Testing spherical surfaces: a fast, quasi-absolute technique

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A technique for measuring the quality of spherical surfaces that provides a quasi-absolute result is presented. It requires only two measurement positions rather than the traditional method of absolute sphere measurement that requires three measurement positions. A measurement is taken with a mirror at the focus of the interferometer diverger lens and is subtracted from a measurement of the sphere tested at its center of curvature. This test assumes that the test sphere does not contain any aberrations with odd symmetry so that these aberrations can be subtracted to provide a fast, quasi-absolute measurement. We describe the new technique and compare measurement results from testing a $\lambda/12$ peak-to-valley sphere (numerical aperture = 0.4) by using a phase-measuring Fizeau interferometer with results from the three-position absolute sphere measurement technique. The repeatability of this measurement technique is ± 0.01 waves peak to valley.

Introduction

A number of techniques have been described in the literature for the absolute measurement of spherical surfaces.[1–5] Absolute measurements are important with optics that are specified to be at least as good as $\lambda/10$ peak to valley (P–V), where λ is the test wavelength. A technique that is widely used with phase-measuring interferometry was first described by Jensen[2] and then further discussed by Bruning,[3] Truax,[4] and Elssner *et al.*[5] This technique has the advantage of giving the absolute shape of the sphere under test independent of the reference surface and diverger optics. The main disadvantage of this technique is the requirement that the test surface be aligned so that it can be rotated about the optical axis with the fringe pattern kept unchanged. A simpler technique has been developed that does not require this precise alignment and requires only two measurements. The result is not exactly an absolute measurement, but as long as the test surface has even symmetry the test can be absolute. We start by describing the Jensen three-position absolute measurement technique and then the new two-position quasi-absolute measurement technique. The results of testing a 0.4-numerical aperture (NA) sphere with both techniques are then presented and compared.

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Three-Position Absolute Measurement Technique

The technique of absolute measurement of spherical surfaces as described by Jensen[2] requires three separate measurements of the surface being tested. These three measurements are depicted in Fig. 1. The first measurement is with the test surface at the focus of the diverger lens (also known as the cat's-eye position). The second measurement is with the test surface positioned so that its center of curvature is at the focus of the diverger lens (also known as the confocal position). The third measurement is taken after rotating the test surface 180° about the optical axis. Mathematically these three measurements can be written as

$$W_{\text{focus}} = W_{\text{ref}} + \tfrac{1}{2}\,(W_{\text{div}} + \overline{W}_{\text{div}}), \tag{1}$$

$$W_{0^\circ} = W_{\text{surf}} + W_{\text{ref}} + W_{\text{div}}, \tag{2}$$

$$W_{180^\circ} = \overline{W}_{\text{surf}} + W_{\text{ref}} + W_{\text{div}}, \tag{3}$$

where W refers to a wave front, surf refers to the test surface, ref refers to the optics in the reference arm of the interferometer and the reference surface, and div refers to the optics in the test arm of the interferometer minus the test surface including the diverger lens. A bar over a wave front indicates a 180° rotation of that wave front. These three measurements can then

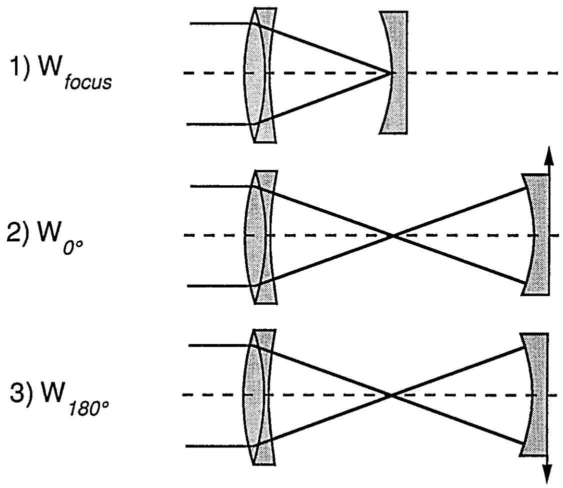


Fig. 1. Measurement setups for absolute testing of spherical surfaces.

be used to solve for the test surface by using

$$W_{\mathrm{surf}} = \tfrac{1}{2}\,(W_{0^\circ} + \overline{W}_{180^\circ} - W_{\mathrm{focus}} - \overline{W}_{\mathrm{focus}}), \tag{4}$$

which is calculated simply with additions, subtractions, and 180° rotations of the three measurements. If a large number of similar spheres are to be tested, the aberrations in the interferometer and errors caused by the reference surface can be obtained by calculating

$$W_{\mathrm{ref}} + W_{\mathrm{div}} = \tfrac{1}{2}\,(W_{0^\circ} - \overline{W}_{180^\circ} + W_{\mathrm{focus}} + \overline{W}_{\mathrm{focus}}). \tag{5}$$

This reference wave front can then be subtracted from measurements of subsequent test spheres as long as the radii of curvature are similar. If there is a significant difference in the radii of curvature, a new reference wave front must be measured. This technique works with both Twyman–Green and Fizeau interferometers.

