Undesired Light in a Reconstructed Hologram Image Caused by the Nonlinearity of the Photographic Process

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Theory and experiment show that for a hologram object of two or more object points, the nonlinearity of the photographic process causes reconstructed images in addition to both the desired reconstructed image and the higher order reconstructed images. It is theoretically and experimentally shown that for a plane object parallel to the hologram plane, some of these undesired images may be focused in the plane of the desired reconstructed image.

Introduction

It is well known that if a nonlinear photographic process is used to record a hologram, higher order reconstructed images of the original object may result. These higher order images will not interfere with the first-order images, if their respective spatial frequency power spectra do not overlap.[1]

There are also other spurious images formed because of the nonlinearity of the photographic process.[2,3] It is the purpose of this paper to determine theoretically and confirm experimentally the position of some of these spurious images and to show that for a plane object parallel to the hologram plane, undesired images may be focused in the plane of the desired reconstructed image. A study has been made to determine under what conditions these undesired images are bright enough to degrade the desired reconstructed image.

Theory

In an earlier paper,[4] the authors showed that for hologram exposures made along the straight line portion of the H&D curve of a photographic plate, T_A, the amplitude modulus of the transmission of a hologram made of N object points exposed simultaneously (conventional holograms) is given by

$$T_A = (E'C)^{\gamma/2}(1 - P + Q - F + W + \ldots), \qquad (1)$$

where

$$P = \tfrac{1}{2}\gamma C(a + b + d),$$

$$Q = \tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma + 1)C^2(1/2!)[a^2 + b^2 + d^2 + 2(ab + ad + db)],$$

$$F = \tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma + 1)(\tfrac{1}{2}\gamma + 2)C^3(1/3!)\{a^3 + b^3 + d^3 + 3[a^2(b + d) + b^2(a + d) + d^2(a + b)] + 6abd\},$$

$$W = \tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma + 1)(\tfrac{1}{2}\gamma + 2)(\tfrac{1}{2}\gamma + 3)C^4(1/4!) \times (\{a^4 + b^4 + d^4 + 4[a^3(b + d) + b^3(a + d) + d^3(a + b)] + 6(a^2b^2 + a^2d^2 + b^2d^2) + (12a^2bd + b^2ad + d^2ab)\} + \ldots),$$

and

$$C = \left[E_o + t\left(I_R + \sum_{n=1}^{N} A_n^2\right)\right]^{-1}, \quad a = tR^*\sum_{n=1}^{N} A_n,$$

$$b = tR\sum_{n=1}^{N} A_n^*, \quad d = t\sum_{i=1}^{N}\ \sum_{\substack{j=1\\ j\neq i}}^{N} A_iA_j^*,$$

where E' is the inertia of the photographic plate, γ is the slope of the straight line portion of the H&D curve of the photographic plate, R is the complex amplitude of the reference beam at the point (X,Y) on the photographic plate, A_n is the complex amplitude at the point (X,Y) due to the nth object point, $I_R = R^2$ is the illuminance of the reference beam, and E_o is any initial exposure (to a uniform illuminance distribution) used to bring the photographic plate to the straight line portion of the H&D curve.

Let the source of the reference wavefront have the coordinates X_R, Y_R, Z_R and the nth object point have the coordinates X_n, Y_n, Z_n. To simplify matters slightly, we will let points in the plane in which the hologram is made have the coordinates X,Y,0. That is, for all points in the hologram plane, Z = 0.

Then at a point (X,Y) in the hologram plane, the amplitude of the reference wave and the amplitude of the wavefront from the nth object point are given by

$$R\exp i\{k[(X_R - X)^2 + (Y_R - Y)^2 + Z_R^2]^{\frac{1}{2}} + \phi\}$$

and

$$A_n\exp i\{k[(X_n - X)^2 + (Y_n - Y)^2 + Z_n^2]^{\frac{1}{2}} + \phi_n\}, \qquad (2)$$

respectively. ϕ_R and ϕ_n are constant phases across the

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hologram plane and can be set equal to zero in the following analysis. As usual, $k = 2\pi/\lambda$. Note that since we are interested only in how the phase depends on position, and not on time, we have omitted the periodic time dependence.

