KEY

1.)

a.) (5 pts.) Determine the refractive index and thickness of a thin film to be deposited on a glass surface with $n = 1.5$ such that no normally incident light of wavelength 600 nm is reflected.

$$n_{film} = \sqrt{1.5} = \boxed{1.22}$$

$$t = \frac{\lambda}{4 n_{film}} = \frac{600}{4 \cdot 1.22} = \boxed{123\ nm}$$

b.) (5 pts.) If a white-light source is used to illuminate the surface, what color is observed in transmission after the film is deposited? Choose between red, green and blue. Justify your answer.

Red portion of the spectrum is transmitted. Eye is not as sensitive to blue.

c.) (5 pts.) If the light illuminating the surface ~~id~~ is incident at 45 degrees, what is the wavelength at which ~~peak~~ minimum reflection occurs?

$$\sin\theta_2 = \frac{1}{1.225}\sin 45^\circ$$

$$\theta_2 = 35.26^\circ$$

$$2nt\cos 35.26^\circ = m\lambda_2 \qquad m = \frac{1}{2}$$

$$\lambda_2 = \cancel{4n}\frac{\lambda_1}{\cancel{4n}}\cos 35.26^\circ = 600 \cdot \cos 35.26^\circ$$

$$= \boxed{489.9\ nm}$$

(Part 1c has a typo in it. It should ask for the wavelength of minimum reflection. If students attempt to find the peak wavelength in a correct way, give them full credit.)

2.) Consider diffraction by a plane screen.

a.) (3 pts.) Write the Green's function G associated with each boundary condition named below.

i.) Kirchhoff

$$\frac{e^{jkr_{01}}}{r_{01}}$$

ii.) Neumann

$$\frac{e^{jkr_{01}}}{r_{01}} + \frac{e^{jkr_{01}'}}{r_{01}'}$$

iii.) Dirichlet

$$\frac{e^{jkr_{01}}}{r_{01}} - \frac{e^{jkr_{01}'}}{r_{01}'}$$

b.) (3 pts.) In order to find the diffraction at an observation point to the right of the screen, what conditions must be placed on U and $\frac{\partial U}{\partial n}$ in the open part of the aperture for each G in (a)?

i) u and $\frac{\partial u}{\partial n}$ same as w/o screen

ii) $\frac{\partial u}{\partial n}$ same as w/o screen

iii) u same as w/o screen

c.) (3 pts.) In order to find the diffraction at an observation point to the right of the screen, what conditions must be placed on U and $\frac{\partial U}{\partial n}$ in the shadow of the screen for each G in (a)?

i) $u = 0$ & $\frac{\partial u}{\partial n} = 0$

ii) $\frac{\partial u}{\partial n} = 0$

iii) $u = 0$

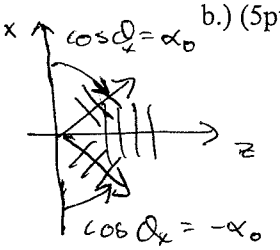
3.) A grating with amplitude transmittance $t(x) = 1 + \cos\left(\frac{2\pi}{\lambda}\alpha_o x\right)$ is illuminated from the left by an on-axis monochromatic plane wave with $\lambda = 0.5$ μm.

a.) (5 pts.) Find the angular spectrum $A_o(\alpha, \beta)$ of the electric field immediately behind the grating.

$$A_o(\alpha,\beta) = \lambda^2 \mathcal{F}\{U(u,v)\}_{\alpha,\beta} \qquad u = \frac{x}{\lambda} \quad v = \frac{y}{\lambda}$$

$$= \lambda^2 \mathcal{F}\{1 + \cos(\tfrac{2\pi}{\lambda} x_o \lambda u)\}_{\alpha,\beta}$$

$$= \lambda^2 \left[\delta(\alpha,\beta) + \tfrac{1}{2}\{\delta(\alpha-\alpha_o) + \delta(\alpha+\alpha_o)\}\delta(\beta)\right]$$



b.) (5pts.) Give a physical interpretation of the result you found in (a).

