11. System Evaluation
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11.1 Resolution Tests

A convenient and useful way of judging the performance of a lens-emulsion combination is to photograph a resolution chart, such as the one shown in Fig. 11-1, and visually determine the frequency for which the resolution bars can no longer be clearly observed. In viewing the photographic image of the target, suitable magnification must be used. A rule generally used here is that the numerical value of the magnifying power must be about equal to the spatial frequency of the target image being viewed in lines per millimeter.

For the USAF resolution test target shown in Fig. 11-1, there are six targets within each group. The sequence of frequencies follows a geometrical progression with a factor of

$$\sqrt[6]{2} = 1.125$$

between successive frequencies. Hence, between corresponding targets in successive groups the spatial frequency changes by a factor of 2. For a test target having the original size specified by the USAF, target 1 in group 0 has a spatial frequency of 1 line/mm. If G is the group number, and P is the target number within a group where P varies from 1 to 6, the spatial frequency of any given target can be found by using the equation

$$\text{Spatial freq. in lines/mm} = 2^{[G+(P-1)/6]}.$$

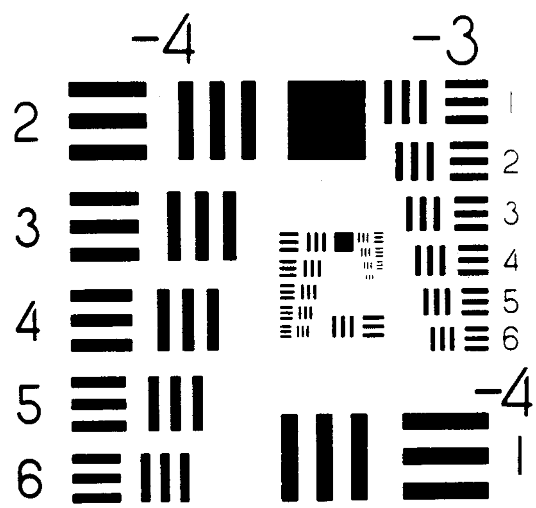


Fig. 11-1. Resolving power test target.

11.2 Veiling Glare

The image-plane of lenses normally receives not only the image forming light, but also unwanted light that may reduce image contrast. This unwanted light is referred to as veiling glare and usually arises from one or more of the following causes:

- Internal multiple reflections between the lens surfaces.
- Scatter from the surfaces of the lens elements due to scratches and other imperfections in the polish, dirt and dust, fingerprints, grease, and poor antireflection coatings.
- Bulk scatter from the interior of the glass and from bubbles and striae.
- Scatter from optical cements.
- Scatter and reflections from the ground edges of the lens elements, from internal lens mounts and from the internal surface of the lens barrel.
- Reflections from the surfaces of diaphragms and shutter blades.
- Fluorescence of the glass or optical cements.

The veiling glare of a lens system on its own may be considerably different from the veiling glare of a lens system and camera body combination. In the latter case reflection of part of the image forming light from the detector in combination with further reflections and scatter from the lens system and camera body contribute significantly to the veiling glare. In other words, the lens must be tested as it is being used.

A convenient method for measuring veiling glare consists of mounting a strip of dead-black material such as velvet across the rear of an integrating sphere or light box, and using the lens to form an image of the black strip on a detector. The veiling glare index, VGI, is the ratio of the illumination at the center of the image of the black area superimposed on an extended field of uniform luminance, to the illumination at the same point of the image plane when the black area is removed. The advantage of using a long strip instead of a small patch is that the veiling glare can be determined at various points across the field of the lens. Since the detector is capable of reflecting light back into the lens, it is essential that the detector be at least as large as the detector that will be used in the final apparatus.

A typical arrangement for measuring VGI is illustrated in Fig. 11-2. Illuminating an integrating sphere with several lamps through suitable portholes produces the extended bright field. The “black area” is either an absorbing cavity in the wall of the integrating sphere or a strip of black material such as velvet. The lens under test is placed with its front end just protruding into an exit port in the integrating sphere that is diametrically opposite the “black area”. The ratio of the detector signal in the above situation to the detector signal when the “black area” is replaced by a section of normal integrating sphere surface gives the veiling glare index. For the situation where the “black area” cannot be replaced with a normal section of the surface the second measurement can be obtained by moving the aperture and detector to a position clear of, but adjacent to, the image of the “black area”.

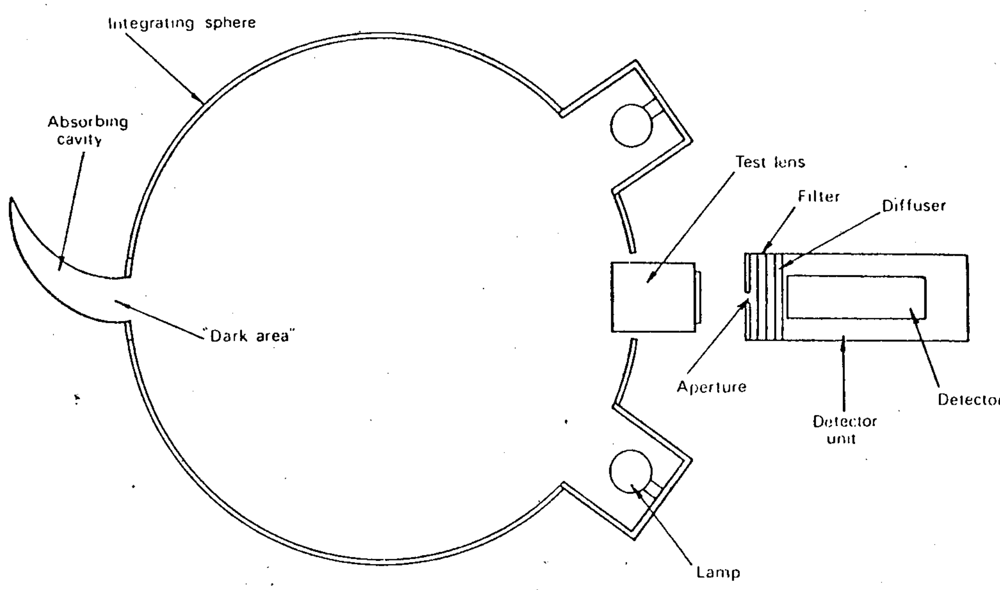


Fig. 11–2. Schematic diagram of typical veiling glare index equipment.

