

Multiple-wavelength phase-shifting interferometry

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This paper describes a method to enhance the capability of two-wavelength phase-shifting interferometry. By introducing the phase data of a third wavelength, one can measure the phase of a very steep wave front. Experiments have been performed using a linear detector array to measure surface height of an off-axis parabola. For the wave front being measured the optical path difference between adjacent detector pixels was as large as 3.3 waves. After temporal averaging of five sets of data, the repeatability of the measurement is better than 25-Å rms ($\lambda = 6328 \text{ \AA}$).

I. Introduction

Conventional single-wavelength phase-shifting interferometry (PSI)¹⁻⁵ is a technique which can perform direct phase measurement by first obtaining three or more intensity readings of fringe patterns with some known amounts of phase shifts and then using these intensity readings to calculate the phase value at each data point. Since the phase distribution across the interferogram is a measured modulo 2π one needs an assumption to remove the 2π discontinuities in the measured phase data. The fundamental assumption for single-wavelength PSI is that the difference of optical path difference (OPD) between any adjacent pixels is $\leq \lambda/2$. This assumption sets a limit to the phase measurement range of single-wavelength PSI. When the slope of the test surface is steep enough that the phase change between any adjacent pixels is larger than π , the 2π ambiguity problem will ruin the result of the phase measurement. By using either a higher resolution detector array or a longer wavelength light source, it is possible to overcome this problem. One can use either an IR light source or two shorter visible wavelengths to synthesize a longer equivalent wavelength as is used in many two-wavelength techniques.^{4,6}

In Ref. 6 two methods were presented to solve the 2π ambiguity problem. For the first method two sets of phase data (with 2π ambiguities) for λ_a and λ_b , are stored in the microcomputer, which will then calculate the phase difference between pixels for a longer equiv-

alent wavelength λ_{eq} according to Eq. (1) with the assumption that the difference of OPD between any adjacent pixels is $< \lambda_{eq}/2$:

$$\Delta OPD_{n+1} = \begin{cases} \frac{1}{2\pi} [\Delta\phi_{(n+1)b} - \Delta\phi_{(n+1)a}] \lambda_{eq} & \text{if } \lambda_a > \lambda_b, \\ \frac{1}{2\pi} [\Delta\phi_{(n+1)a} - \Delta\phi_{(n+1)b}] \lambda_{eq} & \text{if } \lambda_b > \lambda_a. \end{cases} \quad (1)$$

Once these Δ OPDs between pixels were obtained, the relative wave-front (OPD) plot or relative surface height plot could be obtained by integrating all these Δ OPD values. The problem for the first method of the two-wavelength phase-shifting interferometer (TWLPSI) is that the calculated phase data for λ_{eq} are quite noisy due to the error amplification effect,⁶ which makes the measurement precision poorer than that of single-wavelength PSI. In the second more accurate method, the phase data for λ_{eq} obtained by the first method are used as a reference only, so that one can go ahead and use it to correct the 2π ambiguities in the single-wavelength phase data and maintain its measurement precision.

There are several ways to make the 2π ambiguity corrections. A simple way is by finding the difference between the reference data (noisier but without 2π ambiguities) and the data with 2π ambiguities. In general, if the noise in the reference data is not too large, one can get a kind of step function and let one know where an integer number of 2π values should be added or subtracted.

A problem using the first method of the TWLPSI is that as the wave front under test becomes steeper and steeper a longer λ_{eq} is needed to solve the 2π ambiguity problem. Due to the error amplification effect, the amplitude of the high-frequency structure on the calculated phase data for λ_{eq} has a larger amplitude for longer λ_{eq} and makes it harder to correct the 2π ambiguities in single-wavelength phase data. In this paper we propose a method that can overcome this problem

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Received 5 October 1984.

0003-6935/85/060804-04\$0200/0.