The alignment criteria that are necessary to perform this procedure have been outlined by Elssner *et al.*[5] The optical axis is defined by the first measurement in the cat's-eye position with the fringes nulled. The detector in the interferometer should be centered on the optical axis. Next the test surface needs to be aligned relative to the optical axis to rotate the test surface 180° without altering the fringe pattern. This means that the vertex of the sphere must lie on the optical axis, the axis of rotation that is defined by the rotation stage must coincide with the optical axis, and the center of curvature of the test surface must lie at the focus of the diverger lens. Figure 2 is a drawing of the possible misalignments for testing a sphere in a Fizeau interferometer, and Fig. 1 shows the test and reference surfaces after alignment. Elssner *et al.* state that a mount with a minimum of 8 degrees of freedom is required to do this alignment as long as the test surface has been centered in its mount.[6] We have found that six axes (with 4 degrees of freedom)

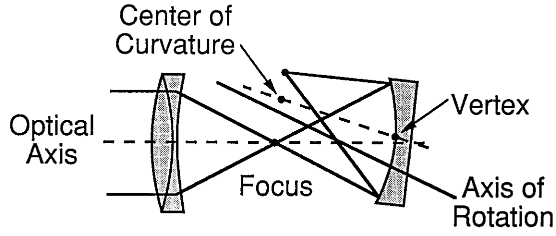


Fig. 2. Possible misalignments include the position of the reference surface, sphere vertex, sphere center of curvature, and axis of rotation, all relative to the optical axis, and the location of the sphere along the optical axis.

are sufficient to rotate the test surface 180° and keep the fringe pattern within two fringes of being nulled. Figure 3 shows a mount that contains eight axes (with 6 degrees of freedom) to test a sphere. The sphere is mounted to an X, Y stage that is used to center the sphere on the rotation axis. A five-axis mount is used to align the axis of rotation with the optical axis of the interferometer. The tip-tilt of the five axes ensures that the sphere is being tested at its center and is not always necessary. In addition it is advantageous to use a flat with a separate mount for the measurement at the cat's-eye position (see Fig. 1). This additional mount needs tip-tilt and z translation for fine positioning. A separate mirror for the cat's-eye position makes it easier to take all the data quickly once the sphere has been aligned relative to the interferometer.

A high-quality rotation stage is required to align the sphere so that it can be rotated 180° without changing the fringe pattern. Stages with aluminum races and ball bearings do not repeatedly return to the same location after rotation. A more expensive stage with good concentricity and repeatability is necessary. It is unclear whether it is possible to align the test surface mechanically and rotate the surface 180° without changing the fringe pattern. However, it is possible to align the surface and keep the pattern to within a few fringes while rotating by 180°.

The alignment of the test surface can be accomplished by looking at the rotation of the return spot from the sphere in a focal plane and comparing it with the return spot from the reference surface as the

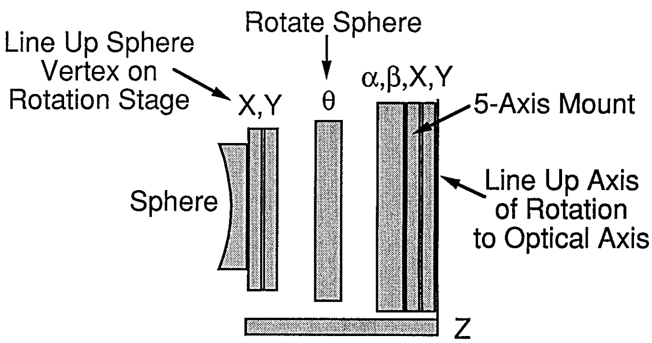


Fig. 3. Mount with the necessary degrees of freedom to align the sphere to rotate 180° and keep the fringes.

sphere is rotated. This procedure starts by adjusting the two spots so that they are on top of one another. After a 180° rotation the sphere X, Y position is adjusted to bring the sphere return spot halfway back to the position of the reference spot. The sphere is then rotated back 180°, and the five-axis X, Y position is adjusted to line up the two spots. This procedure is continued until there is no noticeable movement of the spot as the sphere is rotated. At this point the fringe pattern can be observed, and a similar procedure is followed until the fringe pattern is stationary as the sphere is rotated. With a good rotation stage this alignment procedure is sufficient to measure spherical surfaces with NA's of 0.5 or less to $\lambda/20$ P–V. To perform a high-accuracy measurement good optics in the interferometer (at least $\lambda/10$ P–V) are necessary so that the rays transverse the same path back through the interferometer after reflecting from the sphere.

