OPTICS 380A
Lab 8: Interference

This lab is an introduction to interference of light waves by division of wavefront. In the next lab, we will look at interference by division of amplitude. Three experiments will be done this week--Young's Double Slit (a classic!), Lloyd's Mirror (simple but elegant) and two-point interference. Briefly, we will use the Lloyd's Mirror experiment to measure the wavelength of a He-Ne laser, and Young's Double Slit is cast into a "real-world" type of problem, complete with error analysis.

INTERFERENCE AND INTERFEREROMETERS

An interferometer is an optical device which takes an input beam of light, splits it into two (or more) beams, then recombines the two beams back into one. This process of recombining is known as interference. The beam output by the interferometer will have a spatially-varying pattern of light and dark regions known as fringes. The intensity of a bright fringe may be greater than the sum of the intensities of the individual two beams, and a dark fringe may have zero intensity (totally dark). This process occurs when the light waves of the two beams have a predictable phase relationship with each other--i.e., when the beams are coherent with each other. A laser beam has a high degree of coherency, so we will use a laser as the source in our two experiments. The basic principle of how an interferometer works is shown in Figure 8.1.

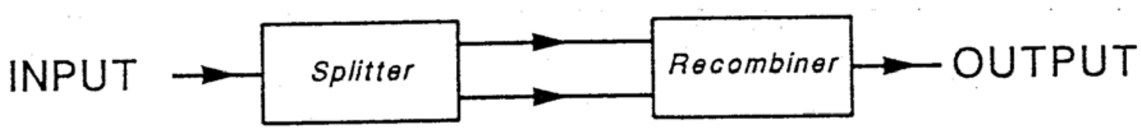

Figure 8.1. Block diagram of an interferometer.

There are two general ways of splitting the input beam into two beams to pass through an interferometer. The first method physically splits the input beam into two separate beams, and is called division of wavefront. This is usually done by placing two apertures in the beam, as shown in Figure 8.2. The second method creates two beams by partially reflecting and partially transmitting the beam, and is called division of amplitude. This is often done using a beamsplitter, as shown in Figure 8.3.

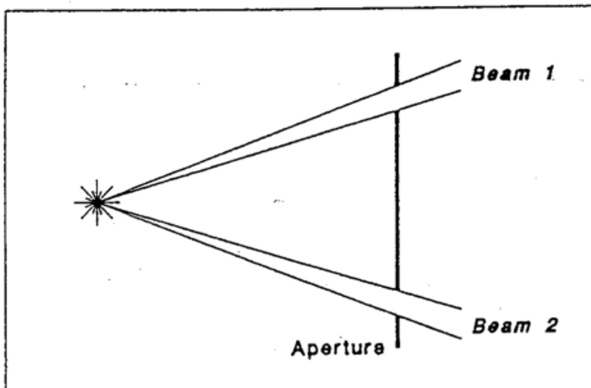

Figure 8.2. Interference by division of wavefront.

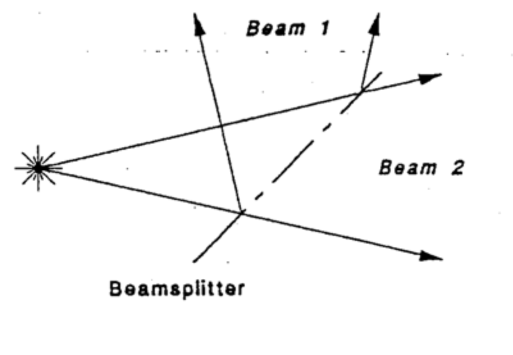

Figure 8.3. Interference by division of amplitude.

Young's Double Slit and Lloyd's Mirror experiments both use the method of division of wavefront. In Young's Double Slit, an aperture with two narrow slits creates two beams, which recombine by expanding as they travel beyond the aperture. Lloyd's Mirror uses a mirror (!) to re-direct the path of part of the beam, in effect creating two beams. As in Young's Double Slit, they recombine by expanding as they travel beyond the mirror.