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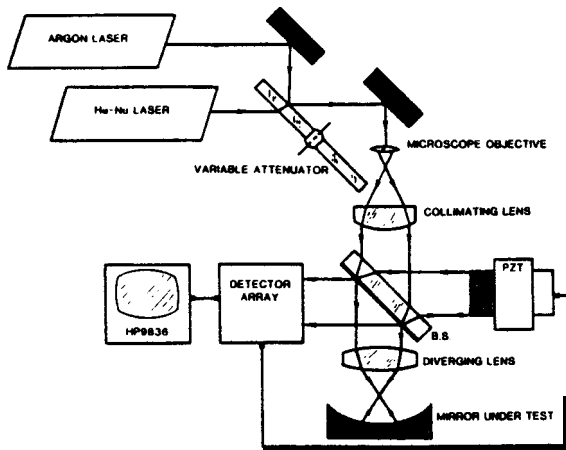


Fig. 1. Experimental setup for the MWLPSI.

by including the phase data of a third wavelength. To select the right wavelengths, one has to make sure that the OPD difference between adjacent pixels is less than half of the longest equivalent wavelength ($\lambda_{eq1}/2$). The idea is to use the input phase data for λ_a , λ_b , and λ_c (assume $\lambda_a < \lambda_b < \lambda_c$) to calculate the phase data for the longest equivalent wavelength (λ_{eq1}) and that for the shortest equivalent wavelength (λ_{eq2}). A good combination of wavelengths is for the ratio of $\lambda_{eq1}/\lambda_{eq2}$ and λ_{eq1}/λ_b to be 3 or 4. Now there are two steps to make the corrections: (1) use the phase data of λ_a to correct 2π ambiguities in the phase data of λ_{eq1} ; (2) use the 2π ambiguity corrected phase data of λ_{eq1} to correct the 2π

ambiguities in λ_a , λ_b , or λ_c . Actually, more correction steps could be applied if one had more phase data from other wavelengths.

II. Experimental Setup and Results

The experimental setup is shown in Fig. 1. An argon-ion laser and a He-Ne laser were used as the light sources so that the equivalent wavelength could be changed from 1.93 to 28.5 μm . Note that the diverging lens in the setup was also used as an imaging lens. Since the mirror under test (an off-axis parabola with apparent $f/5$) is not a very steep aspheric surface, one can defocus it to get a very fine fringe pattern. The fringe patterns of different wavelengths are sampled by a Reticon RC1728H linear array (only 1024 pixels being used); the analog signal is then converted into a 10-bit digitized signal which is fed into a HP9836 microcomputer for processing. Due to the finite width of each pixel's response curve, the largest measurable wave front slope is $-0.8 \lambda/\text{pixel}$ for this particular detector array. To test the capability of the multiple-wavelength phase-shifting interferometer (MWLPSI), one needs a wave front much steeper than that mentioned above and requires a nearly ideal point detector array to sample it. Although this kind of ideal point detector array is not yet available, one can simulate it by using the whole array to take intensity readings and then use every four or five pixels to do phase calculations. In our case, 1024 pixel elements were used to take intensity data for fringe patterns, and only 240 phase values were

