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Effect of the Photographic Gamma on the Luminance of Hologram Reconstructions*

JAMES C. WYANT AND M. PARKER GIVENS
Institute of Optics, University of Rochester, Rochester, N. Y. 14627
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An expression derived for hologram exposures made along the straight-line portion of the H-D curve of a photographic plate shows that the relationship between the luminance of the reconstructed hologram image and the luminance of the original object depends on the value and sign of the gamma of the photographic processes.

To check the theory, several holograms of different exposures were superimposed on Kodak 649-F plates that were pre-flashed with a uniform illuminance so the H-D curve of the photographic process is straight. The calculated and measured luminance ratios of the different reconstructions agree within experimental error. Since the gamma of the usual photographic process is positive, instead of negative, even if the gamma of the photographic process is equal to 2, it is not valid to assume that the relative luminance of the reconstruction of a given superimposed hologram is proportional to the product of the exposure due to the object beam and the exposure due to the reference beam used in the making of the hologram. This assumption would be valid for all fringe contrasts only if gamma were -2.

INDEX HEADINGS: Holography; Photography; Photographic emulsions.

THE role of the photographic process in holography was first studied by Gabor.[1] He showed that the amplitude transmittance and exposure of a hologram could be related linearly if a contact print were made of the hologram and if the resultant gamma of the over-all process were -2. Experimentally, it has been shown that holograms produce good reconstructed wavefronts even if the amplitude transmittance of the hologram and exposure are not related linearly. Work has been done to show that if the effective gamma of the photographic process is not -2, higher-order images will be produced, but these higher-order images will not interfere with the first-order images if their spatial-frequency power spectra do not overlap.[2]

It has also been shown that if the illuminance of the reference beam is much greater than the illuminance of the object beam, the luminance of the image is proportional to the product of the illuminance of the reference beam and the illuminance of the object beam, regardless of the magnitude or sign of the gamma of the photographic process.[3] In most holographic practice the illuminance of the reference beam is not an order of magnitude greater than the illuminance of the object beam. It is the purpose of this paper to show that for the illuminance ratios commonly used, theory predicts and experiments confirm that the luminance of the reconstructed image depends upon the sign and magnitude of the gamma, of the photographic process.

THEORY

Figure 1 illustrates the typical form of the H-D curve showing the density, D, as a function of the logarithm of the exposure, E, for a photographic plate. Between the points P and Q the curve is practically a straight line.

It follows that as long as the photographic plate is used along the straight line portion of the H-D curve, the density, D, is given by

$$D=\gamma \log_{10}(E/E'), \tag{1}$$

where E' is the inertia of the photographic plate.

The density is defined as the logarithm of the reciprocal of the flux transmittance, T. Thus, it follows that

$$\log_{10}(1/T)=\gamma \log_{10}(E/E'), \tag{2}$$

or

$$T=(E/E')^{-\gamma}. \tag{3}$$

In making a hologram, the object and reference beams have the amplitude A, $\exp(i\theta_n)$ and B, $\exp(i\theta)$, respectively. That is, A_n and B_n are the modulus of the amplitude of the two beams and θ and θ_n are the phases. Note that θ and θ_n depend only on position on the plate and not on time. That is, the $\exp(i\omega t)$ time factor has been omitted, Thus, at any given point the illuminance

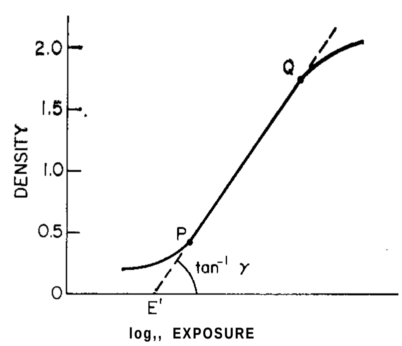

FIG. 1. Typical H-D curve.

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[1] D. Gabor, Proc. Roy. Soc. (London) A197, 454 (1949).