11.3 Spread Function Measurement

The direct measurement of the point-spread function of a lens is one of the most difficult measurements to carry out. It requires the photometry of very small areas in the image plane that inherently contain very little light. In many cases, the dynamic range, i.e. the ratio of magnitudes of luminance levels to be observed, may be very great. If a CCD is used, the CCD must be calibrated and the effect of any optics used to magnify the point-spread function must be well known. If a scanning system is used, precise positioning of the scanning system is generally required. Furthermore, in order to carry out the measurement, the light source must be sufficiently small in extent that the lens under test does not resolve it. If a scanning aperture is used, the size of the scanning aperture must also be small since we are actually measuring the point-spread function convolved with the scanning aperture.

A technique for obtaining a measurement of the spread function with a more favorable illumination situation is to measure the line-spread function. This function is the integral or average of the point- spread function along a line through the image. By use of a slit image and a scanning slit, the amount of light available is increased over the point-scan method by the ratios of the areas of the measuring and source slits to the area of the point aperture. The resulting function is directly representative of the line- spread function for a given azimuth. By measurements in several azimuths, it is possible to reconstruct the point-spread function computationally.

If an interferometric test of the optical system is performed to determine the wavefront produced by the system under test, the point-spread function can be calculated by finding the Fourier transform of the wavefront and squaring the result. This technique generally works well, but it must be remembered that the wavefront is being sampled, and frequency components higher than the sampling frequency will be absent. If the aberration is rotationally symmetric, the point-spread-function can be calculated using the Hankel transform.

If a large amount of aberration is present it may be useful to calculate the geometrical spot diagram from the interferogram data instead of the diffraction point spread function.

11.4 Encircled Energy

Basically, an encircled energy measurement is accomplished by placing a series of circular holes of successively larger radius in the plane of the image of a point. The total amount of light passing through each hole is measured. For normalization, the total flux or energy is measured by a very large hole; the measurements using each of the smaller holes are divided by this measurement to obtain the fraction of the energy within each circle.

The measurement of the flux through the hole must be made when the center of the hole has been located according to some rule. For an image that is symmetrical, the choice or origin is evidently the center of symmetry of the image. If the image is not symmetrical, the hole can be moved in either one or two dimensions to maximize the flux going through the hole. The various size holes can each be moved to maximize the amount of light through each, or they can all have the same center, which is the center for maximizing the amount of light through a given size hole.

If an interferometric test of the optical system is performed, the encircled energy can be calculated from the point-spread function. If a large amount of aberration is present it may be useful to calculate the geometrical-encircled energy from the interferogram data instead of the diffraction-encircled energy.

11.5 Optical Transfer Function Measurement

The optical transfer function (OTF) is the most important function used in image evaluation. After a brief discussion of the meaning of the OTF we will discuss three measurement techniques:

- Scanning Method
- Interferogram Analysis
- Autocorrelation Method

11.5 Optical Transfer Function

Reference: Goodman, *Introduction to Fourier Optics*

Basic Definitions

Let $\text{Abs}[h[\xi, \eta]]^2$ be the point spread function, PSF, and let $I_g[\xi, \eta]$ be the intensity of the geometrical image, then

$$I_i[u, v] = \kappa \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{Abs}[h[u-\xi, v-\eta]]^2\, I_g[\xi, \eta]\, d\xi\, d\eta$$

We will now look at the normalized spatial frequency of I_g and I_i

$$G_g[f_x, f_y] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_g[u, v]\, e^{-i 2\pi (f_x u + f_y v)}\, du\, dv}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_g[u, v]\, du\, dv}$$

$$G_i[f_x, f_y] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_i[u, v]\, e^{-i 2\pi (f_x u + f_y v)}\, du\, dv}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_i[u, v]\, du\, dv}$$

The normalized transfer function of the system is given by

$$H[f_x, f_y] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{Abs}[h[u, v]]^2\, e^{-i 2\pi (f_x u + f_y v)}\, du\, dv}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{Abs}[h[u, v]]^2\, du\, dv}$$

which is the normalized Fourier transform of the PSF.

From the convolution theorem

$$G_i[f_x, f_y] = H[f_x, f_y]\, G_g[f_x, f_y]$$

$H[f_x, f_y]$ is called the optical transfer function, OTF. The modulus of the OTF is called the modulation transfer function, MTF. From the above we see the OTF is the normalized Fourier transform of the PSF.

Relating OTF to pupil function.

The OTF is given by the Fourier transform of the PSF. The PSF is the square of the absolute value of the Fourier transform of the pupil function.

From the autocorrelation theorem we have

$$FT[Abs[g[x, y]]^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G[\xi, \eta]\, G^*[\xi - f_x, \eta - f_y]\, d\xi\, d\eta$$

Therefore if $P[x', y']$ is the pupil function

$$H[f_x, f_y] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P[x', y']\, P^*[x' - \lambda z_i f_x, y' - \lambda z_i f_y]\, dx'\, dy'}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Abs[P[x', y']]^2\, dx'\, dy'}$$

We can do a simple change of variables

$$x = x' - \frac{\lambda z_i f_x}{2} \text{ and } y = y' - \frac{\lambda z_i f_y}{2}$$

$$H[f_x, f_y] = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\left[x + \frac{\lambda z_i f_x}{2}, y + \frac{\lambda z_i f_y}{2}\right] P^*\left[x - \frac{\lambda z_i f_x}{2}, y - \frac{\lambda z_i f_y}{2}\right] dx\, dy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Abs[P[x, y]]^2\, d\xi\, dy}$$

That is, the OTF is given by the autocorrelation of the pupil function.