Young's Double Slit

Thomas Young (1773-1829) was an English physician and physicist who discovered that light waves can interfere with each other, essentially proving the wave theory of light. However, the theory was still not fully accepted by English scientists until many years later, when the work of French physicists Arago and Fresnel confirmed it. Young performed his now-famous experiment in 1801, and aside from the fact that our lab uses a laser as a source, it is the same experiment. An interesting side-note is that while he was a medical student, Young discovered the mechanism by which the eye focuses on objects at various distances known as accommodation.

The setup for Young's Double Slit experiment is shown in Figure 8.4. A laser is used as a source of light to illuminate two slits, S_1 and S_2, separated by a distance d. In practice, the slit widths are on the order of 50 microns, and are equal to each other. The distance d is on the order of a couple hundred microns. The distance D, from the slits to the plane of observation is much larger, on the order of one or two meters. Fringes are created throughout the space where the two beams overlap, but distance D is made large so that the fringes are large enough to be observed clearly.

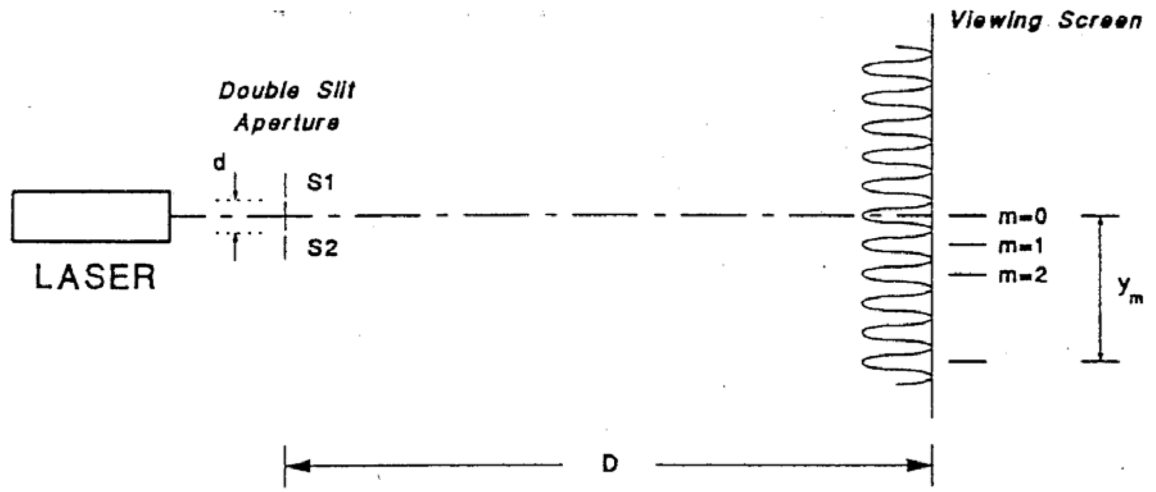

Figure 8.4. Setup for Young's Double Slit experiment (**NOT TO SCALE**).

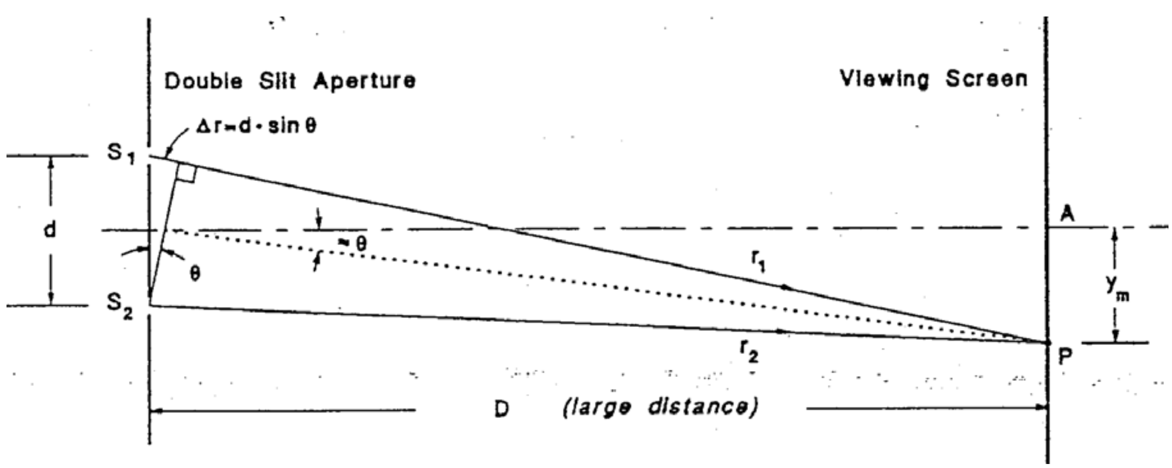

Figure 8.5. Optical paths in Young's Double Slit experiment.