[2] D. G. Falconer, Phot. Sci. Eng. 10, 136 (1966).

[3] J. W. Goodman, J. Opt. Soc. Am. 57, 493 (1967).

of a given hologram exposure is proportional to $A_n^2+B_n^2+2A_nB_n\cos(\theta-\theta_n)$.

Since the exposure is equal to the product of the illuminance and the exposure time, t_n, the hologram exposure is proportional to $t_n[A_n^2+B_n^2+2A_nB_n\cos(\theta-\theta_n)]$.

An initial exposure, E_0, to a uniform illuminance distribution is made to bring the photographic plate to the straight line portion of the H-D curve. Therefore, the total exposure, E, of the photographic plate is equal to the sum of E_0 and the hologram exposures. If there are N different hologram exposures with the phase, θ, of the reference beam the same for all N exposures, $|T_A|$, the amplitude modulus of the transmittance function, is given by

$$|T_A|=E'^{\gamma/2}\Big[E_0+\sum_{n=1}^{N}t_n(A_n^2+B_n^2+2A_nB_n\cos(\theta-\theta_n))\Big]^{-\gamma/2}. \tag{4}$$

If this is expanded using the binomial theorem we obtain

$$\begin{aligned}|T_A|=(E'C)^{\gamma/2}\Big\{1&-\gamma C\sum_{n=1}^{N}t_nA_nB_n\cos(\theta-\theta_n)+\tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma+1)2C^2\sum_{n=1}^{N}\sum_{m=1}^{N}t_nt_mA_nA_mB_nB_m\cos(\theta-\theta_n)\cos(\theta-\theta_m)\\&-\tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma+1)(\tfrac{1}{2}\gamma+2)(\tfrac{4}{3})C^3\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{p=1}^{N}t_nt_mt_pA_nA_mA_pB_nB_mB_p\\&\times\cos(\theta-\theta_n)\cos(\theta-\theta_m)\cos(\theta-\theta_p)+\cdots\Big\},\end{aligned} \tag{5}$$

where

$$C\equiv\Big[E_0+\sum_{n=1}^{N}t_n(A_n^2+B_n^2)\Big]^{-1}.$$

Remembering that $\cos^2x=\frac{1}{2}(1+\cos 2x)$, $\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$, and $\cos^3x=\frac{1}{4}\cos 3x+(\frac{3}{4})\cos x$, we find that

$$\begin{aligned}|T_A|=(E'C)^{\gamma/2}\Big\{1&-\gamma C\sum_{n=1}^{N}t_nA_nB_n\cos(\theta-\theta_n)+\tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma+1)2C^2\Big[\sum_{n=1}^{N}t_n^2A_n^2B_n^2\tfrac{1}{2}(1+\cos 2(\theta-\theta_n))\\&+\sum_{n=1}^{N}\sum_{\substack{m=1\\m\neq n}}^{N}t_nt_mA_nA_mB_nB_m\tfrac{1}{2}(\cos(2\theta-\theta_n-\theta_m)+\cos(\theta_n-\theta_m))\Big]\\&-\tfrac{1}{2}\gamma(\tfrac{1}{2}\gamma+1)(\tfrac{1}{2}\gamma+2)(\tfrac{4}{3})C^3\Big[\sum_{n=1}^{N}t_n^3A_n^3B_n^3(\tfrac{1}{4}\cos 3(\theta-\theta_n)+(\tfrac{3}{4})\cos(\theta-\theta_n))\\&+3\sum_{n=1}^{N}\sum_{\substack{m=1\\m\neq n}}^{N}t_nt_m^2A_nA_m^2B_nB_m^2\tfrac{1}{2}(\cos(\theta-\theta_n)+\tfrac{1}{2}\cos(3\theta-\theta_n-2\theta_m)+\tfrac{1}{2}\cos(\theta-2\theta_m+\theta_n))\\&+\sum_{n=1}^{N}\sum_{\substack{m=1\\m\neq n}}^{N}\sum_{\substack{p=1\\p\neq m\\p\neq n}}^{N}t_nt_mt_pA_nA_mA_pB_nB_mB_p\tfrac{1}{4}(\cos(3\theta-\theta_n-\theta_m-\theta_p)+\cos(\theta-\theta_m-\theta_p+\theta_n)\\&+\cos(\theta-\theta_n+\theta_m-\theta_p)+\cos(\theta-\theta_n-\theta_m+\theta_p))\Big]+\cdots\Big\}.\end{aligned} \tag{6}$$