Three plane waves, One on-axis, Two symmetrically propagating plane waves with direction cosines $\pm\alpha_o$.

c.) (5 pts.) If $\alpha_o = 0.01$, what is the electric field distribution a distance z = 5 mm from the grating?

$$H(\gamma; z) = e^{jk\gamma z} \approx e^{jkz} e^{-j\frac{kz}{2}\sigma^2}$$

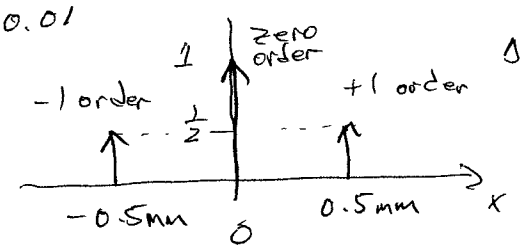
$$A(\alpha,\beta;z) = \lambda^2 e^{jkz}\left[\delta(\alpha,\beta) + \tfrac{1}{2}e^{-j\pi}\{\delta(\alpha+\alpha_o) + \delta(\alpha+\alpha_o)\}\delta(\beta)\right]$$

$$= \lambda^2 e^{jkz}\left[\delta(\alpha,\beta) - \tfrac{1}{2}\{\delta(\alpha-\alpha_o) + \delta(\alpha+\alpha_o)\}\delta(\beta)\right]$$

$$\Rightarrow \quad U(x_o, y_o) = e^{jkz}\left[1 - \cos\left(\tfrac{2\pi}{\lambda}\alpha_o x\right)\right]$$

d.) (5 pts.) A simple lens is placed a distance 10 mm to the right of the grating. Assume that the lens is perfect and is infinitely large. The focal length of the lens is 50 mm. What is the relative distribution of the irradiance in the focal plane of the lens?

for $\alpha_o = 0.01$



$\Delta x = f\alpha_o$
$= 0.5$ mm

e.) (5 pts.) How does the irradiance pattern in part (d) change if the grating frequency $\frac{\alpha_o}{\lambda}$ increases?

The two orders separate away from the zero order.

4.) A 1 mm diameter hole is illuminated with a $\lambda = 600$ nm normally incident plane wave.

a.) (10 pts.) Sketch the on-axis irradiance from $z = 0.1$ m to $z = 0.25$ m, indicating the positions of odd and even Fresnel numbers.

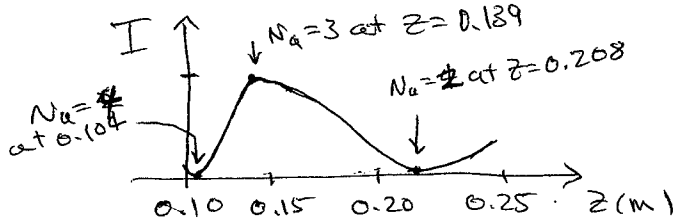
$$z = 0.1\text{ m} \Rightarrow N_a = \frac{(0.5\times10^{-3})^2}{(0.6\times10^{-6})(0.1)} = \frac{0.25}{0.6}\cdot 10 = 4.16$$

$$z = 0.25\text{ m} \Rightarrow N_a = \frac{(0.5\times10^{-3})^2}{(0.6\times10^{-6})(0.25)} = \frac{0.25}{0.6}\cdot 4 = 1.67$$

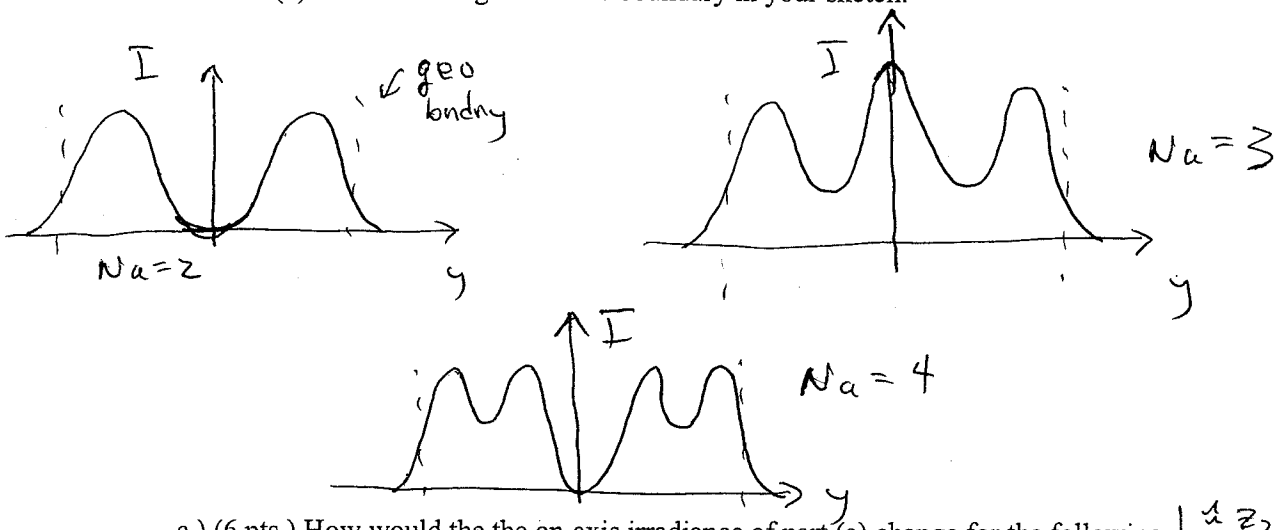
$$N_a = 2 \Rightarrow z = \frac{(0.5\times10^{-3})^2}{(0.6\times10^{-6})(2)} = \frac{0.25}{0.6}\cdot\frac{1}{2} = 0.208$$