General properties of OTF

- **1) H[0,0]=1**

Follows directly from equation for OTF

- **2)** $H[-f_x, -f_y] = H^*[f_x, f_y]$

Fourier transform of real function is Hermitian.

- **3)** $Abs[H[f_x, f_y]] \le Abs[H[0, 0]]$

Follows directly from fact OTF is given by the autocorrelation of the pupil function.

What does OTF mean?

object

$$I[x] = I_o\,(1 + m\,Sin[\nu\, x])$$

Perfect Image

Assume unit magnification

$$I[x] = I_o \left(1 + \frac{m}{2\,i}\,\left(e^{i\,\nu\,x} - e^{-i\,\nu\,x}\right)\right)$$

$$\text{OTF}[\nu] = \text{Abs}[\text{OTF}[\nu]]\, e^{i\,\theta[\nu]}, \text{ Hermitian}$$

Actual image

$$I[x] = I_o \left(1 + \frac{m}{2\,i}\,\text{Abs}[\text{OTF}[\nu]]\,\left(e^{i\,(\nu\,x+\theta[\nu])} - e^{-(i\,\nu\,x+\theta[\nu])}\right)\right)$$

$$I[x] = I_o\,(1 + m\,\text{MTF}[\nu]\,\text{Sin}[\nu\,x + \theta[\nu]])$$

The modulus of the OTF (MTF) changes the contrast of the image and the phase of the OTF shifts the pattern. Since $\theta[\nu]$ depends on spatial frequency, the shift depends upon spatial frequency.

Measurement of OTF -- sinusoidal grating and slit

Can measure the OTF by imaging sinusoidal gratings of different frequencies and scanning the images with a slit or image slit and scan the image using sinusoidal gratings.

One-dimensional case

$$i[x'] = \int_{-\infty}^{\infty} h[x]\, o[x' - x]\, dx$$

$o[x]$ = intensity distribution of incoherently illuminated object
$h[x]$ = image of line source
$i[x']$ = image

To eliminate factor of magnification, the ideal image of the object is considered as the object.

Let object be $o[x'] = b_o\,(1 + \text{Cos}[2\pi\nu x'])$

$$i[x'] = b_o \int_{-\infty}^{\infty} h[x]\,(1 + \text{Cos}[2\pi\nu(x' - x)])\, dx$$

$$= b_o \int_{-\infty}^{\infty} h[x]\,(1 + \text{Cos}[2\pi\nu x]\,\text{Cos}[2\pi\nu x'] + \text{Sin}[2\pi\nu x]\,\text{Sin}[2\pi\nu x'])\, dx$$

From the definition of the OTF we know

$$I[\nu] = H[\nu]\, O[\nu]$$

Let

$$H_c[\nu] = \int_{-\infty}^{\infty} h[x]\,\text{Cos}[2\pi\nu x]\, dx$$

$$H_s[\nu] = \int_{-\infty}^{\infty} h[x]\,\text{Sin}[2\pi\nu x]\, dx$$

$$H[\nu] = \int_{-\infty}^{\infty} h[x]\, e^{-i 2\pi\nu x}\, dx = H_c[\nu] - i\, H_s[\nu]$$

Then

$$i[x'] = b_o \left(\int_{-\infty}^{\infty} h[x]\, dx + \text{Cos}[2\pi\nu x']\, H_c[\nu] + \text{Sin}[2\pi\nu x']\, H_s[\nu] \right)$$

If

$$T[\nu] = \sqrt{H_c[\nu]^2 + H_s[\nu]^2}$$

$$\theta[\nu] = \text{ArcTan}\left[\frac{H_s[\nu]}{H_c[\nu]}\right]$$

$$i[x'] = b_o \left(\int_{-\infty}^{\infty} h[x]\, dx + T[\nu]\, \text{Cos}[2\pi\nu x' - \theta[\nu]]\right)$$

Let

$$\int_{-\infty}^{\infty} h[x]\, dx = 1,$$

then average illumination of image of grating is equal to that of the object

$$i[x'] = b_o (1 + T[\nu]\, \text{Cos}[2\pi\nu x' - \theta[\nu]])$$

11.5.1 Scanning Method

The most straightforward scanning method is to image a target whose luminance varies sinusoidally with distance (sinusoidal grating), and then scan the image with a slit. Equivalently, the image of the slit can be scanned with a sinusoidal grating. By measuring the contrast of the image as a function of spatial frequency, the MTF is determined. The shift of the image as a function of spatial frequency gives the phase of the OTF. The sinusoidal grating can be either of the density type, or the area type, as shown in Fig. 11-3. The area type has the advantage that it is easier to make. In some instances, it is easier to use a square-wave grating and then electronically filter out the first harmonic of the signal.

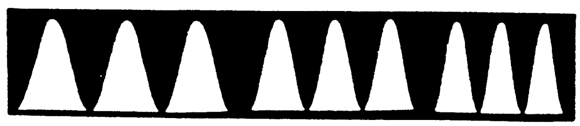


Fig. 11-3. Sinusoidal gratings of the area type.

A convenient method for varying the spatial frequency of the sinusoidal grating is illustrated in Fig. 11-4. A narrow slit is placed over a sinusoidal grating having a period d. If the slit makes an angle of α with respect to the grating lines, the effective period of the grating is d/sin α. Hence the effective frequency of the grating can be made to vary from zero up to 1/d. Being able to obtain zero spatial frequency is very important for calibration purposes. Often, a radial grating is used so it can easily be rotated past the object slit to produce a moving effective grating. The grating moves past the image slit to give an ac output signal. The modulation of this signal gives the MTF.