Since we are interested in only the first-order reconstruction of the original beams, we are interested in only terms containing $\cos(\theta-\theta_n)$. Let $|T_A|_n$ be the absolute value of the part of T_A that contains terms in $\cos(\theta-\theta_n)$. If the amplitude of the reconstructing beam is $D\exp(i\theta)$, the amplitude of the first-order reconstruction of the original object beams is given by

$$|T_A|_n D\exp(i\theta)=D(E'C)^{\gamma/2}(\gamma/2)C\{[\sum_{n=1}^{N} t_nA_nB_n + (\tfrac{1}{2}\gamma+1)(\tfrac{1}{2}\gamma+2)C^2(\tfrac{1}{2}\sum_{n=1}^{N} t_n^3A_n^3B_n^3 + \sum_{n=1}^{N}\sum_{\substack{m=1\\m\neq n}}^{N} t_nt_m^2A_nA_m^2B_nB_m^2)]+\cdots\} \times(\exp i\theta_n+\exp i(2\theta-\theta_n)). \quad (7)$$

Thus, the luminance of the reconstruction of the nth object beam is equal to

$$I_n=D^2(E'C)^{\gamma}C^2(\gamma/2)^2t_n^2A_n^2B_n^2[1+(\tfrac{1}{2}\gamma+1) \times(\tfrac{1}{2}\gamma+2)C^2(\tfrac{1}{2}t_n^2A_n^2B_n^2+\sum_{\substack{m=1\\m\neq n}}^{N} t_m^2A_m^2B_m^2) + \text{higher-order terms}]^2, \quad (8)$$

where

$$C\equiv[E_0+\sum_{m=1}^{N} t_m(A_m^2+B_m^2)]^{-1}.$$

Noting that A_n^2 is equal to the luminance of the nth object beam, it is seen that the luminance of the reconstruction of a given object beam is proportional to the product of the illuminance of the original reference beam and the illuminance of the original object beam if only the first term in Eq. (8) has to be considered.

There are three cases when it is sufficient to consider only the first term in Eq. (8). The first case is when γ = -2 and all the terms with the exception of the first are identically zero because each contains the factor $(1/2\gamma+1)$.

The other two cases are somewhat similar. If either E_0, the exposure to a uniform illuminance distribution required to make the hologram exposure be along the straight-line portion of the H-D curve, is much greater than the hologram exposures, or if the exposure due to either the reference beam or the object beam is much less than the total exposure of the superimposed holograms, all the terms in Eq. (8), with the exception of the first, can be disregarded. These last two conditions are said to be similar since they both represent the condition of low-contrast fringes in the hologram. Practical holograms do not satisfy either of these conditions.

Experimental Setup and Procedure

In an effort to verify the above expression, the apparatus shown in Fig. 2 was used to make holograms of a point source. A Bausch & Lomb laser operating at a wavelength of 6328 Å was used as the coherent light source. A 20X microscope objective was used to produce a diverging beam, which was collimated with a 75cm focal-length objective. The interference fringes in the diverging beam caused by dust particles in the objective were removed by placing a 15-μ pinhole at the focus of the microscope objective. A 2-diopter negative lens was placed in part of the collimated beam to produce a spherical wave. The hologram was produced using the spherical wave as one beam and the collimated wave as the other beam.

