Spatial Synchronous and Fourier Methods

- Both techniques use a single interferogram having a large amount of tilt.

The interference signal is given by

```
irradiance[x_, y_] := iavg (1 + γ Cos[ϕ[x, y] + 2 π f x])
```

Spatial Synchronous

- The interference signal is compared to reference sinusoidal and cosinusoidal signals.

The two reference signals are

```
rcos[x_, y_] := Cos[2 π f x]
```

and

```
rsin[x_, y_] := Sin[2 π f x]
```

Multiplying the reference signal times the irradiance signal gives sum and difference signals.

```
TrigReduce[irradiance[x, y] rcos[x, y]]
```

$$\frac{1}{2}\left(2\,\mathrm{iavg}\,\mathrm{Cos}[2\,f\,\pi\,x]+\mathrm{iavg}\,\gamma\,\mathrm{Cos}[\phi[x,y]]+\mathrm{iavg}\,\gamma\,\mathrm{Cos}[4\,f\,\pi\,x+\phi[x,y]]\right)$$

```
TrigReduce[irradiance[x, y] rsin[x, y]]
```

$$\frac{1}{2}\left(2\,\mathrm{iavg}\,\mathrm{Sin}[2\,f\,\pi\,x]-\mathrm{iavg}\,\gamma\,\mathrm{Sin}[\phi[x,y]]+\mathrm{iavg}\,\gamma\,\mathrm{Sin}[4\,f\,\pi\,x+\phi[x,y]]\right)$$

The low frequency second term in the two signals can be written as

$$s1=\frac{\mathrm{iavg}\,\gamma}{2}\,\mathrm{Cos}[\phi[x,y]]$$

$$s2=-\frac{\mathrm{iavg}\,\gamma}{2}\,\mathrm{Sin}[\phi[x,y]]$$

$$\mathrm{Tan}[\phi[x,y]]=\frac{-s2}{s1}$$

The only effect of having the frequency of the reference signals slightly different from the average frequency of the interference signal is to introduce tilt into the final calculated phase distribution.

Fourier Method

- The interference signal is Fourier transformed, spatially filtered, and the inverse Fourier transform of the filtered signal is performed to yield the wavefront.

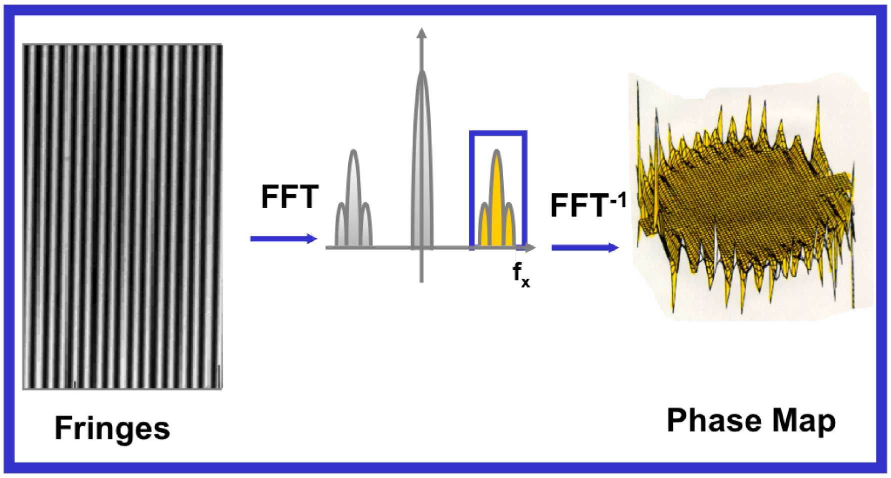
The Fourier analysis method is essentially identical to the spatial synchronous method. The irradiance can be written as

$$\text{irradiance}[x, y] = \text{iavg}\,(1 + \gamma\,\text{Cos}[\phi[x, y] + 2\,\pi\,f\,x])$$

We can rewrite this as

$$\text{irradiance}[x, y] = \text{iavg}\left(1 + \frac{1}{2}\,e^{-2\,i\,f\,\pi\,x - i\,\phi[x,y]}\,\gamma + \frac{1}{2}\,e^{2\,i\,f\,\pi\,x + i\,\phi[x,y]}\,\gamma\right)$$

We can take the Fourier transform of this irradiance signal and spatially filter to select the portion of the Fourier transform around the spatial frequency f, and then take the inverse Fourier transform of this filtered signal to give the wavefront.



Note that both the spatial synchronous method and the Fourier method require a large amount of tilt be introduced to separate the orders. Since a spatially limited system is not band limited, the orders are never completely separated and the resulting wavefront will always have some ringing at the edges. Also, the requirement for large tilt always limits the accuracy of the measurement. The advantage of the techniques is that only a single interferogram is required and vibration and turbulence cause less trouble than if multiple interferograms were required.