Plotting errors measurement of CGH using an improved interferometric method

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An improved interferometric method is described for measuring plotting errors of desk-top computer plotters used to make computer-generated holograms. The plotting errors are measured using moiré fringes formed using Young's fringes and straight lines drawn by the plotters. The Young's fringes are produced by laser beams originating from two single-mode optical fibers. Using this method, plotting errors of Hewlett-Packard 7225A and 7470A plotters are measured.

I. Introduction

Computer-generated holograms[1] (CGH) are very useful for testing aspherical lenses or mirrors since there is no need for using costly null lenses. Accurately drawn CGHs are required for precision tests of aspherical lenses or mirrors. Wyant *et al.*[2] presented the interferometric method for measuring distortion in CGH. In this method, a computer and its plotter draw the simplest CGH, i.e., equispaced straight lines or dots, and this CGH is reduced to a small photo film. Then the distortion in the CGH is measured from interferograms taken by the setup shown in Fig. 1.

A CGH made by a desk-top Hewlett-Packard computer model 85 and a Hewlett-Packard type 7225A plotter was tested by the interferometric method. Figure 2 shows an interferogram taken by the setup shown in Fig. 1. Fringes in Fig. 2 were obtained by interfering plus and minus third-order diffracted beams from the CGH photo film. In advance the setup was checked to have $<1/10\lambda$ wave-front error, so the interferogram was not considered to be seriously influenced by this error.

In Fig. 2 we can see rather distorted fringes having a maximum distortion of ~3/4 fringe. This 3/4 fringe distortion corresponds to 0.15-mm plotting errors in the original CGH. Since the HP 7225A plotter has a 0.025-mm positioning resolution, the 3/4 fringe distortion is larger than expected. The additional distortion is most likely a result of distortion in the photoreduction lens. The conclusion is that it is difficult to measure net plotting errors with the interferometric method unless a distortion-free lens or a known distortion lens is used.

The contribution of this paper is an improved interferometric test method for measuring plotting errors in CGH. The photoreduction step of earlier methods is eliminated by testing the CGH directly using moiré fringes. An innovative way of generating these fringes is to use lines in the CGH and Young's interference fringes produced by two point light sources from single-mode optical fibers. As an application of this new technique, the Hewlett-Packard 7225A and 7470A plotters are tested.

II. Optical Setup

The optical setup of the improved method is shown in Fig. 3. A laser beam from a 1-mW He–Ne laser is expanded by a lens and divided into two beams by a beam splitter (BS). Two beams are condensed by two object lenses and fed into two 5-μm core diam single-mode fibers. The laser beams coming out of the single-mode fibers can be considered as light from ideal point light sources which produce Young's fringes. Two point light sources located near each other can also be obtained by Fresnel's biprism, Billet's split lens, and so on, but these optical elements may have aberrations, and the fringe spacing cannot be easily changed. When Young's fringes are combined with lines in the CGH, moiré fringes are produced.

The CGH was made by drawing straight lines on a transparent sheet with the plotter. To test the plotter error precisely, lines were drawn by dotting-in the orthogonal direction to the lines. The CGH tested was 150 mm in diameter with 125 lines of 1.2-mm line spacing. When the distance between the point light

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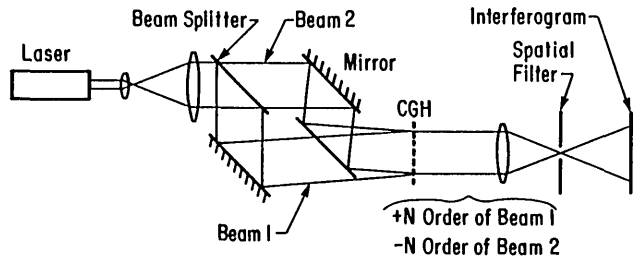

Fig. 1. Optical setup of interferometric method.