$$N_a = 3 \Rightarrow z = \frac{0.25}{0.6}\cdot\frac{1}{3} = 0.139$$

$$N_a = 4 \Rightarrow z = \frac{0.25}{0.6}\cdot\frac{1}{4} = 0.104$$



b.) (5 pts.) Sketch the transverse irradiance profiles at each integer Fresnel number in part (a). Indicate the geometrical boundary in your sketch.



c.) (6 pts.) How would the the on-axis irradiance of part (a) change for the following cases. Draw a rough sketch indicating the envelope of the modulation. Also indicate whether the peak locations shift toward the hole or away from it.

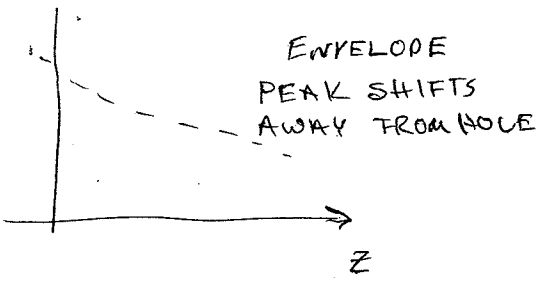
i.) diverging illumination ($z_1 = 10$ m)?

ii.) converging illumination (z_1=-10 m)?

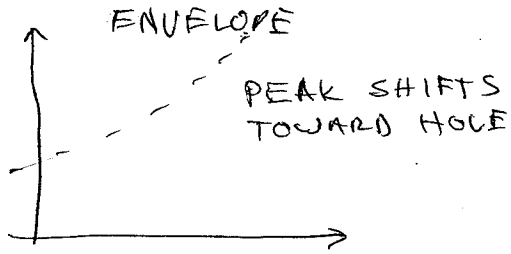
$L \approx z_2(1 - 1/z_1)$

$z_2 \approx \frac{a^2}{N_a\lambda}(1 + 1/z_1)$

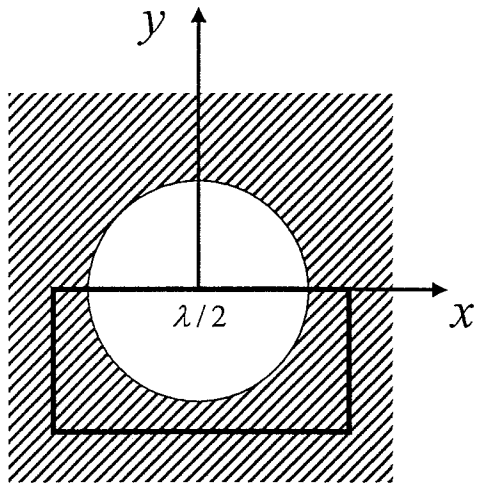
i)



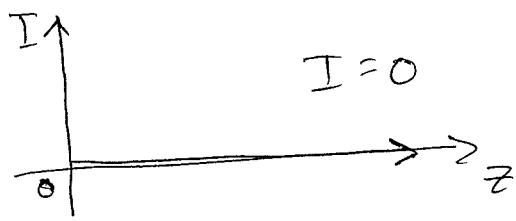
ii)



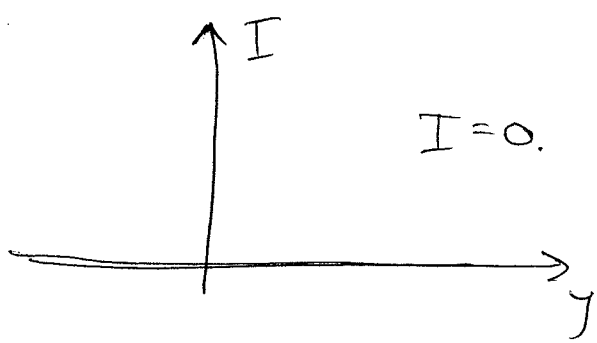
5.) Consider the 1 mm diameter hole of Problem (4), where a $\lambda/2$ plate is placed so that it covers exactly half of the open area, as shown below.



a.) (10 pts.) Sketch the on-axis irradiance from $z = 0.1$ m to $z = 0.25$ m.



b.) (5 pts.) Draw a rough sketch of the transverse irradiance profile along $y = 0$ at $z = 0.1$ m.