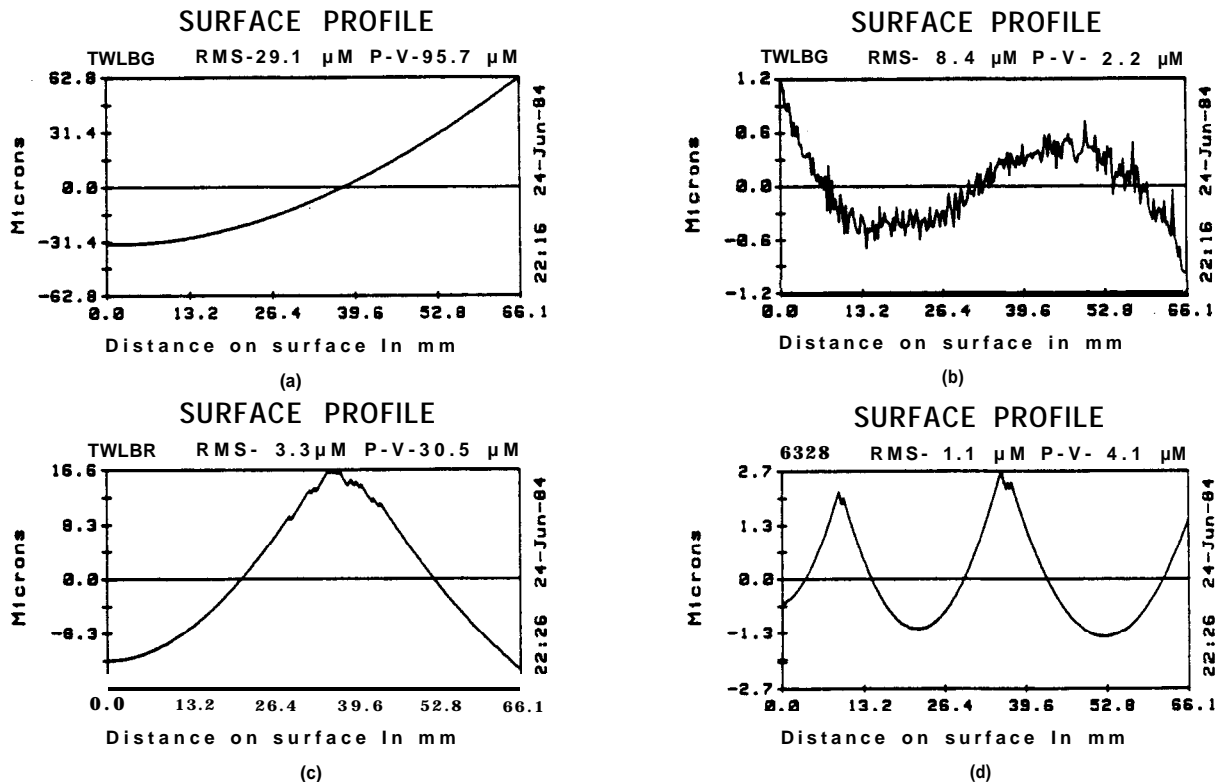
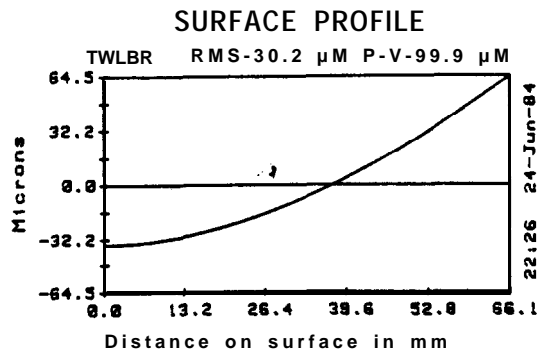
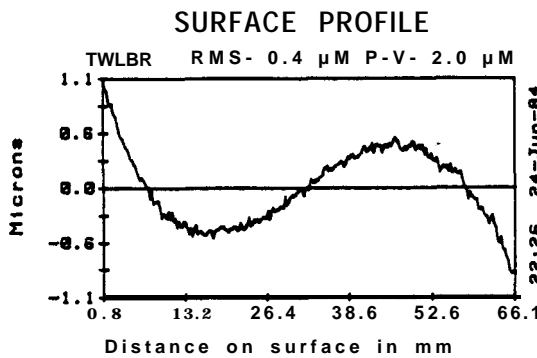


Fig. 2. (a) Two-wavelength surface height plot for λ_{eq1} , where $\lambda_{eq1} = 6.45 \mu\text{m}$. (b) Some data as in Fig. 2(a) but with both tilt and focus removed. (c) Two-wavelength surface height plot for λ_{eq1} , where $\lambda_{eq1} = 1.93 \mu\text{m}$. (d) Single wavelength surface height plot for $\lambda = 6328 \text{ \AA}$ obtained by single-wavelength PSI.



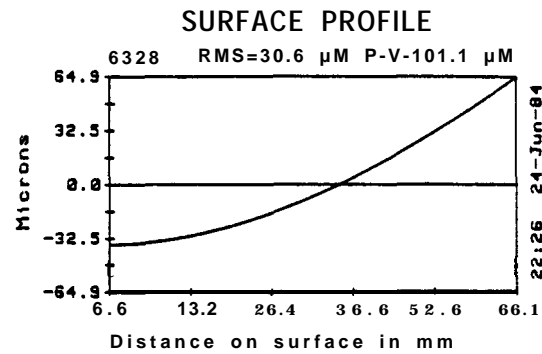
(a)



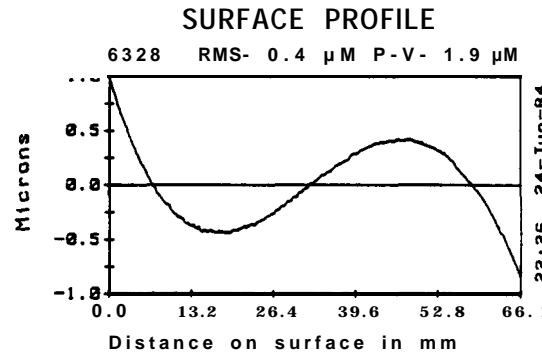
(b)

Fig. 3. (a) 2π ambiguity corrected surface height plot for λ_{eq} . (b) Same data as in Fig. 3(a) but with both tilt and focus removed.