The physics behind the experiment may be explained with the help of Figure 8.5. Note that Figure 8.5 is a top-view of the situation, showing a horizontal slice through the geometry involved. Two slits are illuminated by some wavefront, perhaps a plane wave from a laser. Huygens principle says that each point in the slits emits a spherically-expanding wavefront. Taken as a collection of point sources, each slit may be thought as emitting a cylindrical wavefront. The phases of these two wavefronts at each slit, at any instant in time, are the same because the illuminating beam from the laser has a constant phase across its wavefront. In other words, the laser beam has a high degree of spatial coherence.

The distances r_1 and r_2, from the center of each slit to an arbitrary point P in the observation plane, are nearly the same but differ by an amount Δr. This difference is a function of the slit separation d, and the angle θ between the on-axis point A and observation point P. The geometry shows that Δr is equal to $d\sin\theta$. As a result of this path difference, the phase difference δ between the two beams, at Point P, is equal to

$$\begin{aligned} \delta &= 2\pi\left(\frac{\Delta r}{\lambda}\right) \\ &= 2\pi\left(\frac{d\cdot\sin\theta}{\lambda}\right) \end{aligned} \tag{8.1}$$

where δ is given in radians.

Constructive interference occurs when the path difference equals $m\lambda$, an integer number of wavelengths, where m=0, 1, 2… In this situation, the two wavefronts arriving at point P have an absolute phase difference equal to an integer number times 2π. Because the waves are sinusoidal functions, this statement is equivalent to saying that the <u>relative</u> phase difference is zero, that crests from both waves arrive at P, and that the waves are in-phase. A bright fringe of maximum intensity is then observed at point P. Note that m=0 describes the on-axis observation point A, where the path difference is zero.

Destructive interference occurs when the path difference equals $(m+½)\lambda$, an odd number of half-wavelengths, where m=0, 1, 2… In this situation, the two wavefronts arriving at point P have an <u>absolute</u> phase difference equal to an integer number times π. Again, because the waves are sinusoidal functions, this statement is equivalent to saying that the <u>relative</u> phase difference is 180 degrees, that a crest and a trough from the waves arrive at P, and that the waves are out-of-phase. A dark fringe of minimum (zero) intensity is then observed at point P.

At points between these maxima and minima, the two waves arriving at the observation point have a relative phase difference between 0 and 180 degrees. The intensity of the fringe produced at this point is correspondingly between the maximum and minimum values as described above.

The distance y_m, from the on-axis point to an observation point of maximum intensity, may be written as follows. Because angle θ is small, we may write that

$$y_m \approx D \cdot \theta = \frac{mD\lambda}{d} \tag{8.2}$$

By measuring y_m, d, and D, one can calculate λ, the wavelength of light. This is exactly what Young did to measure the seven wavelengths of light that Newton had earlier predicted from his experiments with prisms and refraction. Conversely, one can know λ, measure y_m, and D to calculate the slit width d. This situation is the basis for our experiment that follows.

Lloyd's Mirror was first described in 1834, and is another classic demonstration of interference by division of wavefront. In our experiment, as in the original description, we will use the method to determine the wavelength of a source of light. Figure 8.6 shows the setup used for Lloyd's Mirror.