If Z_R and Z_n are large compared to X_R-X, Y_R- Y, X_n-X and Y_n- Y, the phase factors can be expanded using the binomial theorem and the amplitudes of the reference wave and the nth object wavefront are given by

$$R \exp ik\left[Z_R + \frac{(X_R - X)^2 + (Y_R - Y)^2}{2Z_R}\right]$$

and

$$A_n \exp ik\left[Z_n + \frac{(X_n - X)^2 + (Y_n - Y)^2}{2Z_n}\right]. \quad (3)$$

We will assume that the phase distribution of the reconstructing beam across the hologram is identical with the reference beam used to make the hologram. If the complex amplitude of the reconstructing beam is ***D***, the total amplitude of all the light transmitted by the hologram is $\boldsymbol{DT_A}$. The amount of light in the reconstruction of the kth object point is given by the terms of $\boldsymbol{DT_A}$ equal to a real quantity times R*AK.[4]

At the present time we are not interested in the light in the reconstruction of the original object points, but rather we are interested in undesired reconstructed points focused near the desired reconstructed object points. In particular, we want to show that if we have a plane object parallel to the hologram plane, there are undesired reconstructed images focused in the same plane as the desired reconstructed image.

We know from previous work[4] that 2ad is the only part of term Q in Eq. (1) that gives us a contribution to the desired reconstructed image. It gives us a contribution only if j is equal to ***n***. When j is not equal to n we have

$$2ad = 2t^2R^* \sum_{n=1}^{N} \sum_{i=1}^{N} \sum_{\substack{j=1\\ j\neq i\\ j\neq n}}^{N} A_nA_iA_j^*$$

$$= 2t^2R^* \sum_{n=1}^{N} \sum_{i=1}^{N} \sum_{\substack{j=1\\ j\neq i\\ j\neq n}}^{N} A_nA_iA_j \exp ik\left\{Z_n + Z_i - Z_j + \frac{[(X_n - X)^2 + (Y_n - Y)^2]}{2Z_n} + \frac{[(X_i - X)^2 + (Y_i - Y)^2]}{2Z_i} - \frac{[(X_j - X)^2 + (Y_j - Y)^2]}{2Z_j}\right\}. \quad (4)$$

If the wavefront of the reconstructing beam has the same phase distribution across the hologram plane as the original reference beam wavefront, the phase of ***R**** will be canceled when the hologram amplitude transmission is multiplied by the amplitude of the reconstructing wavefront. Thus, any reconstructed point represented by Eq. (4) will be focused at a point (X',Y',Z'), if there exists an X',Y', and Z', such that for all X and Y

$$Z_n + Z_i - Z_j + \frac{[(X_n - X)^2 + (Y_n - Y)^2]}{2Z_n} + \frac{[(X_i - X)^2 + (Y_i - Y)^2]}{2Z_i} - \frac{[(X_j - X)^2 + (Y_j - Y)^2]}{2Z_j} = \text{const} + \frac{[(X' - X)^2 + (Y' - Y)^2]}{2Z'}. \quad (5)$$

Since Eq. (5) must be true for every point (X,Y) on the photographic plate, Eq. (5) can be broken into separate equations for X and Y. The equation involving Y is

$$\frac{(Y_n - Y)^2}{2Z_n} + \frac{(Y_i - Y)^2}{2Z_i} - \frac{(Y_j - Y)^2}{2Y_j} = \frac{(Y' - Y)^2}{2Z'}. \quad (6)$$

The equation involving X has the same form as Eq. (6). It will not be necessary to use the equation involving X, since the equations giving Y' and X' both have the same form.

If Eq. (6) is true for all values of Y, the derivatives of the two sides of the equation with respect to Y must be equal. Thus,

$$\frac{Y_n - Y}{Z_n} + \frac{Y_i - Y}{Z_i} - \frac{Y_j - Y}{Z_j} = \frac{Y' - Y}{Z'}. \quad (7)$$

Again, since this is true for all Y, the terms involving the first power of Y and the terms not involving Y must be satisfied independently, leading to two equations:

$$\frac{1}{Z_n} + \frac{1}{Z_i} - \frac{1}{Z_j} = \frac{1}{Z'} \quad (8a)$$

and

$$\frac{Y_n}{Z_n} + \frac{Y_i}{Z_i} - \frac{Y_j}{Z_j} = \frac{Y'}{Z'}, \quad (8b)$$

where it should be remembered that j # n. and j # i.