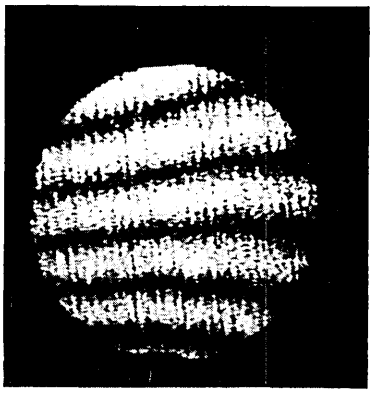
Fig. 2. Interferogram taken by the setup shown in Fig. 1.

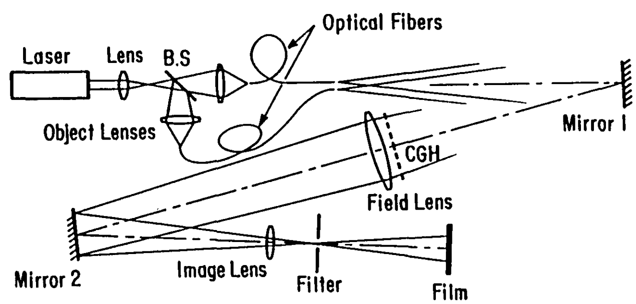

Fig. 3. Optical setup of improved interferometric method.

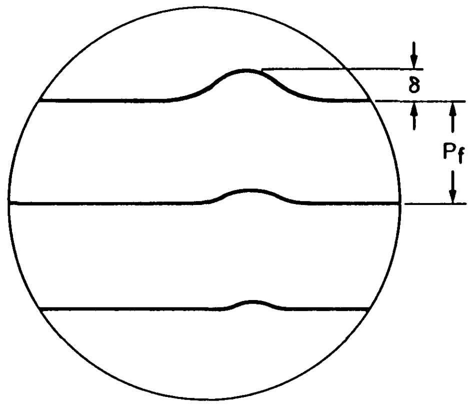

Fig. 4. Model pattern of interference fringes.

sources and the CGH is large enough (see next paragraph), the Young's fringes at the CGH position are straight. If the lines in CGH have plotting errors, the lines are not exactly straight and the moiré fringes are distorted. The plotting errors can be measured from the moiré fringe distortion.

In Fig. 3 the laser beams passed through the CGH are converged by a field lens and an image lens. A spatial filter (small aperture) is placed at the beam convergent point. The filter position and the space between two fibers are adjusted so that the $+N$th-order diffraction beam produced by the laser beam from one of the fibers and the $-N$th-order diffraction beam produced by the laser beam from another fiber may pass through the filter.

After passing through the filter, the $\pm N$th-order diffraction beams reimage the CGH image on a photo film and interfere with each other. We can also see some fringes even if the filter is removed or a ground glass is placed just behind the CGH, but these fringes do not have good visibility since they are moiré fringes between Young's fringes and a high-frequency component ($2N$ times) of the CGH lines. Since a filter eliminates the high-frequency CGH line image and the Young interference fringes, the fringes at the photo film have better visibility than the above moiré fringes at the CGH. Figure 4 shows a model pattern of the interference fringes at the photofilm. If one of the fringes has distortion of $\delta(x,y)$ at the (x,y) position, the plotting error ε (X,Y) can be expressed by

$$\varepsilon(X,Y) = \frac{\delta(x,y)}{P_f} \cdot \frac{P_c}{2N}, \tag{1}$$

where (x,y) are the coordinates on the photo film, (X,Y) are the coordinates on the CGH, P_c is the line spacing on the CGH, and P_f is the fringe spacing on the photo film.

The diameter of the field lens shown in Fig. 3 has to be at least as large as the CGH. The diameter of the CGH depends on the plotter sheet size. In general the sheet size for a desk-top computer plotter is 28 × 21 cm (11 × 8½ in.), so the diameter of the CGH may be from 100 to 180 mm. The field lens is required to have a fairly large diameter, but it is not difficult to get such a large field lens since it does not need to be good quality. Both of the $\pm N$th-order diffraction beams pass through the field lens at the same position, thus thickness variation in it does not affect the interference fringes. Also, aberration of the field lens affects the reimage little. In the setup in Fig. 3, a telescopic lens which has a 150-mm diam and a 1500-mm focal length was used for the field lens. Mirrors 1 and 2 in Fig. 3 were used to make the setup more compact.