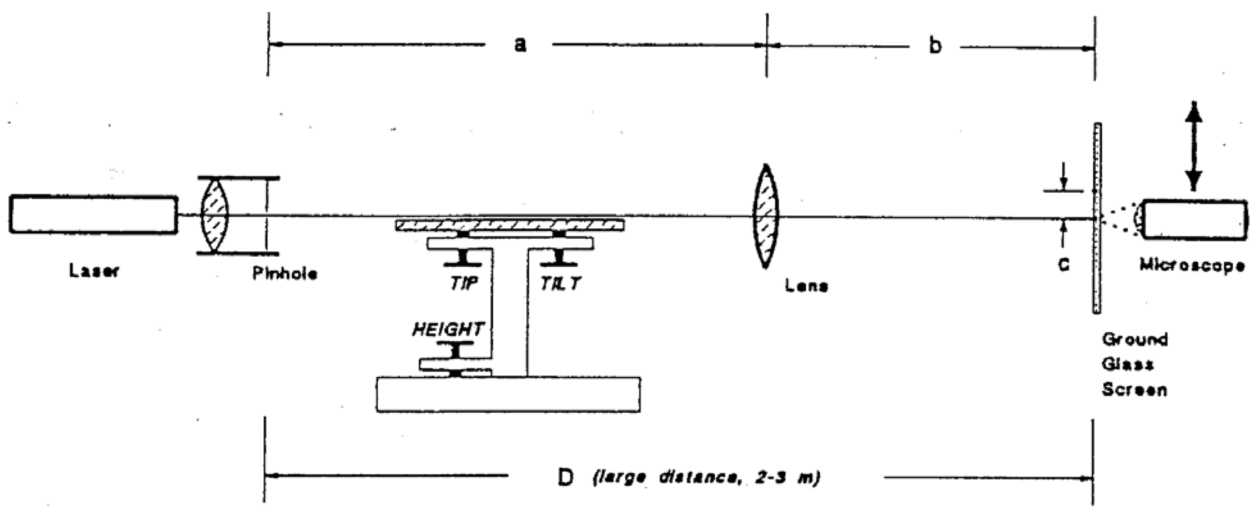

Figure 8.6. Setup used for Lloyd's Mirror (**NOT TO SCALE**).

A laser is used as the source of monochromatic light. The beam is focused through a small pinhole, on the order of 50 microns in diameter. This pinhole becomes the point source for the remainder of the experiment. A positive lens is placed a distance **a** away from the pinhole, and focuses the point source onto a screen a further distance **b** away. The overall distance from the pinhole to the screen is on the order of 1-2 meters. Next, a plane front-surface mirror is brought up in to the beam, so that its surface lies just below the optical axis. This proper location is found when a second point image appears a small distance c away from the original point image. The height of the mirror is adjusted so that distance c is between 2-5 mm. At this point, the lens is removed, and straight-line interference fringes are observed on the observation screen, just as in Young's Double Slit experiment.

The physics behind Lloyd's Mirror is essentially the same as in Young's Double Slit, as is shown in Figure 8.7. In Lloyd's Mirror, the mirror is used to divide the original wavefront into two, in effect creating a second, "virtual" point source PH". Ray #1 travels directly from the pinhole PH to the observation point P. Ray #2 reflects from the mirror, arrives at point P but appears to originate from a second pinhole PH. The height of the mirror changes the apparent separation of these two point sources--the closer the mirror surface is to the optical axis, the closer the point sources appear to be to each other.

Correspondingly, this also changes the fringe separation. The closer the point sources, the more widely separated are the fringes. The farther apart the point sources, the more closely spaced are the fringes. (This is exactly the same relationship between slit separation and fringe spacing in Young's Double Slit.)

As in Young's Double Slit, the path difference Δr is again equal to dsinθ, and fringes are formed in exactly the same manner as described earlier. However, there is one difference with this arrangement. Fresnel reflection from the mirror produces a phase change in the second beam of 180 degrees. The result is that the on-axis fringe is a dark fringe of maximum intensity. In other words, the fringe pattern of Lloyd's Mirror is complementary to that found in Young's Double Slit. In practice, the beam leaving the pinhole PH "overfills" the mirror, striking its front edge. This produces another set of fringes due to diffraction, which are seen at the bottom of the fringe pattern from Lloyd's Mirror. Be careful not to include these (more widely-spaced) fringes when measuring the fringes from Lloyd's Mirror.