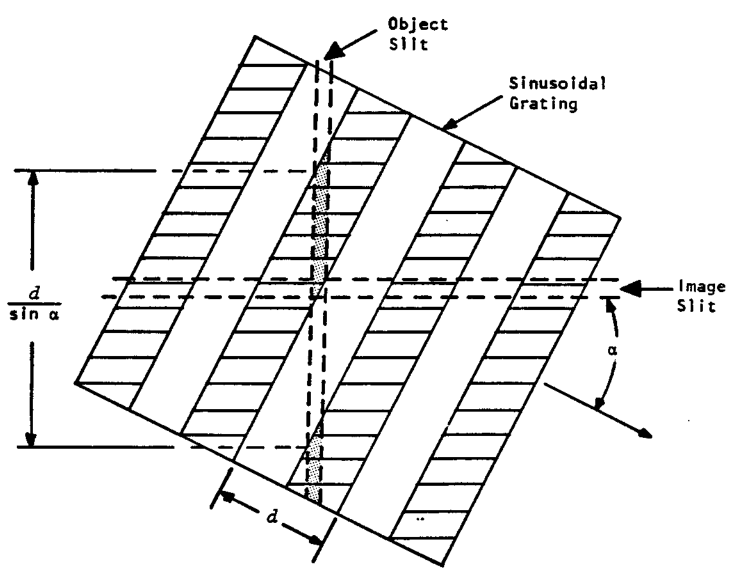


Fig. 11-4. Convenient technique for varying effective spatial frequency of target.

Figure 11-5 shows a schematic of an OTF measuring instrument for measuring the OTF at discrete frequencies. A slit is imaged onto the gratings, which are mounted on a drum that can be rotated to change the spatial frequency. For each spatial frequency, two gratings are used; the two gratings are shifted a fourth of a period so one grating can be thought of as a sinusoidal grating and the other as a cosinusoidal grating. Measuring the amount of light going through the two gratings, and taking the ratio of light through the sinusoidal grating to the light through the cosinusoidal grating gives the tangent of the phase of the OTF, while the square root of the sum of the squares is proportional to the MTF.

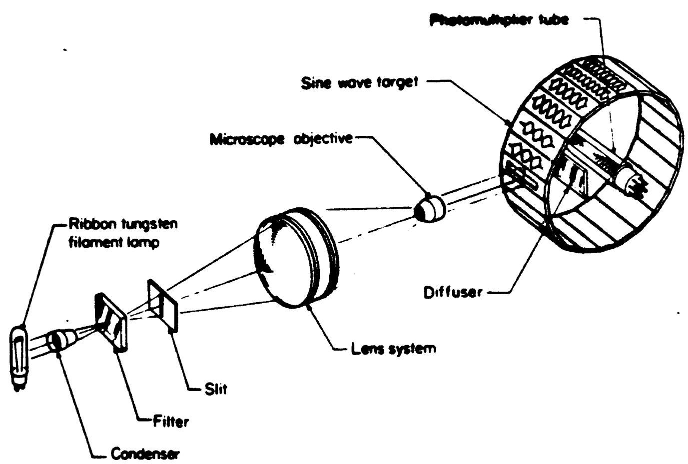
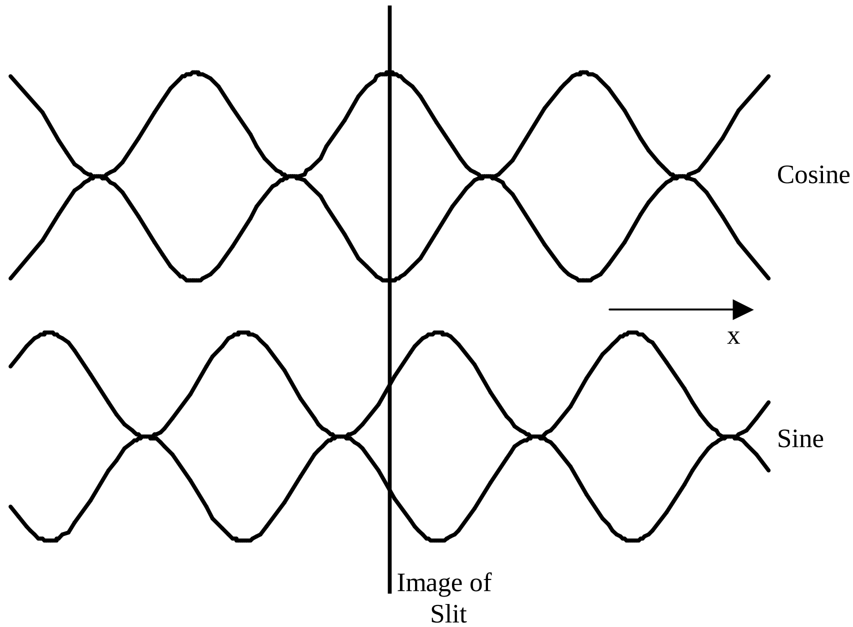


Fig. 11-5 A schematic diagram of one type of optical-transfer function measurement device

The OTF is given by the Fourier transform of the point-spread function, or in one dimension the line-spread function. It is possible to obtain the OTF by scanning a slit image and using a computer to obtain the Fourier transform. Since a line-spread function can be obtained from an edge image by differentiation, it is also possible to obtain the OTF by scanning an image of an edge and then differentiating and Fourier transforming.

MTF Measurement

Slit Image Falling on Cosine and Sine Area Grating



$$h(x) = \text{image of line source}$$

$$H_c(\nu) = \int_{-\infty}^{\infty} h(x)\cos 2\pi\nu x dx \qquad H_s(\nu) = \int_{-\infty}^{\infty} h(x)\sin 2\pi\nu x dx$$

$$H(\nu) = \int_{-\infty}^{\infty} h(x)e^{-i2\pi\nu x}dx = H_c(\nu) - iH_s(\nu) = T(\nu)e^{i\theta(\nu)}$$

$$T(\nu) = \sqrt{H_c^2(\nu) + H_s^2(\nu)} \qquad \theta(\nu) = \tan^{-1}\left(\frac{H_s(\nu)}{H_c(\nu)}\right)$$

11.5.2 Interferogram Analysis

As stated above, the point spread function can be obtained from an interferogram by taking a Fourier transform of the wavefront and squaring the result. The optical transfer function can then be obtained from the point spread function by taking another Fourier transform. The modulus of the OTF gives the MTF.