If we have a plane object parallel to the hologram plane, $Z_n = Z_i = Z_j$ and Eq. (8a) shows that $Z' = Z_j$. That is, if we have a plane object parallel to the hologram plane, the nonlinearity of the photographic process causes undesired reconstructed image points focused in the same plane as the desired reconstructed image points. For this case, Eq. (8b) reduces to

$$Y_n + Y_i - Y_j = Y', \quad (9)$$

where j # ***i*** and j # n. Y' locates the undesired image in the reconstruction plane.

As given earlier,[4] the only part of term ***F*** that gives us the desired reconstructed image points are the terms a^2b and d^2a:

$$a^2b = t^2I_RR^* \sum_{n=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} A_nA_iA_j^* \quad (10)$$

and

$$d^2a = t^3R^* \sum_{i=1}^{N} \sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{u=1}^{N} \sum_{\substack{v=1\\ v\neq u}}^{N} \sum_{n=1}^{n} A_iA_j^*A_uA_v^*A_n. \quad (11)$$

For Eq. (10) to give us a desired reconstructed image point, j has to be equal to either ***i*** or n. If j is not equal to either i or n, we have an equation with the same phase factors as Eq. (4). Therefore, the undesired reconstructed image points will be at the locations given by Eq. (9).

Equation (11) gives a desired reconstructed image point if j is equal to either u or n and v is equal to i or if j is equal to u and v is equal to either i or n. If these conditions are not satisfied, Eq. (11) gives us an undesired reconstructed image point. By comparing Eq. (11) with Eq. (4) and looking at Eqs. (8a) and (8b), it is seen that the coordinates (Y',Z') of the undesired reconstructed image points are given by the equations:

$$\frac{1}{Z_i} - \frac{1}{Z_j} + \frac{1}{Z_u} - \frac{1}{Z_v} + \frac{1}{Z_n} = \frac{1}{Z'} \qquad (12a)$$

and

$$\frac{Y_i}{Z_i} - \frac{Y_j}{Z_j} + \frac{Y_u}{Z_u} - \frac{Y_v}{Z_v} + \frac{Y_n}{Z_n} = \frac{Y'}{Z'}, \qquad (12b)$$

where $j \neq i$, $v \neq u$, and the subscripts are such that for at least one term with a minus sign in front of it there is not an identical term with a plus sign in front of it.

Again we see that if we have a plane object parallel to the hologram plane there are undesired reconstructed images focused in the same plane as the desired reconstructed images. The coordinate Y' is found by solving the equation

$$Y_i - Y_j + Y_u - Y_v + Y_n = Y', \qquad (13)$$

where Eq. (13) has the same restrictions on the subscripts as Eqs. (12).

In a similar fashion, the position of the undesired images due to term W in Eq. (1) can be determined. The only part of term W that gives desired reconstructed image points are the terms $12a^2bd$ and $4d^3a$.[3] The images produced by term $12a^2bd$ have the coordinates given by Eq. (12), and the images produced by term $4d^3a$ have coordinates given by the equations :

$$\frac{1}{Z_i} - \frac{1}{Z_j} + \frac{1}{Z_u} - \frac{1}{Z_r} + \frac{1}{Z_s} - \frac{1}{Z_t} + \frac{1}{Z_n} = \frac{1}{Z'} \qquad (14)$$

and

$$\frac{Y_i}{Z_i} - \frac{Y_j}{Z_j} + \frac{Y_u}{Z_u} - \frac{Y_r}{Z_r} + \frac{Y_s}{Z_s} - \frac{Y_t}{Z_t} + \frac{Y_n}{Z_n} = \frac{Y'}{Z'}.$$

where $j \neq i$, $i' \neq u$, $t \neq s$, and again the subscripts are such that for at least one term with a minus sign in front of it, there is not an identical term with a plus sign in front, of it.

For a plane object' parallel to the hologram plane, Y' is given by

$$Y_i - Y_j + Y_u - Y_v + Y_s - Y_t + Y_n = Y', \qquad (13)$$

where Eq. (15) has the same restrictions on the subscripts as Eq. (14).

Hence, it is seen that all the terms, except, P, of Eq. (1) that contribute to the light in the desired reconstructed points in the image plane may also cause undesired reconstructed points focused in the image plane.

The above analysis was made for holograms made of several object points exposed simultaneously (conventional holograms. Similar results are obtained for holograms made if several object points exposed sequentially (synthetic holograms).