III. Distortion of Young's Fringes

In this method the Young's interference fringes have to be straight and sufficiently equispace, because they are standard lines for measuring plotting errors. Figure 5 shows an optical schematic of Young's interference by two point light sources S_1 and S_2. If a distance l between point sources and a screen is not sufficiently large, the Young's fringes may not be straight, but rather they may be curved as shown in Fig. 6. Distortion of Young's

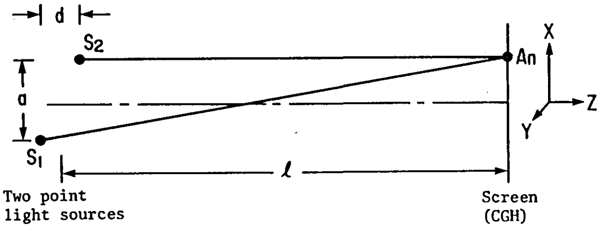

Fig. 5. Optical schematic of Young's interference.

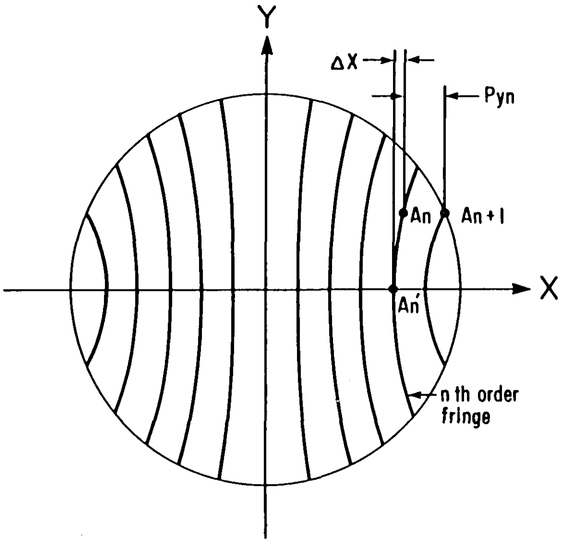

Fig. 6. Distortion of Young's fringes.

fringes was estimated as shown below, and the limit of the distance l was obtained from this estimate.

When point A_n is of the nth-order bright Young's fringe, an optical pass difference (OPD) between S_1 to A_n and S_2 to A_n is written as[3]

$$S_1A_n - S_2A_n = n\lambda \simeq \frac{aX + dl}{\sqrt{X^2 + Y^2 + l^2}}, \tag{2}$$

where a and d are distances between two point sources in the X and Z directions, respectively (see Fig. 5). The spacing P_Y of the Young's fringes is

$$P_y = \frac{\lambda l}{a}, \tag{3}$$

when $X = 0$ and $Y = 0$. It is assumed that point $A_n(X_n,Y_n)$ is of an nth-order fringe and point $A'_n(X'_n,0)$ is at an intersection of this fringe with the X axis as shown in Fig. 6. The distortion of Young's fringe is defined as ΔX, which is a difference between X_n and X'_n ($\Delta X = X_n - X'_n$). From Eq. (2),

$$\frac{Ax_n + dl}{\sqrt{X_n^2 + Y_n^2 + l^2}} = \frac{aX'_n + dl}{\sqrt{X_n'^2 + l^2}} \tag{4}$$

is obtained. Substituting $X'_n = X_n - \Delta X$ into Eq. (4) yields

$$\frac{aX_n + dl}{\sqrt{X_n^2 + Y_n^2 + l^2}} = \frac{a(X_n - \Delta X) + dl}{\sqrt{(X_n - \Delta X)^2 + l^2}}. \tag{5}$$

By expanding Eq. (5) and neglecting small terms of ΔX^2,ΔX is written as

$$\Delta X = \frac{\left(X_n + \frac{d}{a} l\right) Y_n^2}{2\left(l^2 - \frac{d}{a} X_n l\right)}. \tag{6}$$

Usually $(d/a)X_n l \ll l^2$; then ΔX is approximately

$$\Delta X = \frac{\left(X_n + \frac{d}{a} l\right) Y_n^2}{2l^2}. \tag{7}$$

The distortion of Young's fringe at point $A(X_n,Y_n)$ can be calculated by Eq. (7).