Two-Position Quasi-Absolute Measurement Technique

Because the alignment of the test surface becomes much more difficult as the NA becomes larger, a simpler technique was developed. This technique only requires two measurements. These two measurements are given by Eqs. (1) and (2). When the first measurement is subtracted from the second measurement, the result is the wave front caused by the surface plus an error term because of the diverger. This result is written mathematically as

$$W_{0^\circ} - W_{\text{focus}} = W_{\text{surf}} + \tfrac{1}{2}\,(W_{\text{div}} - \overline{W}_{\text{div}}). \tag{6}$$

For aberrations with even symmetry such as defocus, spherical, and astigmatism, the error term is zero because these aberrations cancel out ($W_{\text{div}} = \overline{W}_{\text{div}}$). The difference in the two measurements then becomes

$$W_{0^\circ} - W_{\text{focus}} = W_{\text{surf}}. \tag{7}$$

For aberrations with odd symmetry such as coma, $W_{\text{div}} = -\overline{W}_{\text{div}}$. The difference between the two measurements is

$$W_{0^\circ} - W_{\text{focus}} = W_{\text{surf}} + W_{\text{div}}. \tag{8}$$

Most spherical surfaces do not have a coma in them, and, because a misalignment of the spherical test surface would not induce a coma into the measurement, it can be assumed that any coma in the measurement should be a result of the interferometer or the diverger lens. As long as the coma is assumed to be in the interferometer and not in the test surface, it can be subtracted from the measurement to yield the test surface independently of the interferometer. For higher-order aberrations those with even symmetry will cancel, while those with odd symmetry will not cancel and should be subtracted from the measurement result as long as they are not in the test surface.

The coma resulting from both the interferometer

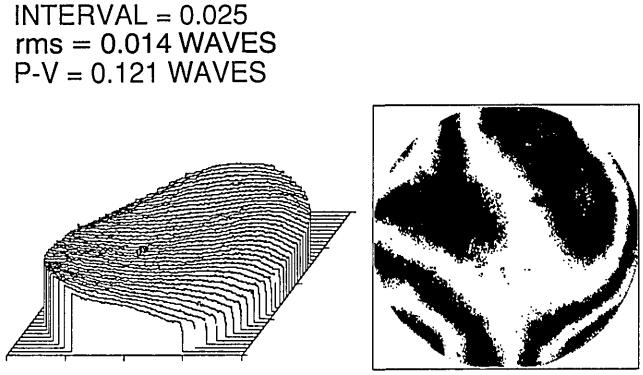


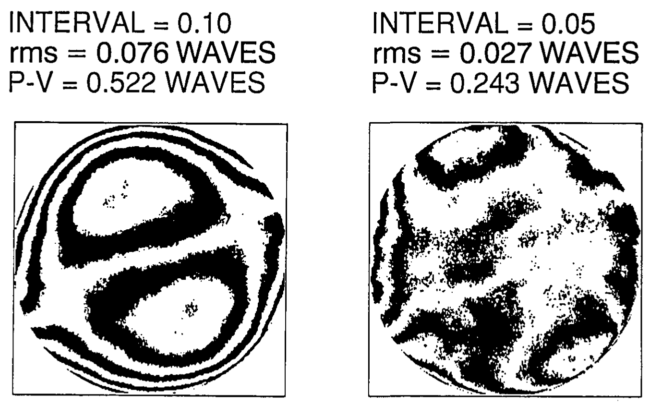
Fig. 4. Single measurement of the sphere. The tilt and power have been removed.

and the test surface can be found either by a least-squares fit or by rotating the final data set by 180° and subtracting this from the data set before the rotation (this causes even aberrations such as defocus, spherical, and astigmatism to cancel and leaves twice the odd aberrations such as coma in the entire system). Coma and other odd aberrations resulting from the test surface can only be found by including a third measurement as in the Jensen technique. By subtracting the second and third measurements (see Fig. 1) the result is twice the coma (and other odd aberrations) in the test surface. Without the first measurement of Fig. 1 the surface profile cannot be obtained. Thus it takes three measurement positions to determine the surface completely.