11.5.3 Use of lateral shear interferometer to measure the OTF

H. H. Hopkins was the first to show that a lateral shearing interferometer can be used to measure the OTF of an optical system. (Ref: H. H. Hopkins, Opt. Acta 2, 23 (1955).)

The one-dimensional OTF of an optical system $H[f_x]$ is given by the autocorrelation of the pupil function

$$H[f_x] = \frac{1}{c} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\left[x + \frac{s}{2},\ y\right] P^*\left[x - \frac{s}{2},\ y\right] dx\, dy$$

If z_i is the image distance, D is the pupil diameter, and λ is the wavelength

$$f_x = \frac{s}{\lambda z_i} = \frac{s}{D} \frac{1}{\lambda} \frac{D}{z_i} = \frac{s}{D} \frac{1}{\lambda f^{\#}}$$

For simplicity let the two interfering beams in a lateral shear interferometer have the same intensity. Let δ be the phase difference between the two sheared interfering wavefronts due to path difference in the interferometer. Generally δ is made to vary linearly with time, i.e. $\delta = \omega t + \phi_o$.

Then the total flux in the interference pattern is

$$F[\delta, s] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{Abs}\left[P\left[x + \frac{s}{2},\ y\right] + P\left[x - \frac{s}{2},\ y\right] e^{-i\delta}\right]^2 dx\, dy$$

$$= 2 \int\int \text{Abs}\left[P\left[x + \frac{s}{2},\ y\right]\right]^2 dx\, dy +$$

$$\int\int P\left[x + \frac{s}{2},\ y\right] P^*\left[x - \frac{s}{2},\ y\right] e^{i\delta} dx\, dy + \text{complex conjugate}$$

$$= 2c\,(1 + \text{Abs}[H[f_x]]\, \text{Cos}[\delta - \Theta[f_x]])$$

2c is the average amount of flux in the two interfering wavefronts.

11.5.3 Autocorrelation Method

It can be shown that the OTF is given by the autocorrelation of the pupil function. The auto- correlation process can be done either digitally or in an analog fashion. The most common technique for determining the OTF at the present time involves measuring the pupil function using an interferometer, such as a Twyman-Green, and then using a computer to digitally find the autocorrelation of the pupil function and hence, both the phase and magnitude of the OTF. It should be noted that this technique for finding the OTF does not take into account any scattering in the system; it accounts only for pupil shape and wave- front aberrations. Generally measurements are performed at a single wavelength, although if dispersions are known, the OTF can be calculated for white light.

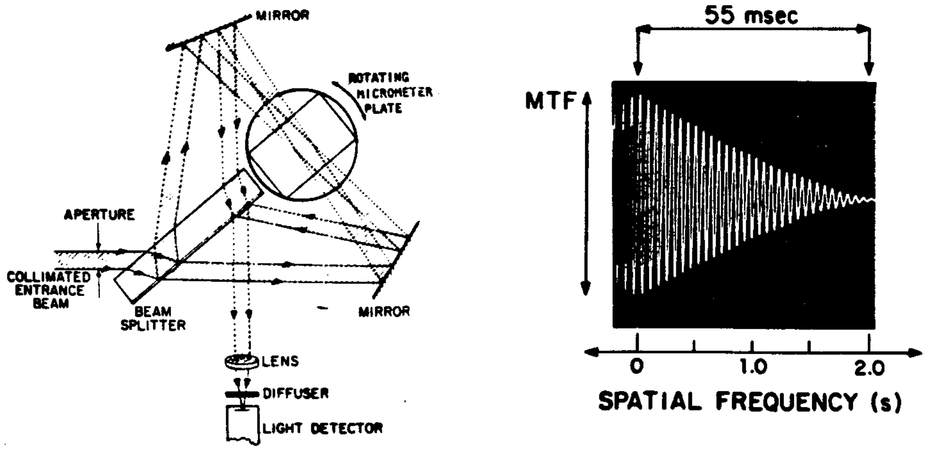
Although the method is not commonly used, a lateral shear interferometer can be used to calculate the autocorrelation of the pupil function in an analog fashion. It can be shown that if the two interfering beams in a lateral shear interferometer have the same intensity, the total flux in the interference pattern is given by

$$F = 2c\{1 + |OTF(\nu,\psi)|\cos[\delta - \theta(\nu,\psi)]\}$$

where 2c is the average amount of flux in the two interfering wavefronts, and δ is the phase difference between the two sheared wavefronts due to path differences in the interferometer. $\theta(\nu,\psi)$ is the phase of the OTF. If δ is made to vary linearly with time, the modulation of the output signal as a function of shear gives the MTF, and the phase of the signal gives the phase of the OTF. The relationship between shear and spatial frequency is given by

$$\nu = \frac{S}{D}\frac{1}{\lambda f\#}$$

where (S/D) is the ratio of the shear distance to the exit pupil diameter, and f# is the f/number of the optical system. Figure 11-6 shows a convenient lateral shear interferometer for performing MTF measurements using monochromatic light. In the interferometer, both the amount of shear and the varying phase shift between the two interfering beams are accomplished by rotating the plane parallel plate as shown in the figure. The amount of light in the shearing pattern is detected as illustrated. The detector output is ac-coupled to an oscilloscope, which displays the MTF as a function of shear (or equivalently spatial frequency), as shown in Fig. 11-6.



a) Triangular interferometer schematic. b) MTF display.

Fig. 11-6. Lateral shear interferometer for measuring MTF (Ref: Kelsall, Appl. Opt., 12, 1398 (1973)).

A SIMPLE INTERFEROMETRIC MTF INSTRUMENT

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A simple law interferometric technique for measuring the modulation transfer function of an optical system is described.

1. Introduction

It is well known that the optical transfer function of an optical system can be measured using a lateral shear interferometer [1]. Several interferometers for measuring the optical or modulation transfer function have been described previously [2-10], including an ingeneous interferometric technique invented by Kelsall [8]. For all these interferometric techniques a lateral shear interferometer is used in which the shear is varied and, at the same time, the optical path difference between the two interfering beams is varied so as to produce a time varying amount of flux in the shearing interferogram. The resulting time varying signal obtained by measuring the total amount of flux in the lateral shear interferogram is proportional to the magnitude of the OTF (MTF), while the phase of the OTF can be obtained from the phase of this time varying signal. This letter describes a similar interferometric MTF measurement technique which has the unique features of simplicity and minimum number of optical components required. The technique requires the use of temporally and spatially coherent light, such as provided by most lasers.