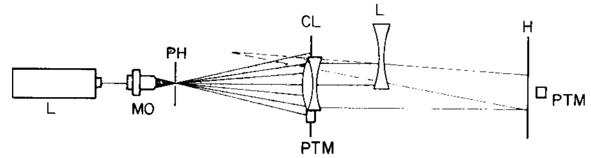

FIG. 2. Apparatus used to make holograms. L, laser; MO, 20X microscope objective; PH, 15-μ pinhole; PTM, photomultiplier tube; CL, collimating lens; L, -2 diopter negative lens; and H, hologram plane.

Before the hologram was made, the photographic plate was preflashed to a General Electric AG-1 flashbulb placed 2.5 m from the photographic plate. A larger portion of the photographic plate was exposed to the uniform illuminance distribution than was exposed to the hologram exposures, so after development, the pre-exposed density could be determined.

Four different hologram exposures were superimposed on each photographic plate. Between each exposure the photographic plate was moved approximately 1/4 cm. Thus, if a collimated beam is used for the reconstruction of the holograms, the first-order real-image reconstruction is four dots of different brightness separated by approximately 1/2 cm.

In the reconstruction, the hologram was stopped down to approximately f/125 and a photomultiplier tube with a 70-μ-diam pinhole in front of it was used to measure the luminances of the four reconstructed dots, so the relative luminances of the dots could be compared with the relative exposures of the hologram used in recording each dot.

To determine the H-D curve of the photographic process, a photographic plate was preflashed with a General Electric AG-l flashbulb in the same manner as the photographic plate used to make the hologram was preflashed. Several different exposures were made using the laser source, and this plate, as well as all the other plates, was developed in D-19 at 68°F for 5 min. The density of the different exposures was measured using an Ansco-Sweet densitometer.

To determine the pre-exposure, E_0, another photographic plate that was not preflashed was given several different exposures to a uniform illuminance distribution from the laser. The H-D curve of this unpreflashed plate was obtained. From it, E_0, the exposure due to the laser light needed to produce the same density as was produced by the AG-l flashbulb, could be determined.

The pre-exposure was made using a flashbulb instead of light from the laser itself, since a very short intense

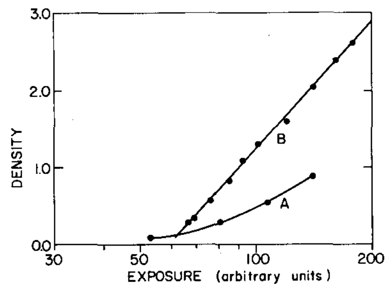

FIG. 3. Curve A is H-D curve of 649-F plates without pre-flashing. Curve B is H-D curve of 649-F plates preflashed with an AG-l flashbulb to a density of 0.14.

pre-exposure gives a more nearly straight H-D curve than does a longer less intense pre-exposure.[4]

Experimental Measurements and Results

Equation (8) gives the luminance of the reconstruction of the nth object beam for the general case of N superimposed holograms. In our experimental case, N=4. Also, in the experiment, the illuminance of the object and reference beams was held constant, and only the exposure time was varied. Therefore, Eq. (8) reduces to

$$I_1=D^2(E'C)^\gamma C^2(\gamma/2)^2t_1^2A^2B^2[1+(\tfrac{1}{2}\gamma+1)\times(\tfrac{1}{2}\gamma+2)C^2A^2B^2(\tfrac{1}{2}t_1^2+t_2^2+t_3^2+t_4^2)+\cdots]^2, \quad (9)$$

where

$$C=[E_0+(A^2+B^2)(t_1+t_2+t_3+t_4)]^{-1}.$$

Similar equations exist for I_2, I_3, and I_4, the luminances of the reconstructions of the other three object beams. Thus, the ratio of the luminances of the reconstructions of the different beams can be calculated if the pre-exposure, E_0, the hologram exposures, and the γ of the photographic process are known.