For quick and easy-to-set-up measurements that yield surface shape in the range from $\lambda/10$ to $\lambda/15$ P–V, the two-position technique is quite useful. It provides a simple test without the need for expensive mounts and many minutes of alignment. If greater accuracy is required or if there are odd aberrations in the test surface, the Jensen technique is better to use.

Results

To compare the two techniques a 0.4-NA sphere was tested in a Fizeau interferometer with a diverger lens



(a) (b)

Fig. 5. Cat's-eye measurement (a) with the tilt and power removed and (b) with the tilt, power, and coma removed.

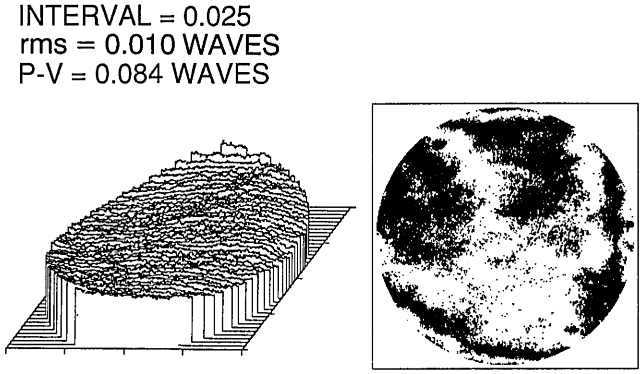


Fig. 6. Three-position absolute reference showing errors in reference and diverger. The tilt and power are removed.

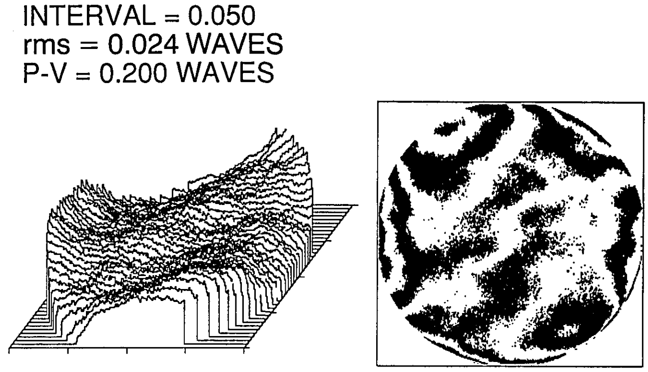


Fig. 8. Two-position measurement with the tilt, power, and coma removed.

having an $F/1.1$ spherical reference surface. The source was a He–Ne laser operating at 0.6328 μm. Both algorithms were implemented by using phase-measurement interferometry techniques with a CCD TV camera and a 68030-based computer. A single measurement of the sphere is shown in Fig. 4 with an rms of 0.014 waves and a P–V of 0.121 waves. Tilt and power have been subtracted from the measurements because they are functions of the alignment that are not part of the test surface. All measurement results have been evaluated over 95% of the aperture to eliminate diffraction effects at the edge of the pupil. A flat placed at the focus of the diverger lens in the cat's-eye position shows that there are 0.522 waves P–V of coma present in the interferometer and diverger lens as seen in Fig. 5(a). With third-order coma subtracted from this measurement [Fig. 5(b)], there is a noticeable odd aberration having a three-point symmetry with a P–V of 0.243 waves. This aberration can be expressed in polynomial form by using the ninth and tenth Zernike polynomials that have a functional form given by

$$\rho^3 \cos 3\theta \text{ and } \rho^3 \sin 3\theta,$$

where ρ is the normalized radius and θ is the azimuthal angle. By using the three-position measurement technique of Jensen [Eqs. (4) and (5)], the errors in the interferometer showing the quality of the collimating lens, the diverger lens, and the reference surface are 0.084 waves P–V as seen in Fig. 6. This means that the interferometer optics are good to $\lambda/12$ P–V. The spherical test surface is shown in Fig. 7 and has 0.081 waves ($\lambda/12$) P–V. Using the two-position measurement technique that is described here, we show in Fig. 8 a measurement with tilt, power, and third-order coma subtracted. It has a P–V of 0.200 waves and an error present with the same noticeable three-point symmetry as seen in the measurement taken at the cat's-eye position. This error is obviously not in the test surface and can be subtracted. Figure 9 shows a wave front generated from the Zernike 9 and 10 polynomial coefficients of the wave front that is shown in Fig. 8. This error term has a P–V of 0.102 waves. Once this error term is subtracted from the two-position absolute measurement of Fig. 8, the test sphere has a P–V of 0.089 waves as shown in Fig. 10. This compares quite favorably with the three-position measurement of Fig. 7. The orientation of the test surface is the same for both measurements. Notice the roll-off in the lower right-hand corner of both results. Both techniques show that the sphere is better than $\lambda/10$ P–V; and although the numbers are not exactly the same,