2. Optical system

A common lateral shear interferometer for use with temporally coherent sources is a plane parallel plate where the beams reflected off the two parallel surfaces are interfered to give a lateral shear interferogram. The amount of shear is selected by the thickness and tilt of the plate. Likewise, the beam transmitted through the plate and the beam reflected off the two surfaces of the plate can be interferred to give a lateral shear interferogram as illustrated in fig. 1. The resulting interferogram has low fringe visibility; however, it does have the good feature that, unlike the case for the commonly used two reflected beams, without the use of any additional optics the interfering beams do not change direction as the plate is tilted to vary the amount of shear. As illustrated below, for our application the low fringe visibility causes no problem.

As the tilt of the plane parallel plate in fig. 1 is varied, the shear, S, between the beams reflected off the two surfaces is given by

$$S = \frac{t \sin 2\theta}{\sqrt{n^2 - \sin^2\theta}} , \qquad (1)$$

where t is the plate thickness, and θ is the angle of incidence as shown in fig. 1. Fig. 2 gives a plot of the

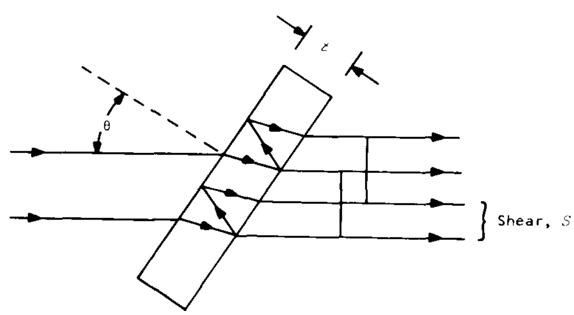

Fig. 1. Plane parallel plate lateral shear interferometer used in transmission.

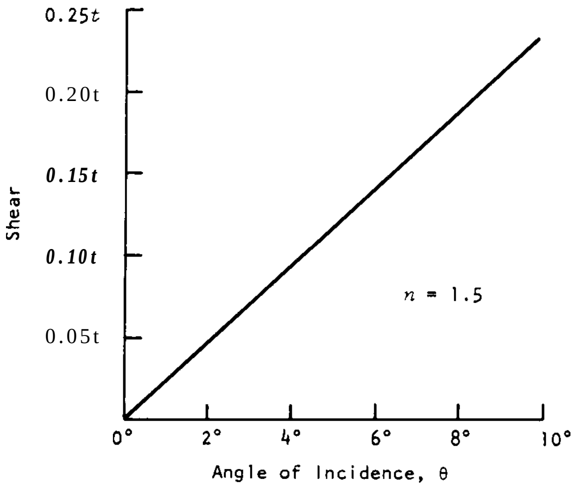

Fig. 2. Shear versus angle of incidence.

shear versus angle of incidence for refractive index n = 1.5.

As the plane parallel plate is tilted, the optical path difference for the two interfering beams changes. If δ is the change in the OPD from its value of 2nt for $\theta = 0^\circ$, δ is given by

$$\delta = 2nt\left[\sqrt{1-\frac{\sin^2\theta}{n^2}}-1\right]. \qquad (2)$$

The technique for using a plane parallel plate for measuring the MTF of an optical system is straightforward. Essentially, collimated light from the system under test is incident upon the plane parallel plate. The light transmitted through the plate is focused onto a detector, whose output is displayed on an oscilloscope as shown in fig. 3. As the plate is rotated, the amplitude of the time varying signal displayed on the oscilloscope and illustrated in fig. 4, is directly proportional to the MTF of the system under test. This time varying signal is reduced since the two interfering beams have different intensities; however, since the only quantity of interest is the variation of the

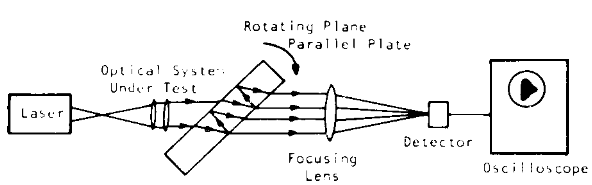

Fig. 3. Interferometric setup for measuring MTF

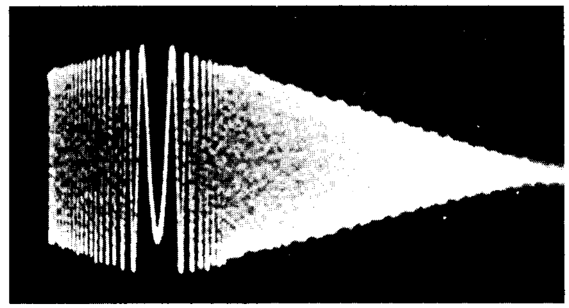

Fig. 4. MTF as displayed on oscilloscope. Vertical axis: MTF; horizontal axis: spatial frequency.

signal as a function of shear or equivalently spatial frequency, this reduction in the time varying signal is unimportant as long as the signal to noise is acceptable. For reasonable angles of incidence the reflection is essentially constant, so no appreciable error is introduced by changes in the intensities of the interfering beams.

The disadvantage with this interferometric technique for measuring MTF is that the modulation frequency of the signal, given by the time rate of change of $(2\pi/\lambda)\delta$, is not constant as a function of θ, but rather it varies as indicated in fig. 4. Also, unless 2nt is equal to a multiple of the wavelength λ, a maximum is not obtained for zero shear, i.e. zero spatial frequency. For this reason, it is often necessary to normalize at a frequency other than zero. For some applications this may not be acceptable; however, when it is acceptable this very simple interferometric technique, which consists only of a plane parallel plate, a motor to rotate the plate, an inexpensive focusing lens, a detector and oscilloscope, is very convenient for measuring the MTF of an optical system.

