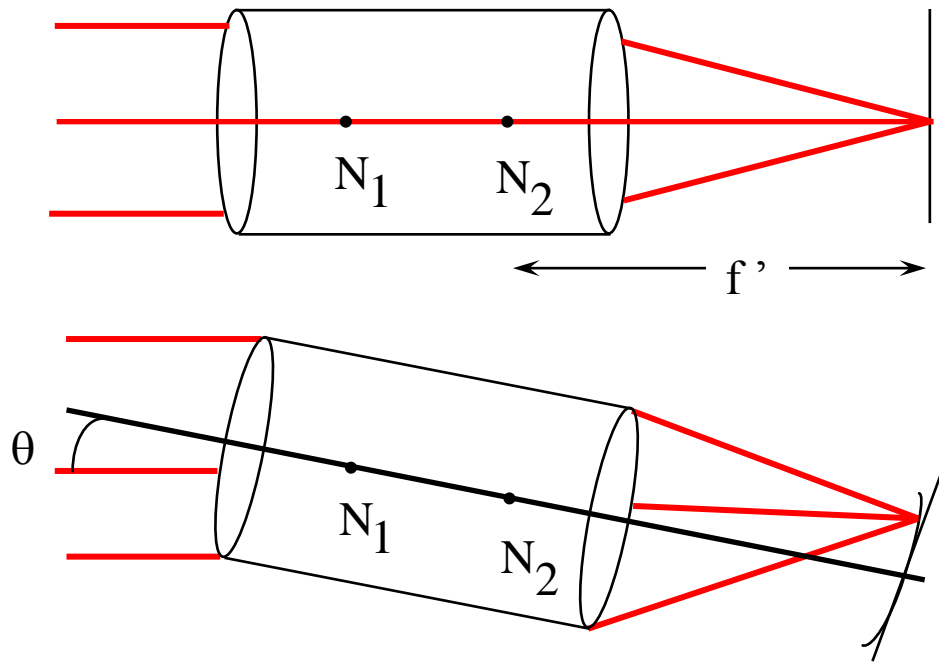


Fig. 1-8. Rotation about the second nodal point.

We see from Fig. 1-8 that as the lens is rotated about the second nodal point, the distance from the lens to the focus is effectively shortened. If the image of an infinitely distant source is observed, the image moves along a circular arc whose radius of curvature is equal to the focal length of the lens. Most lenses are designed to form an image on a plane surface, and in order to inspect the image on such a surface, it is necessary to withdraw the observation surface as the lens is rotated by an amount

$$\varepsilon_z = f'(\sec\theta - 1) \quad (1.11)$$

A lens bench that automatically compensates for this field curvature is the Kingslake lens bench, illustrated in Fig. 1-9. The Kingslake lens bench consists essentially of a nodal slide for holding the lens to be tested, a microscope for viewing the image formed by the lens, and a T-bar arrangement that automatically keeps the viewing microscope focused on a flat field.

In the T-bar construction of the Kingslake lens bench, the viewing microscope is mounted on a carriage that rides on two longitudinal support rails. As the lens is rotated about its nodal point, the T-bar swings around with it, and the microscope carriage, which is pulled against the T-bar by a tensioning weight, moves back by the proper amount to keep the viewing microscope focused on a flat field.

The lens bench is designed to be used with a pinhole collimator light source. If the bench is to be used for a detailed study of the image of the test lens, the quality of the collimator lens should be extremely high. In addition, the diameter of the collimator lens must be

larger than the diameter of the lens to be tested. This must be especially true if telephoto or retrofocus lenses are to be measured, since with these types of lenses the nodal points often lie outside the lens.

The procedure for using the lens bench to find the location of the cardinal points of a lens is straightforward. Assuming that the bench itself has been previously aligned and calibrated, the lens to be tested should be mounted in the holder and the coarse focus adjusted until an image is formed in the plane of the microscope focus. To locate the image, it will generally be necessary to experiment with different centering positions of the lens in its holder. The initial adjustment of the lens in its holder should be done with the eyepiece removed from the viewing microscope, and the aerial image of the source through the microscope objective examined directly, so that the largest possible field of view is obtained.

Once the image is located and focused in the center of the filar eyepiece field, the nodal slide should be rotated back and forth by a few degrees and the longitudinal position of the pivot point adjusted until no motion of the image is observed as the slide is rotated. When this condition is obtained, the nodal point of the lens coincides with the pivot point of the nodal slide, and the focal point of the lens is directly over the axis of the roller on the microscope carriage. Provided the bench is calibrated, the focal length of the lens can then be read directly from the scale on the side of the bench.