Next the maximum value of $|\Delta X|$ will be estimated. The Young's fringe distortion $\Delta X(X,Y)$ is considered only in the CGH area of diameter $2R$. The value of $|\Delta X|$ is maximum when the values of X and Y satisfy Eqs. (8) and (9):

$$f(X,Y) = X^2 + Y^2 - R^2 = 0, \tag{8}$$

$$\frac{\partial \Delta X(X,Y)}{\partial X}\frac{\partial f(X,Y)}{\partial Y} - \frac{\partial \Delta X(X,Y)}{\partial Y}\frac{\partial f(X,Y)}{\partial X} = 0. \tag{9}$$

From Eqs. (8) and (9) the following is obtained:

$$X = -\frac{1}{3}\frac{dl}{a} + \frac{1}{3}\sqrt{\frac{d^2}{a^2} l^2 + 3R^2},$$
$$Y^2 = \frac{2}{3}R^2 - \frac{2}{9}\frac{d^2}{a^2} l^2 + \frac{2}{9}\frac{d}{a} l \times \sqrt{\frac{d^2}{a^2} l^2 + 3R^2} \quad \text{when } d > 0; \tag{10}$$

$$X = -\frac{1}{3}\frac{dl}{a} - \frac{1}{3}\sqrt{\frac{d^2}{a^2} l^2 + 3R^2},$$
$$Y^2 = \frac{2}{3}R^2 - \frac{2}{9}\frac{d^2}{a^2} l^2 - \frac{2}{9}\frac{d}{a} l \times \sqrt{\frac{d^2}{a^2} ;l^2 + 3R^2} \quad \text{when } d < 0. \tag{11}$$

Then $|\Delta X|$ max is obtained from Eqs. (7), (10), and (11):

$$|\Delta X|\ \max = \left|\frac{1}{3}\frac{d}{al} R_2 - \frac{1}{27}\frac{d^3}{a^3} l\right| + \left(\frac{1}{27}\frac{d^2}{a^2} + \frac{1}{9}\frac{R^2}{l^2}\right) \times \sqrt{\frac{d^2}{a^2} l^2 + 3R^2} \tag{13}$$

Here the diameter of the CGH is 150 mm, so $R = 75$ mm, the line spacing P_c is 1.2 mm, diffraction order N is selected to be 3, and the Young's fringe space P_y is $P_c/2N = 0.2$ mm. When $\lambda = 0.633 \times 10^{-3}$ mm, a can be calculated from Eq. (3) as

$$a = \frac{\lambda l}{P_y} = 3.17 \times 10^{-3} l, \tag{13}$$

The value of d is an alignment error of setting the point sources (see Fig. 5). The value of $|d|$ could be <0.05 mm in the setup shown in Fig. 3. Substituting these values for R, a, and d into Eq. (12) yields

$$|\Delta X|_{\max} = \frac{1.12 \times 10^5}{l_2}. \tag{14}$$

Both plotters under test have a resolution of 0.025 mm. To test plotting errors precisely, $|\Delta X|_{max}$ must be less than the plotter resolution. From Eq. (14), the distance l between the point sources and the CGH is required to be more than 2120 mm. In the actual setup, l was 2550 mm. In this case, $|\Delta X|_{max}$ may be <0.017 mm in the whole CGH area.

If the CGH is plotted with this distortion of Young's fringes taken into account, the distance l can be short and the setup becomes more compact. This case requires the field lens to have a shorter focal length than before. It is difficult to get such a field lens which has a large diameter and a short focal length.