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OTF measurements with a white light source: an interferometric technique

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The use of lateral shear interferometers for measuring the optical transfer function of an optical system for a white light source is investigated. It is shown that grating lateral shear interferometers fulfill the requirements necessary to perform measurements of both the optical transfer function and the optical coherence function for a white light source. Several possible grating lateral shear interferometers are described.

Introduction

Since Hopkins showed that a lateral shear interferometer can be used to measure the optical transfer function of an optical system,[1] several types of shearing interferometers for measuring either OTF or MTF have been presented.[2-9] The techniques illustrated previously have been useful only in situations where a quasi-monochromatic light source (or equivalently a white light source and a narrowband spectral filter) could be used, while the technique described in this paper can be used with a white light source, making the interferometric measurement of OTF more useful in the testing of a general optical system.

Theory

To determine the requirements for a lateral shear interferometer to measure the OTF of an optical system for a white light source, the relationship between a shearing interferometer output and OTF must be investigated. The one-dimensional optical transfer function of an optical system $D(\nu_x,\psi)$ is represented by the autocorrelation of the pupil function $f(x,y)$:

$$D(\nu_x,\psi) = \frac{1}{C}\iint_A f\left(x+\frac{1}{2}S,y\right)f^*\left(x-\frac{1}{2}S,y\right)dxdy = |D(\nu_x,\psi)|\,\exp[i\theta(\nu_x,\psi)], \quad (1)$$

where the azimuth angle ψ is the direction of shear.

S, the shear of the wavefront, is related to the spatial frequency ν_x, by the expression

$$S = D\lambda(f_{no})\nu_x. \quad (2)$$

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D is the diameter of the pupil, λ is the wavelength of light, f_{no} is the f number of the system, A is the region of overlap of the apertures, and C is the value of the integral when the shear S is zero.

As shown by Hopkins,[1] one method of obtaining the value of the integral in Eq. (1) is to measure the total flux in the interference pattern obtained using a lateral shear interferometer. If δ is the phase difference between the two sheared interfering wavefronts at the center of the interference area and the interfering wavefronts have equal amplitude, the total flux in the interference pattern is

$$F(\delta,S,\psi) = \iint\left|f\left(x+\frac{1}{2}S,y\right) + f^*\left(x-\frac{1}{2}S,y\right)\exp(-i\delta)\right|^2 dxdy = 2C\{1+|D(\nu_x,\psi)|\cos[\delta-\theta(\nu_x,\psi)]\}. \quad (3)$$

The term 2C is the average amount of flux in the two interfering wavefronts. Generally δ is made to vary linearly with time, i.e.,

$$\delta = \omega t + \phi_0, \quad (4)$$

so the second term varies sinusoidally with time having an amplitude $2C|D(\nu_x,\psi)|$ and phase $\phi_0 - \theta(\nu_x, y)$. Therefore, by changing the shear S, both the modulus $|D(\nu_x,\psi)|$ and the phase $\theta(\nu_x,\psi)$ of the OTF can be determined.

If a white light source is to be used with a lateral shear interferometer in an OTF measurement, three conditions must be satisfied. The first two conditions are that ω, the frequency shift between the two interfering wavefronts, and ϕ_0, the constant phase shift between the two interfering wavefronts, must be independent of wavelength. The third requirement is that the lateral shear introduced by the interferometer must be proportional to wavelength. This condition follows from Eqs. (2) and (3), which show

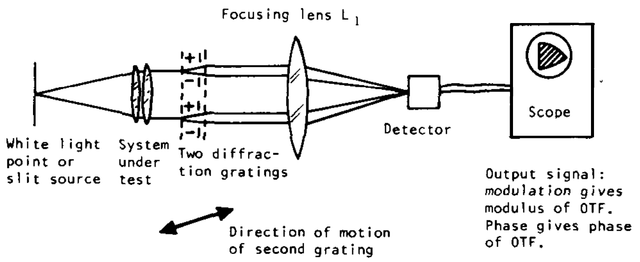

Fig. 1. Grating shearing OTF measuring interferometer for use with white light.

that only if the shear is proportional to wavelength does the flux in the interference pattern give the OTF at a unique spatial frequency ν_x.

Interferometer Details

There exist several grating lateral shear interferometers that possess the three above characteristics. One example is shown in Fig. 1. Collimated light from the optical system under test is incident upon a diffraction grating, which produces +1 and -1 diffraction orders. (The situation where other diffraction orders are produced is discussed below.) A second grating placed some distance from the first grating converts the +l and -1 orders into beams traveling in the same direction as the beam incident upon the first grating. (In general there are beams leaving the second grating at different angles; however, these other beams can be spatially filtered out of the system at the focus of lens L_1.) For small diffraction angles the two beams leaving the second grating are sheared or laterally displaced from one another an amount S given by the equation

$$S = (2\lambda/d)L, \quad (5)$$

where d is the grating period, and L is the distance between the gratings. It should be noted that the shear is proportional to wavelength, a requirement necessary if the OTF measurement is performed using a white light source.

If one grating has a constant velocity v perpendicular to both the grating lines and the direction of propagation of the incident light, one of the first diffraction orders will experience a frequency shift of v/d, and the other first diffraction order will be frequency shifted by an amount -v/d. Therefore, Eq. (4) becomes

$$\delta = 2\pi(2v/d)t + \phi_0, \quad (6)$$

where ϕ_0 depends only on the relative position of the two gratings at the time when t = 0. Therefore, δ is independent of wavelength, and all three conditions required to enable the OTF measuring interferometer to work with white light are satisfied. (It should be noted that if unsymmetrical diffraction orders such as 0 and +l are used, a path difference would exist between the two interfering beams, and hence ϕ_0 would depend upon the wavelength.)