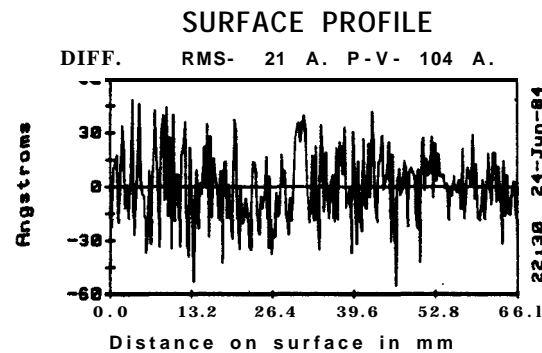
calculated so that the largest wave front slope becomes 3.3λ /pixel, and the TWLPSI stops working when λ_{eq} is shorter than $3.3 \mu\text{m}$. Three typical wavelengths used in this experiment are $\lambda_a = 4765 \text{ \AA}$, $\lambda_b = 5145 \text{ \AA}$, and $\lambda_c = 6326 \text{ \AA}$ [where $\lambda_{eq1} = \lambda_a \lambda_b / (\lambda_b - \lambda_a) = 6.45 \mu\text{m}$, $\lambda_{eqs} = \lambda_a \lambda_b / (\lambda_b - \lambda_a) = 1.93 \mu\text{m}$]. The computer did the calculations for the phase data for λ_a , λ_b , and λ_c , then calculates the phase data for λ_{eq1} and λ_{eqs} , according to Eq. (1). After integration, the surface profile data for λ_{eq1} are shown in Fig. 2(a). Figure 2(b) shows the same data but with both tilt and focus removed. The surface profile data for λ_{eqs} are shown in Fig. 2(c). For comparison, Fig. 2(d) shows the result of single-wavelength PSI for $\lambda = 6328 \text{ \AA}$; one can see how the 2π ambiguity problem limits the phase measurement range of the conventional single-wavelength PSI. The result shown in Fig. 2(a) shows the correct wave front under test because λ_{eq1} is long enough, but that shown in Fig. 2(c) contains 2π ambiguities since λ_{eqs} is too short. The first step is to use the phase data of λ_{eq1} as a reference to correct the 2π ambiguities in the phase data of λ_{eqs} by the method mentioned earlier. Figure 3(a) shows the corrected phase data for λ_{eqs} , and Fig. 3(b) shows the same data but with both tilt and focus removed. This corrected phase data for λ_{eqs} can be used as intermediate reference data so that one can go ahead and perform the second step 2π ambiguity correction work, i.e., correct the 2π ambiguities in any single-wavelength phase data. Figure 4(a) shows the final result for $\lambda = 6328 \text{ \AA}$. Figure 4(b) shows the same result but with



(a)



(b)



(c)

Fig. 4. (a) $2x$ ambiguity corrected single-wavelength surface height plot for $\lambda = 6328 \text{ \AA}$. (b) Same data as in Fig. 4(a) but with both tilt and focus removed. (c) Repeatability of the surface height measurement by the MWLPSI.

both tilt and focus removed. After temporal averaging of five sets of data to get rid of air turbulence problems, the repeatability of the surface height measurement by the MWLPSI is better than 25-\AA rms ($\lambda = 6328 \text{ \AA}$), as shown in Fig. 4(c).

III. Conclusion

A method to enhance the capability of TWLPSI has been developed. By introducing the phase data of a third wavelength, one can measure the phase of a very steep wave front. Since all necessary phase calculations have been done inside a computer, no hologram is required as in two-wavelength holography. A sample test measuring the surface height of a defocused off-axis parabola (wave front sag = $200 \mu\text{m}$) has been performed to verify the capability of the MWLPSI. After tem-

poral averaging of five sets of data, the repeatability of the measurement is better than 25-Å rms ($\lambda = 6328$ Å).

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