For synthetic holograms the equation giving the amplitude transmission of the hologram is of the same form as Eq. (l), but C, a, b, and d become

$$C = \left[E_o + \sum_{n=1}^{N} t_n(I_{Rn} + I_n)\right]^{-1},$$
$$a = \sum_{n=1}^{N} t_n R_n A_n^*, \qquad (16)$$
$$b = \sum_{n=1}^{N} t_n R_n^* A_n,$$
$$d = 0,$$

where R_n and A_n are the complex amplitudes of the reference and object beams, respectively, for the nth hologram exposure, t_n is the exposure time for the nth exposure, $I_n = A_n^2$ and $I_{Rn} = R_n^2$.[4]

The values a, b, and C have the same form for both synthetic and conventional holograms. The value d, which is not zero for conventional holograms, is equal t zero for synthetic holograms. Thus, the undesired image points that were a result of term d being in he transmission function for conventional hologram not present for synthetic holograms. The other undesired image points found above for conventional holograms are also present for synthetic holograms.

The undesired images we have been concerned with here are a result of both $\gamma \neq -2$ and the hologram being a hologram of more than one object point,. It should be remarked again that these undesired images are not the higher order images that result from $\gamma \neq -2$ that are often mentioned in the literature.[1] The higher order images usually mentioned are given by such terms as the a^2 and a^3 terms of Eq. (10).

Experimental

The procedure used for making the holograms has been described in detail earlier.' Briefly, it should be mentioned that the holograms were made using Kodak 649-F photographic plates. The plates were developed in D-19 at 68°F for 5 min. Before each hologram was made, the 649-F plate was preflashed to a General Electric AG-1 flashbulb to obtain a straight H&D curve. As far as the locations of the undesired reconstructed image points are concerned, preflashing the plates to the AG-1 flashbulb is unimportant.

A collimated wavefront was used for both the reference and reconstructing beam for the holograms. When the hologram object, consisted of point sources, all the point sources were put in a plane parallel to the hologram plane. Likewise, for the experiment where the hologram object was a piece of ground glass, the ground glass was in a plane parallel to the hologram plane.

Figure 1 shows the reconstructions of three conventional holograms. For the case of the hologram of two object points, the desired reconstructed image points have the Y' coordinates 1 and - 1. For the reconstructions of the hologram of three object points

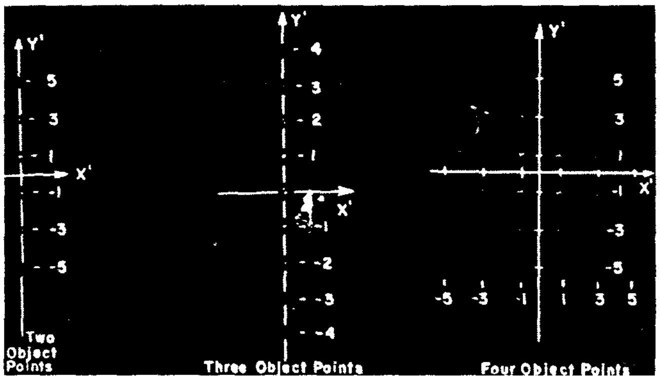

Fig. 1. Reconstructed image of hologram of two, three, and four object points.

the desired reconstructed image points have the Y' coordinates of approximately 1, 0, and - 1, and for the hologram of four object points the desired reconstructed points have the (X', Y') coordinates of approximately (l,l), (1,-l), (-1,-l), and (-1,l). For all three holograms the beam balance ratio (reference beam illuminance divided by object beam illuminance) was nearly one, the preexposed density was about 0.2, and the average hologram density was approximately 1.

Figure 1 shows that as the theory predicted, for a plane object parallel to the hologram plane, there are undesired reconstructed points focused in the same plane as the desired reconstructed image. It will now be shown that Eqs. (9); (13), and (15) correctly predict the X' and Y' coordinates of the undesired reconstructed points.

As Eqs. (9), (13), and (15) predict,, all the reconstructed image points in the reconstruction of the hologram of two object points are in a straight line. Also, as the theory predicts, the spacing between the reconstructed image points is equal to the spacing of the two original object points. Equations (9), (13), and (15) predict the undesired reconstructed points at the Y' coordinates 3 and -3. The points at Y equal to 5 and -5 are predicted by both Eq. (13) and Eq. (15). That is, terms Q, F, and W in Eq. (1) are responsible for the undesired reconstructed points having the Y' coordinates 3 and -3, but only terms ***F*** and W are responsible for the undesired reconstructed points having the Y' coordinates 5 and -5. Equation (15) also predicts two undesired reconstructed points having the Y' coordinates 7 and -7. Although the two points cannot be seen in Fig. 1, they were observed in the laboratory.