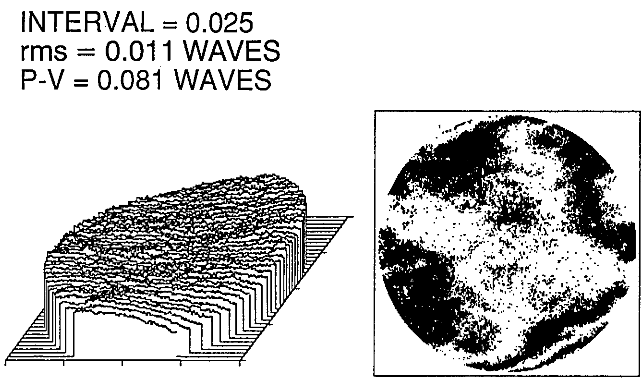


Fig. 7. Three-position absolute measurement of the sphere. The tilt and power have been removed.

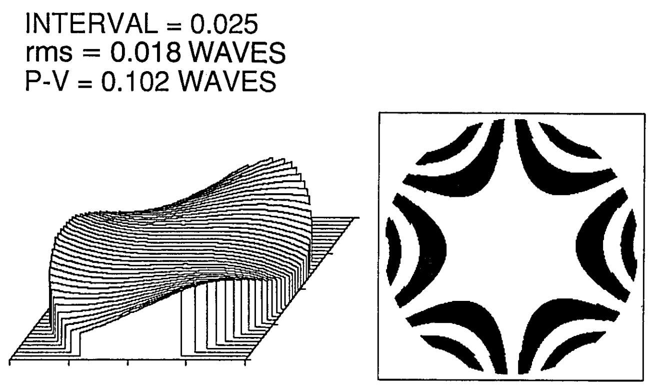


Fig. 9. Zernike 9 and 10 coefficients of the two-position measurement.

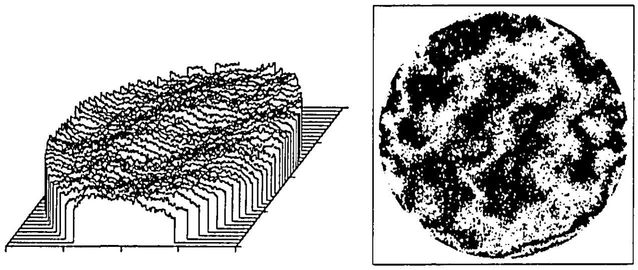


Fig. 10. Two-position measurement with the tilt, power, coma, and Zernike 9 and 10 removed.

gross errors on the surface are the same in both measurements. Both measurements are repeatable to ±0.01 waves P–V.

Conclusions

The three-position measurement technique for absolute measurement of spherical surfaces requires critical alignment of the test surface and an extremely good rotation stage. It theoretically has a high precision and accuracy but is hard to do. A faster and simpler technique for quasi-absolute measurement of spherical surfaces has been introduced that does not require the precise alignment of the Jensen technique. It requires only two measurements instead of three, and a complex mount for rotating the test object and retaining fringes is not required. The test assumes that no coma (or higher-order aberrations with odd symmetry) is introduced by the test surface so that odd aberrations may be subtracted from the measurement. This is not strictly an absolute test, but for the measurement of high NA surfaces that need to be at least $\lambda/10$ P–V, it is sufficient in most cases. For higher-quality surfaces $\lambda/20$ P–V can be measured, but it must be done with care, and high-quality diverging optics (better than $\lambda/10$ P–V) must be used.

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References and Notes

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6. The ultimate mount to perform this measurement contains 10 axes with 8 degrees of freedom. The sphere is mounted on a tip-tilt stage on an X, Y translation stage that is mounted on the rotation stage. This assembly is then mounted on a tip-tilt stage on an X, Y, Z translation stage. The rotation and Z axes are not considered as degrees of freedom in aligning the sphere and rotation axes relative to the interferometer optical axis.