It is important when measuring the location of the cardinal points of a lens not to rotate the nodal slide through large angles, as distortion in the lens will then cause a motion of the image even when the lens is pivoted about its nodal point. In fact, by measuring this image motion, the amount of distortion present in a lens can easily be measured. One of the principal uses of the Kingslake lens bench is the observation and measurement of lens aberrations.

In order for the bench to function properly, a number of adjustments and calibrations must be made. First, the focus of the microscope must be precisely on the axis of the roller on the microscope carriage. On the Kingslake bench, the microscope is mounted on the carriage so it can be pivoted about the axis of the roller. Thus the focus of the microscope can be set on this point by the following procedure: A small object, such as a vertical scratch on a piece of film, is mounted on a stage that permits precision motion both along the axis of the bench, and horizontally perpendicular to this axis (the nodal slide itself can be used for such a stage). The microscope is focused on the scratch. If the scratch is not located precisely on the axis of the roller, its image will appear to move when the microscope is pivoted back and forth about the axis of the roller. If the scratch is displaced along the axis, the image will move from side to side, while if the scratch is displaced transversely, the image will appear to move longitudinally. Adjustments are made until the scratch does not move as the microscope is pivoted back and forth; the microscope is given a final focus adjustment and the vernier scales on the transverse and longitudinal microscope slides are set to zero.

The optical axis of the microscope must intersect the axis of the pivot on the nodal slide. To make this adjustment, the film with the vertical scratch is placed on the lens mount, and the coarse focus is adjusted until it is in focus in the microscope (on the Kingslake bench it is necessary to remove the T-bar to make this adjustment). The nodal point adjustment and lens holder lateral adjustment are then used to center the scratch by observation of the image motion as the nodal slide is rotated back and forth. When the image does not move, the focus of the microscope lies on the axis of the nodal point pivot. The focal length scale should then be set to zero.