The desired reconstructed image points in the reconstruction of the hologram of three object point's shown in Fig. 1 have the (X',Y') coordinates of (0,-l), (0,0), and (A,l+B), where A and B are small compared to 1. Table I gives the coordinates of the undesired reconstructed points calculated using Eqs. (9) and (13). The coordinates of the undesired reconstructed points obtained from Eq. (15) will not be given because there are so many different coordinates and the reconstructed points are too dim to be seen in Fig. 1.

Table I. (X',Y') Coordinates of Undesired Reconstructed Points for Hologram of Three Object Points with Coordinates (0,-1), (0,0), and (A,1+B)

Coordinates predicted by Eq. (9)		
$(0,-2)$	(A,B)	$(A,2+B)$
$(-A,-3-B)$	$(0,1)$	$(2A,3+2B)$
$(-A,-2-B)$	$(-A,-1-B)$	$(2A,2+2B)$
Coordinates predicted by Eq. (13)		
$(0,-3)$	$(2A,1+2B)$	$(-A,-B)$
$(0,-2)$	$(-2A,-5-2B)$	$(A,2+B)$
$(A,-1+B)$	$(-2A,-2-2B)$	$(2A,4+2B)$
$(-A,-4-B)$	$(-A,-1-B)$	$(2A,3+2B)$
$(-A,-3-B)$	$(-2A,-3-2B)$	$(-2A,-2-2B)$
$(-A,-2-B)$	$(0,1)$	$(3A,5+3B)$
(A,B)	$(0,2)$	$(3A,4+3B)$
$(2A,2+2B)$	$(A,3+B)$	$(3A,3+3B)$

Table II. (X',Y) Coordinates of Undesired Reconstructed Points Predicted by Eq. (9) for Holograms of Four Object Points with Coordinates (l,l), (-l,l+A), (-1-B-1-G), and (1,-l)

$(3,1-A)$	$(3+B,1+G)$	$(-3-2B,-3-2G)$
$(3+B,3+G)$	$(-3,1+2A)$	$(-1-2B,-3-A-2G)$
$(1,3)$	$(-1+B,3+2A+G)$	$(-3-2B,-1-2G)$
$(1+B,3+A+G)$	$(-3,3+2A)$	$(-1-B,-3-G)$
$(-1,3+A)$	$(-3-B,-1+A-G)$	$(1-B,-3-A-G)$
$(1-B,-1-A-G)$	$(-3-B,1+A-G)$	$(1,-3)$
$(-1-B,1-G)$	$(-1,-1+A)$	$(3,-3-A)$
$(3,-1-A)$	$(1+B,1+A+G)$	$(3+B,-1+G)$

Table I shows that, all the undesired reconstructed images shown in Fig. 1 for the reconstruction of the hologram of three object points are predicted by theory. It should be noted that the coordinates shown in Table I are just the different coordinates predicted. At some of the coordinates shown in Table I there are actually many reconstructed points predicted and at other coordinates only one point predicted. This is the reason image points can be seen at, some, but not all, the coordinates predicted by Table I.

The desired reconstructed image points in the reconstruction of the hologram of four object points shown in Fig. 1 have the (X',Y') coordinates (l,l), (-l,l+A), (-l-B,-1-G), and (1,-l), where A, B, and G are

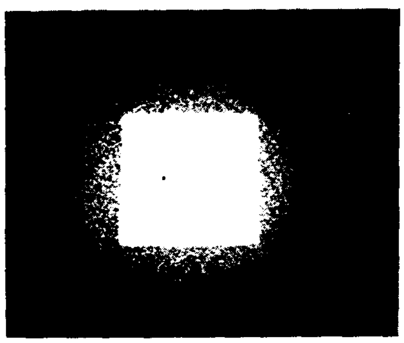

Fig. 2. lieconstruction of hologram of a piece of ground glass approximately 7 mm square. Flare around square is due to undesired reconstructed points.