By moving one grating with constant velocity in the direction shown in Fig. 1, both the shear and the phase difference between the two sheared beams are made to vary linearly with time. If the light is directed to the detector the AC portion of the signal coming from the detector is proportional to the magnitude of the OTF, and the phase is related to the phase of the OTF. If an oscilloscope is AC coupled to the detector, the oscilloscope traces out a curve whose envelope is the magnitude of the OTF of the system under test. The phase of the OTF can be measured directly if the grating is moving with constant velocity. Otherwise, the phase can be determined by measuring the difference between the phase of the signal from a known high quality beam and phase of the test beam.

The assumption was made above that the gratings produced only plus and minus first orders, i.e., the gratings have sinusoidal amplitude transmission. The modulation frequency produced by interfering diffraction order m_1 with diffraction order m_2 is equal to ($\nu\,|m_2 - m_1|$). Hence, unless the difference between the two diffraction order numbers is equal to 2, the modulation frequency is different from the modulation frequency of interest, and the unwanted signal can be filtered out in the electronics. In a breadboard constructed using holographically produced bleached gratings, no problems with unwanted signals were experienced. However, if required, unwanted orders could be eliminated as shown in Fig. 2, in which an afocal optical system is placed between the two diffraction gratings, and an aperture stop is placed in the focal plane of the first lens to eliminate unwanted orders. As mentioned above, unwanted orders produced by the second diffraction grating are eliminated by placing the appropriate aperture stop at the focus of the lens focusing the light beams onto the detector.

Many other grating shearing interferometers can be used to measure the OTF of an optical system using a white light source. One example is a double frequency grating described previously[10] and illustrated in Fig. 3. If the two grating periods are d_1 and d_2, the angular shear can be approximated as

$$\Delta\alpha = \lambda\left(\frac{1}{d_2} - \frac{1}{d_1}\right). \quad (7)$$

If the difference between the two grating spatial frequencies is made to vary across the grating, the

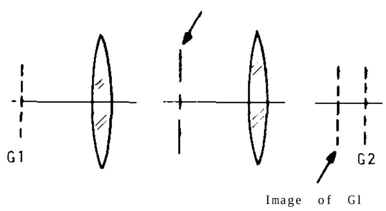

Fig. 2. Afocal lens system to eliminate undesired diffraction orders.

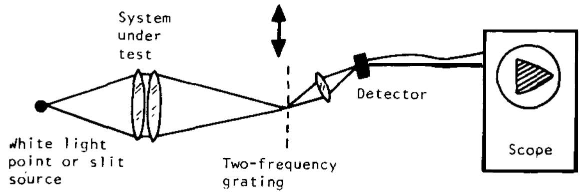

Fig. 3. Two frequency diffraction interferometer having variable difference frequency for measuring OTF.

shear is varied as the grating is translated in a direction perpendicular to the grating lines. At the same time, the phase difference between the interfering beam varies such that the irradiance of the interference pattern is modulated at a frequency

$$\omega = 2\pi v\left(\frac{1}{d_2} - \frac{1}{d_1}\right). \tag{8}$$

Hence, all the conditions for obtaining white light operation are satisfied. As long as the grating spatial frequencies are sufficiently high that the various diffraction orders do not overlap, additional diffraction orders pose no problems. The biggest drawback of the technique is that while the modulation frequency is independent of wavelength, the modulation frequency is a function of the grating spatial frequency difference and thus varies as the OTF spatial frequency is varied. This makes measuring the phase of the OTF more difficult, but certainly not impossible.

A third grating shearing interferometer for measuring the OTF is described in Refs. 11 and 12 for making wavefront measurements. The experimental setup is the same as shown in Fig. 3, except the two-frequency grating is replaced with two identical single frequency gratings placed in close contact. In this situation the shear is produced by rotating the two diffraction gratings with respect to one another. If d is the grating line spacing, f_{no} is the f number of the beam converging upon the gratings, and α is the angle one grating is rotated with respect to the second grating, the percentage shear, i.e., the ratio of the shear distance to the beam radius can, for small diffraction angles, be approximated as

$$\text{percentage shear} = [4f_{no}\sin(\alpha/2)]\lambda/d. \tag{9}$$

The principal advantage of the rotating diffraction grating OTF measuring interferometer over the two interferometers described above is that since the variable shear and modulation are achieved using rotational mechanical motion, instead of translational motion, data can be taken at higher rates.

Discussion

Equation (3) was written for the situation where the light source is either a point or a narrow slit source. For the general situation of an incoherent extended source, Eq. (3) becomes

$$F(\delta, S, \psi) = 2C\{1 + |\gamma(\nu_x, \psi)|\,|D(\nu_x, \psi)| \times \cos[\delta - \beta(\nu_x, \psi) - \theta(\nu_x, \psi)]\}, \tag{10}$$

where $|\gamma(\nu_x, \psi)|$ is the magnitude of the optical coherence function, and $\beta(\nu_x, \psi)$ is its phase. It should be noted that for the same reasons a grating shearing interferometer measures the OTF for a white light source, the grating shearing interferometer can also be used to measure the optical spatial coherence function for a white light source.[13,14]

It is interesting, but not surprising, to note the similarity between the autocorrelation method of measuring the OTF of an optical system as described in this paper and scanning methods for measuring the OTF (see, for example, Ref. 15). In the scanning method for OTF measurement either a sinusoidal grating is used as a test object and the image of the grating is scanned by a slit or point detector, or, equivalently, a slit or point source is used as a test object and the image of the slit is scanned by a sinusoidal grating. Square wave gratings can be used instead of a sinusoidal grating if the fundamental harmonic is selected electronically. Thus, a close relationship is seen between the scanning method of OTF measurement and grating shearing interferometer OTF measurement, especially for the last two grating shearing interferometers described in this paper. The two-frequency diffraction grating lateral shear interferometer is equivalent to using a variable frequency sinusoidal grating in a scanning method measurement. (The shearing interferometer could actually have been a variable frequency sinusoidal diffraction grating; however, it is believed it is easier to make the two-frequency grating than a sinusoidal grating.) Likewise the rotating diffraction grating lateral shear interferometer OTF measurement technique is equivalent to using Moiré fringes in a scanning mode, as described in Ref. 15.

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