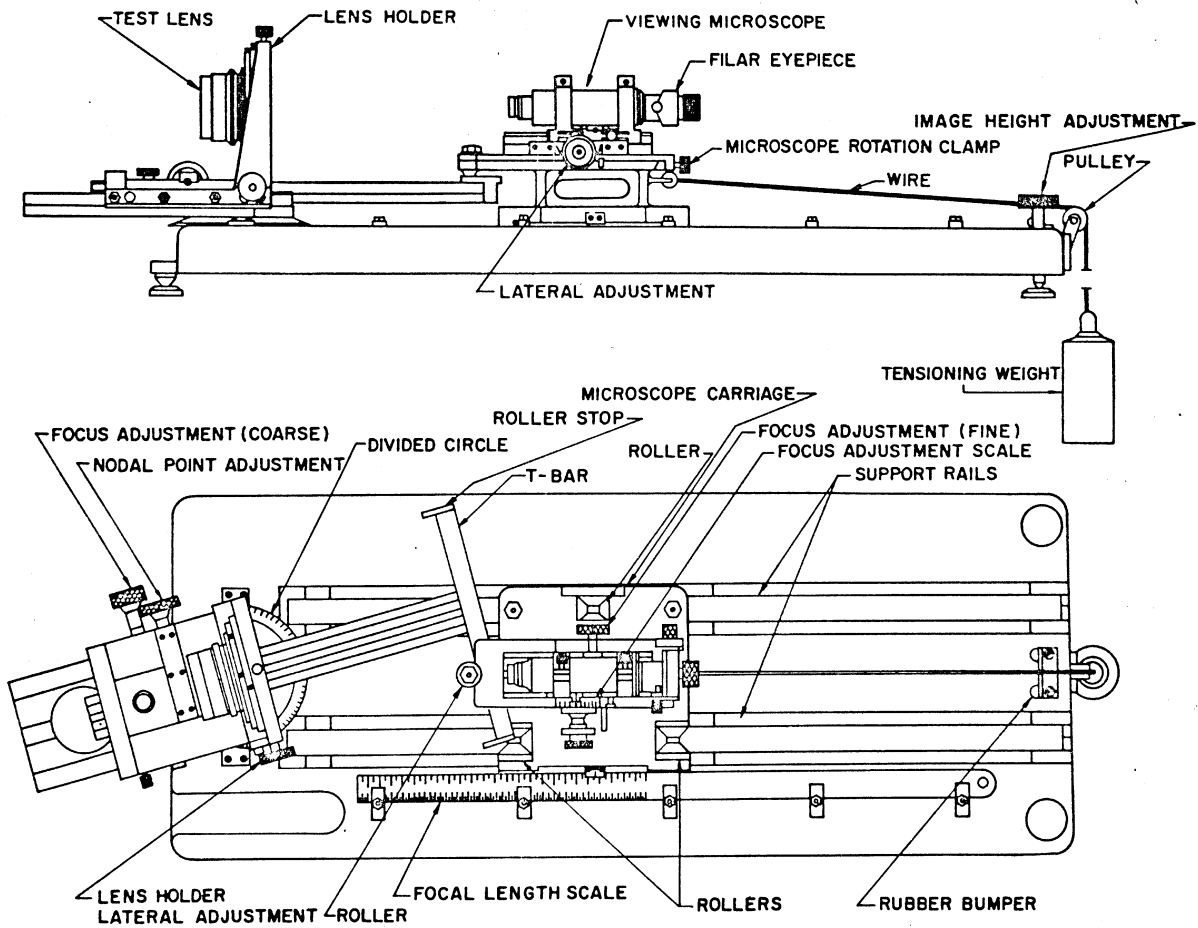
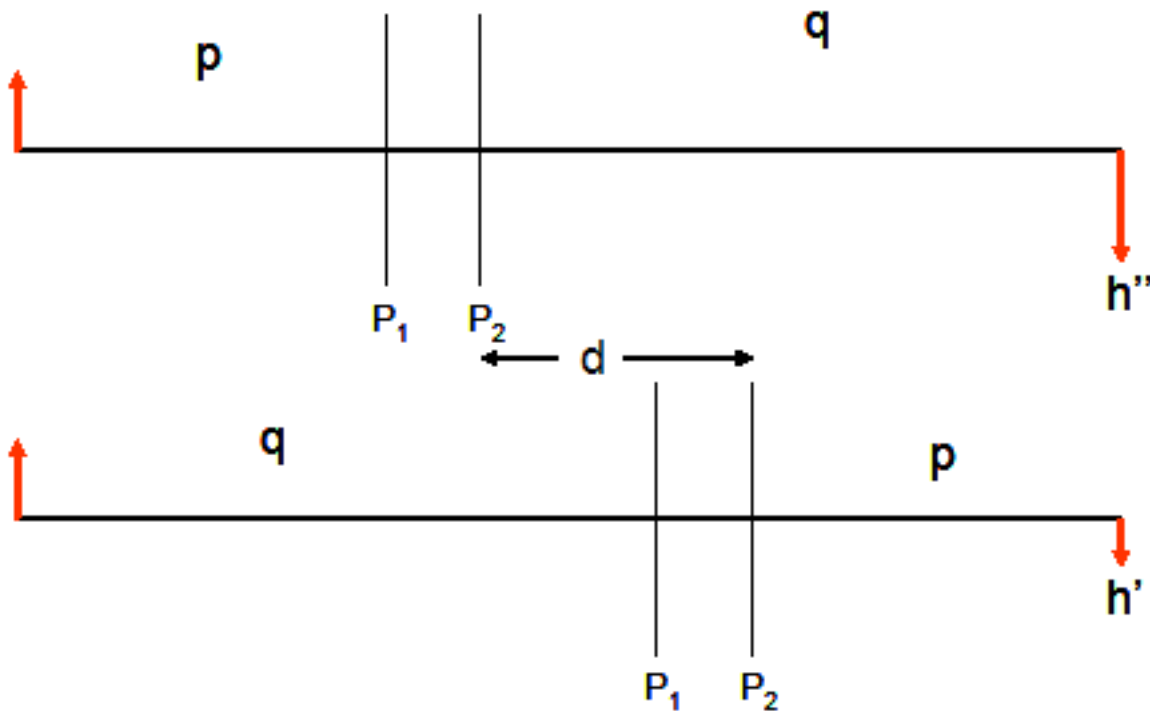


Fig. 9. The Kingslake lens bench (after R. Kingslake, *J. Opt. Soc. Am.* 22, 207-222 (1932)).

Appendix

Reciprocal Magnification Derivation

P_1 and P_2 are the two principal planes. Let p and q be the object and image distances.



For given positions of the object and image there are two possible positions of the lens as shown in the figure. Let d be the distance between the two lens positions.

$$d = q - p \quad (1)$$

We know that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}; \quad m = \frac{q}{p} = \text{magnitude of magnification} \quad (2)$$

It follows that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{p m} = \frac{1}{p} \left(1 + \frac{1}{m} \right) \quad (3)$$

But

$$d = p m - p = p (m - 1) \quad (4)$$

$$\frac{1}{f} = \frac{m - 1}{d} \left(\frac{m + 1}{m} \right) = \frac{m^2 - 1}{m d} \quad (5)$$

Therefore

$$f = \frac{d}{m - \frac{1}{m}}; \quad p = \frac{d}{m - 1}; \quad q = \frac{d}{1 - \frac{1}{m}} \quad (6)$$