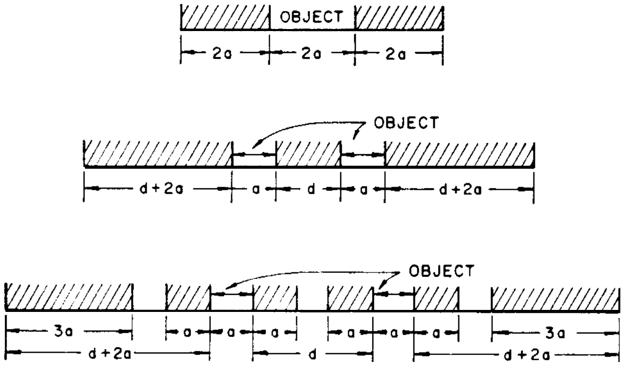

Fig. 3. Location of undesired images as predicted by Eq. (9) for object of length 2a and two objects, each of length a, separated by distance d; extent of objects shown by arrows; extent of external flare due to undesired reconstructions shown by shaded regions.

small compared to 1. Table II gives the coordinates of the undesired reconstructed points as predicted by Eq. (9).

Table II gives the location of most of the undesired images shown in Fig. 1 for the reconstruction of the hologram of four object points. Equation (13) predicts all the undesired images seen in Fig. 1 for the reconstruction of the hologram of four object points. Due to the large number of images predicted by Eq. (13) the coordinates calculated will not be given here.

Thus, it is seen that the theory correctly predicts the location of the undesired reconstructed images for conventional holograms. For the case of synthetic holograms, the same undesired reconstructed images are observed. However, these images are dimmer than the images observed for conventional holograms, because term ***d*** in Eq. (1) is zero for synthetic holograms.

Figure 2 shows the undesired images for the reconstruction of a conventional hologram of many object points. The object for the hologram was a piece of ground glass approximately 7 mm square. The beam balance ratio for the hologram was about 0.9, the preexposed density was approximately 0.2, and the average hologram density was approximately 1.

The flare around the reconstruction shown in Fig. 2 of the square piece of ground glass is an example of the undesired reconstructed points. There are, of course, undesired reconstructed points within the square itself, but for an object like the piece of ground glass they cannot be seen.

There are three reasons why the undesired images show up so well in Fig. 2. First, in the making of the hologram the beam balance ratio was about one and a relatively dense hologram exposure was made. Secondly, the photograph of the reconstruction was purposely overexposed so the undesired light is more evident. The third reason is that the object was of uniform brightness and as will be shown below, for an extended object containing a given amount of light, the undesired reconstructed images will be more noticeable if the object has uniform brightness than if the object is not of uniform brightness.

The total amount of light in the undesired reconstructed images depends only upon the number of object points and the illuminance of each object point and not upon the position of the object points. Thus, if we cut an extended object into two or more pieces and separate the pieces, the total amount of light in the undesired reconstructed points does not change. However, as shown in Fig. 3, as the various pieces of the object are moved apart, the undesired reconstructed images are spread over more area, and thus the amount of light per unit area in the undesired reconstructed image is decreased, while the amount of light per unit area in the desired reconstructed image remains constant.

The top drawing in Fig. 3 shows the locations of the undesired images given by Eq. (9) for an object of length 2a. The two bottom drawings show the location of the undesired images given by Eq. (9) for the same object cut into two pieces, each of length a, and the two pieces separated by a dark region of length d.

Thus, cutting the original object into two pieces and separating the pieces spreads out the undesired reconstructed images whose locations are given by Eq. (9). Similarly, the images predicted by Eqs. (13) and (15) are spread out even more when the object. is divided up into smaller objects and the smaller objects are separated.

Thus, we see that if an extended object is cut into smaller pieces and these pieces are separated (i.e., we no longer have a uniform object), the undesired reconstructed images cause less trouble because the ratio of the luminance in the desired reconstructed image to the luminance in the undesired reconstructed image increases.

Thus, for most holograms the undesired reconstructed images do not greatly degrade the desired reconstructed image because the undesired images are much dimmer than the desired image. However, for certain hologram objects, the undesired images can be trouble some. For example, if we have an extended object of uniform brightness the reconstruction of the extended object may be surrounded by a haze due to the undesired reconstructed images. Or if the hologram object contains a few bright points, the nonlinearity of the photographic process may cause additional bright points in the reconstruction. This explanation does not involve scattering by the grains of the photographic emulsion.

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