IV. Uniformity of the Young's Fringe Spacing

The Young's fringes are not absolutely equispaced when the distance l is small. Variation in spacing as a function of fringe position was estimated as the following: In Fig. 6, point A_{n+1} is of the nth-order Young's fringe and has a coordinate of (X_{n+1}, Y_n). The ordinate Y_n of the point A_{n+1} is the same as that of point A_n. Let P_{yn} be the distance between the nth- and n + 1th-order Young's fringe. The spacing P_{yn} is written as

$$P_{yn} = X_{n+1} - X_n, X_{n+1} = X_n + P_{yn}. \quad (15)$$

From Eq. (2), an equation of

$$\frac{aX_{n+1} + dl}{\sqrt{X_{n+1}^2 + Y_n^2 + l^2}} - \frac{aX_n + dl}{\sqrt{X_n^2 + Y_n^2 + l^2}} = \lambda \quad (16)$$

can be obtained. Equation (15) is substituted into Eq. (16) as

$$\frac{a(X_n + P_{yn} + dl)}{\sqrt{(X_n + P_{yn})^2 + Y_n^2 + l^2}} - \frac{aX_n + dl}{\sqrt{X_n^2 + Y_n^2 + l^2}} = \lambda. \quad (17)$$

From Eq. (17), P_{yn} is derived by neglecting small terms:

$$P_{yn} = \frac{\lambda\sqrt{X_n^2 + Y_n^2 + l^2}}{a} + \frac{\lambda X_n(aX_n + dl)}{a^2 l}. \quad (18)$$

When $X_n = 0$, $Y_n = 0$. P_{yn} is equal to P_y of Eq. (3). Let ΔP_y be a difference between P_{yn} and P_y. From Eqs. (18) and (3), ΔP_y is obtained by neglecting small terms:

$$\Delta P_y = P_{yn} - P_y = \frac{\lambda a(3X_n^2 + Y_n^2) + 2\lambda d X_{nl}}{2a^2 l}. \quad (19)$$

The value of $|\Delta P_y|$ is a maximum when $X_n = R, Y = 0$ for $d > 0$ or $X_n = -R, Y = 0$ for $d < 0$ in the CGH area with a diameter of $2R$. The same values as given above (i.e., $R = 75$ mm, $\lambda = 0.633 \times 10^{-3}$ mm, $d = 0.05$ mm, $l = 2550$ mm, $a = 3.17 \times 10^{-3}\, l$) are substituted into Eq. (19); then

$$|\Delta P_y|_{max} = 0.0003 \text{ mm} = 0.3\ \mu\text{m}. \quad (20)$$

This value is smaller than the plotter's resolution of 0.025 mm, so the uniformity of the Young's fringe space does not have to be considered.

According to distortion and space uniformity of Young's fringes and ambiguity of reading interference fringe position, the accuracy of measuring plotting error was expected to be 0.02 mm. This accuracy is sufficiently precise, since 0.02 mm is 1/60 of CGH line spacing and is smaller than the plotter's resolution.

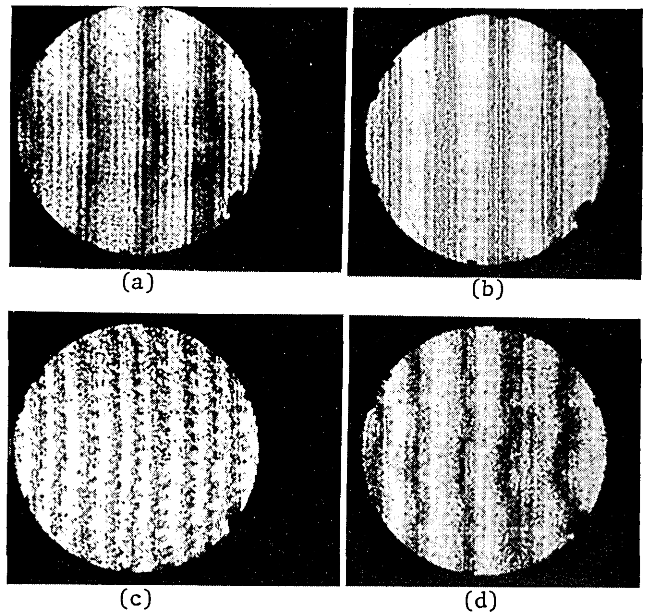

Fig. 7. Interferograms with fringes parallel to CGH lines.