In making the holograms, the photographic plates were first preflashed with an AG-l flashbulb to a density of 0.14. Curve A in Fig. 3 shows the H-D curve of 649-F plates exposed to laser illumination with no preflashing. Curve A indicates that the pre-exposure, E_0, in the same arbitrary units as the hologram exposures are measured, is equal to approximately 66 to correspond to the preflash density of 0.14.

Curve B in Fig. 3 shows the H-D curve of 649-F plates preflashed with an AG-1 flashbulb to a density of 0.14. It is seen that this H-D curve is nearly a straight line with a slope of 5.5.

Thus, since E_0 is known to be equal to 66 in the same arbitrary units as the hologram exposures are measured, and γ is measured to be 5.5, Eq. (9) can be used to calculate the image luminance ratios. Table I shows the results of these calculations, and the measured luminance ratios, as well as what the luminance ratios would have been, as calculated using Eq. (9) with gamma equal to -2, and with gamma equal to +2. It is noted that only three reconstructed luminances are given for the second photographic plate. This is because the fourth hologram exposure was so low that the reconstruction was lost in the noise.

[4] G. Bird (private communication).

TABLE I. Ratio of reconstructed image luminances of superimposed holograms.

Hologram exposure ratio	Reconstructed luminance ratio: measured	calculated using measured value of gamma $\gamma=5.5$	calculated if $\gamma=-2$	calculated if $\gamma=+2$
Plate 1				
$E_1/E_2=10$	77	78	100	91
$E_1/E_3=50$	1780	1940	2500	2275
$E_1/E_4=100$	7600	7750	10 000	9100
$E_2/E_3=5$	23	25	25	25
$E_2/E_4=10$	98	100	100	100
$E_3/E_4=2$	4.27	4	4	4
Plate 2				
$E_1/E_2=10$	89	87	100	96
$E_1/E_3=50$	2020	2100	2500	2400
$E_2/E_3=5$	23	25	25	25

It is seen that there is good agreement between the measured and the calculated luminance ratios. The maximum discrepancy of approximately 10% between the observed and calculated luminance ratios is believed to be within the experimental error of the measurements.

Table I shows that if $\gamma=-2$, calculations made using Eq. (9) show that the luminance ratio of the reconstructed images of the superimposed holograms is equal to the square of the exposure ratio of the superimposed holograms, for all hologram fringe contrasts. That is, if $\gamma=-2$, the ratio of the luminances of the reconstructed images of the superimposed holograms is equal to' the product of the exposures of the object beam and the reference beam, regardless of the hologram fringe contrast.

If gamma is equal to +2, or to the measured value of 5.5, Table I shows that the reconstructed luminance ratios are not generally equal to the square of the hologram exposure ratio. For the holograms having low fringe contrast, the ratio of reconstructed image luminances is equal to the square of the hologram exposure ratios for all reasonable values of gamma, but if the hologram has a reasonably high fringe contrast, the ratio of the reconstructed image luminance is not equal to the square of the hologram exposure ratios.

Since in the usual photographic process gamma is positive, and not negative, even if gamma were equal to 2, the luminances of the images of the superimposed holograms would not be proportional to the product of the exposures of the object beam and reference beam. unless the hologram fringe contrast were very low.

Although the above work has all been done for holograms exposed sequentially to different object points, we believe that the work clearly calls attention to the fact that the often quoted statement that the luminance of the reconstructed image is independent of the sign of gamma rests upon the assumption that the object beam is of low illuminance as compared with the reference beams, i.e., the hologram fringes are of low contrast. This assumption is not valid in most experimental work.

If the hologram had been made by exposure to the several points simultaneously rather than sequentially, the binomial expansion which gave Eqs. (5) and (6) would have contained all of the terms found there and others in addition. The second-order terms do not cancel out in the simultaneous-exposure case so that here also the luminance of the reconstructed image depends upon the magnitude and sign of gamma unless the fringe contrast is low.

ACKNOWLEDGMENTS

The authors are indebted to Dr. George Bird for a fruitful discussion on the influence of short preflashing exposure times on the linearity of H-D curves.