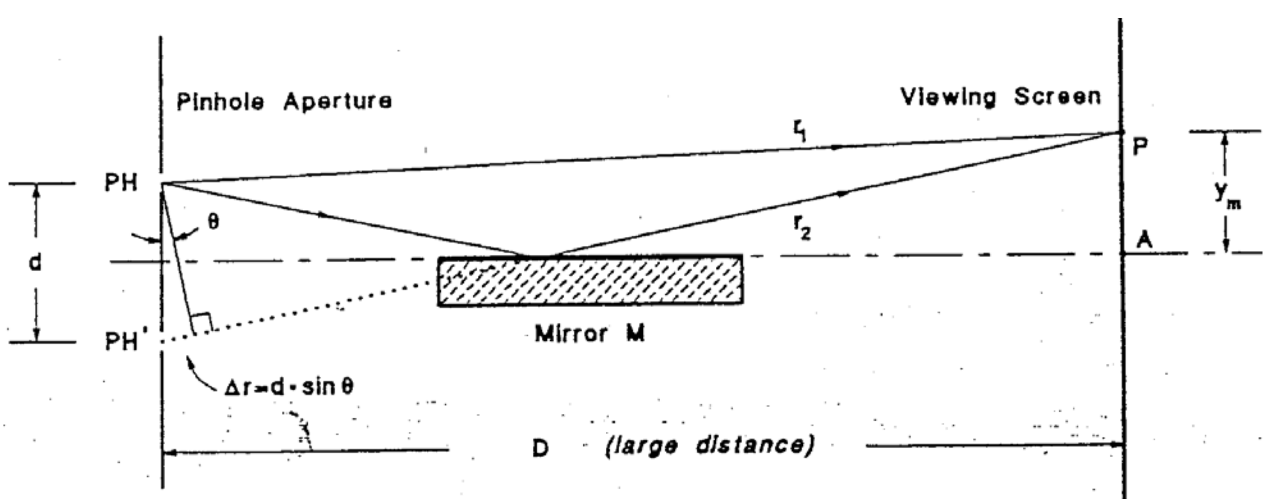

Figure 8.7. Optical paths in Lloyd's Mirror experiment.

Part A: Young's Double Slit Interferometer (YDSI)

We will use this experiment as an example of a real-world problem in acceptance testing. Suppose you have just started working for an optical laboratory, and your boss hands you a package that she purchased from Company P. Of interest are the double slits, as they will be used your company's next generation of optical spectrometer. You are asked to measure the spacing between the two slits, as this is the important parameter needed in the company's current initial design effort. Company P has supplied information that the "A" slit on slide 9165-B has a center-to-center spacing of 250 microns, with a tolerance of ±5 microns. She wants to know if this information is accurate or not. Depending on your results, the design engineers will know whether or not to trust the data from this particular company. If this information is valid, you are to go ahead and order 500 of these slits (at considerable cost) of the first production run of the spectrometer. You are asked to do this as soon as you can, with as little expenses as possible… (usually that means by tomorrow). All that is asked of you is to make the most accurate measurement that you can with equipment available.

You quickly think to yourself, "this is an optics lab, not a measurements lab…how does she expect me to do this?" The only tools you have available are a laser, some optical mounts (not even as nice as you had in optics lab in school), a small microscope, and a translation slide. If only you had one of those brand new, expensive measuring microscopes… or how about an electron microscope…that's really what's needed, you think. After a few cups of coffee, you begin to remember something about what you learned in Physical Optics, some relationship between slits and fringes. After reviewing Young's Double Slit experiment, you realize that maybe this is possible after all…

You begin to think…I need to know the wavelength of this laser and the angle subtended by the dark fringes at the slit for a particular order of fringe (the center of a dark fringe may be easier to locate than the center of a bright fringe). Ok…I can calculate that angle if I know something about the distance between the slits and the fringes, and also the fringe spacing. Before starting work, you realize that she wants to know HOW ACCURATE your measurement is…for simplicity you'll assume all errors to be independent, allowing them to add in a RSS fashion. After some quick math, you realize that you'll need to consider or estimate the errors involved in your measurements. As a starting point, you decide the following:

- The error in knowing the laser wavelength is so small that you'll set it equal to zero.
- Given the optical bench that you have, you will consider the error in measuring the distance from the slit to the fringes to be ± 1mm.