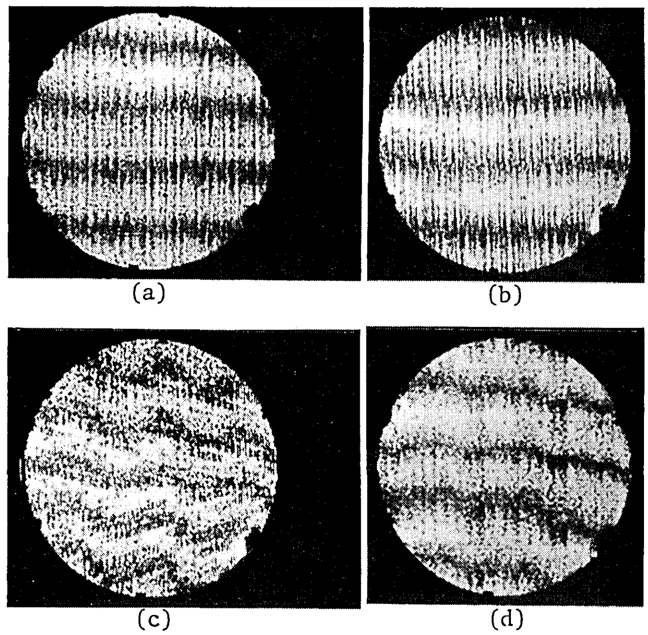

Fig. 8. Interferograms with fringes perpendicular to CGH lines.

V. Experiment

Plotting errors of the CGHs drawn by the HP 7225A and 7470A plotters were tested using this improved interferometric method. Figures 7 and 8 show interferograms of experimental result. Conditions for taking these interferograms are shown in Table I. As mentioned above, the line spacing of the CGHs is 1.2 mm, and the diameter is 150 mm.

Table I. Conditions for Taking Interferograms

Interferogram number	CGH was drawn by	CGH lines are parallel to	Interference fringes are
Fig. 7 (a)	7225A	Y axis	Parallel to CGH lines
(b)		X	
(c)	7470A	Y	
(d)		X	
Fig. 8 (a)	7225A	Y	Perpendicular to CGH lines
(b)		X	
(c)	7470A	Y	
(d)		X	

Table II. Straightness of CGH Lines

Interferogram number (Fig. 7)	Maximum distortion of fringes (δ_{max}/Pf)	Straightness of CGH lines (ε_{max})
(a)	0.16 fringe	0.03 mm
(b)	<0.1	<0.02
(c)	0.25	0.05
(d)	0.32	0.06

Table IV. Maximum Distortion

Interferogram number (Fig. 8)	Fringe distortion (δ_{max}/P_f)	CGH line distortion ε_{max} (mm)
(a)	0.17	0.034
(b)	0.18	0.036
(c)	0.40	0.080
(d)	0.27	0.054

Table III. Difference of Plotter's Scale Deviation From 7225A X-Axis Scale

Interferogram number (Fig. 7)	Scale	Mean spacing of fringes (Pf) (mm)	Difference of deviation for 150 mm (D) (mm)
(a)	7225A X axis	35	—
(b)	7225A Y axis	33	0.05/150
(c)	7470A X axis	15	1.14/150
(d)	7470A Y axis	30	0.14/150

Table V. Line Space Fluctuation

Interferogram number (Fig. 8)	Fringe slope		Line space fluctuation ΔP_c (μm)
	θ_{max}	θ_{min}	
(a)	0	−0.003	0.7
(b)	0	−0.009	2.2
(c)	0.01	−0.014	5.8
(d)	0.01	−0.009	4.6

In Fig. 7 interference fringes are parallel to the CGH lines, so straightness of fringes corresponds to that of the lines and can be calculated using Eq. (1). Measurement results are shown in Table II. From Table II it is seen that the CGH lines of Fig. 7(b) had the best straightness and those of Fig. 7(d) were the worst. Even the worst straightness is only a few times that of the plotting resolution of 0.025 mm, so all the CGHs are considered to have lines of good straightness.