- Let's see…the microscope and stage are pretty good, and after some practice in measuring the fringes, you decide that you can locate the center of a dark fringe to within ±10% of the width of on a dark band or fringe.

- For more accuracy, you decide to measure across more than one fringe with the microscope.

- The angles involved are so small that sinθ = θ.

With that, (and with the equipment in front of you), you get started. What do you report to your boss the next day? **(This answer and how well you support it is how this part of the lab will be graded.)**

Part B: Lloyd's Mirror

The purpose of this experiment is to use the interference pattern created by the divided wavefront to measure the wavelength of the He-Ne laser. For purposes of reference, the wavelength (in air) of the laser is 0.6328 microns. The experiment is interesting in that it shows how such a small quantity as wavelength can be found by measuring much larger quantities such as distances and fringe spacings. As it stands on the optical table, the experiment is set up and ready for data-taking.

The equation for dark fringe centers is

$$y_m = \frac{mD\lambda}{d} \quad . \qquad (8.3)$$

Fringe spacing is given by

$$\Lambda = \frac{D\lambda}{d} \quad . \qquad (8.4)$$

Equation (8.5) can be rewritten in terms of wavelength such that

$$\lambda = \frac{d\Lambda}{D} \quad . \qquad (8.5)$$

We can measure *D* and Λ, but we have trouble measuring *d*. However, we can measure *changes* in *d* accurately with a dial indicator. The dial indicator used in this lab changes one tic mark on the dial for every 10 microns of actuator movement. Starting with Eq. (8.4), rearranging for *d* and using difference calculus to relate changes in Λ to changes in *d*, we find

$$\Delta d = \frac{D\lambda}{\overline{\Lambda}^2}\Delta\Lambda \quad , \qquad (8.6)$$

where $\bar{\Lambda}$ is the average of two fringe spacing measurements. Equation (8.6) can be arranged to yield λ in terms of the other variables, where

$$\lambda = \left(\frac{\Delta d}{D}\right)\left(\frac{\bar{\Lambda}^2}{\Delta\Lambda}\right). \tag{8.7}$$

(A) Measure the distance *D* and fringe spacing Λ_1. To measure fringe spacing, it will probably be more accurate to measure over a distance of *N* fringes and divide the result by *N*.

(B) Change the vertical height of the mirror by about 200 microns. How does a change in the mirror height correspond to a change in *d*?

(C) Measure the fringe spacing Λ_2.

(D) Calculate the wavelength from Eq. (8.7), and comment on the accuracy.

(E) What measurements are most prone to errors?

(F) Devise one method to reduce a measurement error.

[Parts (G) and (H) were removed from this procedure.]

Part C: Two-Point Interference

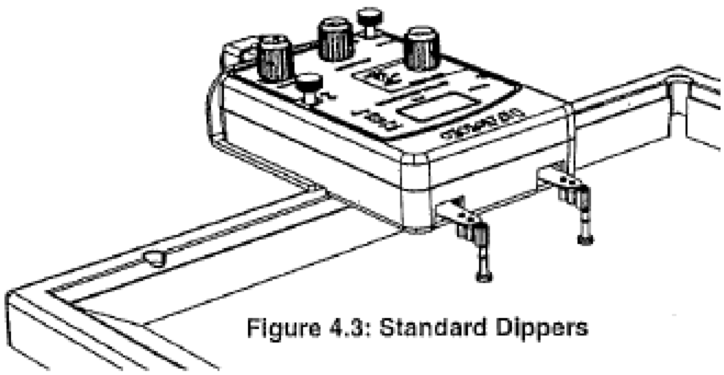

Figure 8.8 Arrangement for two-point interference.

(I) Setup the ripple tank as shown above. The dippers represent two point sources. Set the phase of the dippers to be "in phase". Change the source frequency until the dippers are separated by 1, 2 and 4 wavelengths, recording the frequency settings. For each wavelength, draw the interference lines on the shadow paper below the tank.

(J) Compare your fringe lines with theoretical curves given in class.

(K) Change the phase of the dippers to "out of phase". Draw the interference lines on the shadow paper for each of the three wavelengths in Part (I). Explain the shape of the interference pattern.