Since the four interferograms shown in Fig. 7 were taken under the same conditions, the difference in the interference fringe spacing for the different interferograms show that there was a difference in the mean line spacing of the CGHs. This difference was caused by a small change in the plotter's scale. For example, in CGH coordinates, Fig. 7(a) has a 35-mm fringe spacing and Fig. 7(b) has 30-mm fringe spacing. From Eq. (1), the difference of scale deviation D is written as

$$D = 2R\left(\frac{1}{P_f'} - \frac{1}{P_f}\right)\frac{P_c}{2N} = 150 \times \left(\frac{1}{30} - \frac{1}{35}\right) \times 0.2 = 0.14 \text{ mm}. \tag{21}$$

This 0.14 mm means that the Y-axis scale of the 7470A plotter is 0.14 mm longer than the X-axis scale of the 7225A plotter for 150 mm length. (In this case the Young's fringe spacing was adjusted to be a little smaller than the CGH line spacing.)

Table III gives plotter scale deviation. Table III shows that 7225A X and Y axes and a 7470A Y axis have almost the same deviation and may have accurate scales, but the deviation of a 7470A X axis is rather different from the others. This deviation is so large that it could be checked using conventional techniques. This deviation will cause significant errors in drawing a CGH. Usually a plotter pen is positioned by a sliding arm, but the positioning of the 7470A X axis is done by shifting a sheet of paper with a couple of rollers. The diameter of the rollers is ~15 mm. If the diameter is 0.11 mm larger than its designed size or a sheet slips from the roller's movement, the 1.14-mm/150-mm deviation will happen.

In Fig. 8 the interference fringes are perpendicular to the CGH lines. The CGH line distortion can be calculated from the fringe distortion using Eq. (1). Table IV shows data of the maximum fringe distortion $\delta_{\max}/P_f$ and the maximum line distortion $\varepsilon_{\max}$ in an interferogram. Figure 8(a) has the smallest distortion, and Fig. 8(c) has the largest. The difference between the smallest and the largest value is not so large, but there is a great difference in fringe undulation. Fringes in Fig. 8(a) are smooth, but those in Fig. 8(c) are very rough. A differential of fringe distortion has to be taken into account.

The differential of fringe distortion is equal to the slope of the fringe, and the slope of the fringe corresponds to the CGH line spacing, so fringe undulation corresponds to a line-spacing fluctuation. The difference ΔP_c between the maximum and minimum line spacing will be obtained. Line slope θ is defined as fringe distortion divided by a distance converted into the CGH dimension. Let $\theta_{\max}$ and $\theta_{\min}$ be the maximum and the minimum values of the fringe slope θ, respectively, in an interferogram. The line-spacing fluctuation ΔP_c is calculated by

$$\Delta P_c = (\theta_{\max} - \theta_{\min}) \frac{P_c^2}{2N} . \tag{22}$$

Table V shows data of the line-spacing fluctuation. In Table V there is a big difference between the values of Figs. 8(a) and (c). Since the fluctuation accumulates with many lines being drawn, it yields a large distortion.

In Fig. 8(c) the interference fringes curve periodically with a period of ~48 mm. Since this length is almost the same as the circumference length of the plotter rollers, the fringe undulation may be caused by the off-centered axles of the rollers. A distance from the off-centered axle to the exact center can be estimated at 0.018 mm from the value of ΔP_c.

From these results it is seen that, at least for the two plotters tested, the HP 7225A type plotter is somewhat better than the 7470A plotter for drawing CGHs.

VI. Conclusion

An improved interferometric method for testing plotters used to make CGHs was presented. In this method, plotting errors were measured using moiré fringes between straight lines in the CGH and Young's fringes produced by laser beams originating from two single-mode optical fibers. It was shown that, for 150-mm diam CGHs, Young's fringes are sufficiently straight if the distance between fibers and a CGH is longer than 2120 mm. This method was used to measure CGH line straightness, distortion, spacing fluctuation, and plotter's scale deviation for Hewlett-Packard 7225A and 7470A plotters. From the measurements it was revealed that, at least for the two plotters tested, the 7255A type plotter is somewhat better than the 7470A plotter for drawing CGHs.

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