6.) A collimated 1 W/cm^2 laser beam of wavelength 632.8 nm illuminates a Fresnel zone plate.

a.) (5 pts.) The primary focus occurs at z = 100 mm. What is the radius of the first open area in the zone plate?

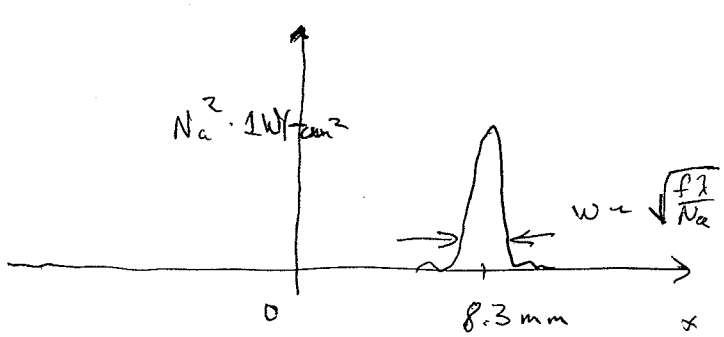
$$\rho_1 = \sqrt{\lambda f_1} = \boxed{0.252 \text{ mm}}$$

b.) (5 pts.) If the zone plate is heated, it uniformly expands so that the open and closed areas of the plate increase their diameter. If the plate is heated so that the diameter of the first area increases by 1% , where is the new position of the primary focus?

$$f_1' = \frac{\rho^2 (1.01)^2}{\lambda} = f_1 (1.01)^2 = \boxed{102 \text{ mm}}$$

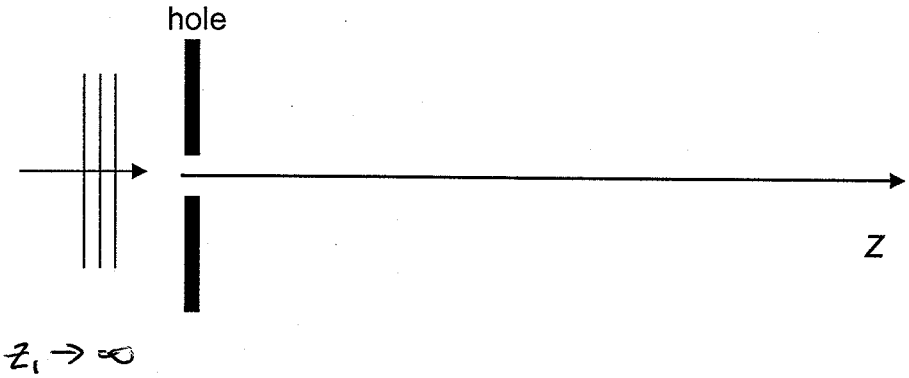
c.) (5 pts.) If the x-angle of the illuminating plane wave is changed to 5 degrees with respect to the z axis in part (a), sketch the transverse irradiance at z = 100 mm along the x axis.

$$\Delta x \approx \frac{5}{60} \cdot 100 = 8.3 \text{ mm}$$



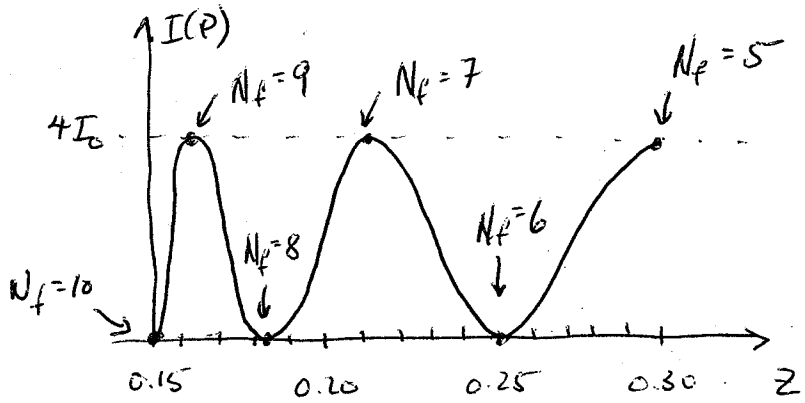
KEY

1.) A round hole of diameter 2 mm is illuminated with an on-axis $\lambda = 0.667$ μm plane wave as shown below.

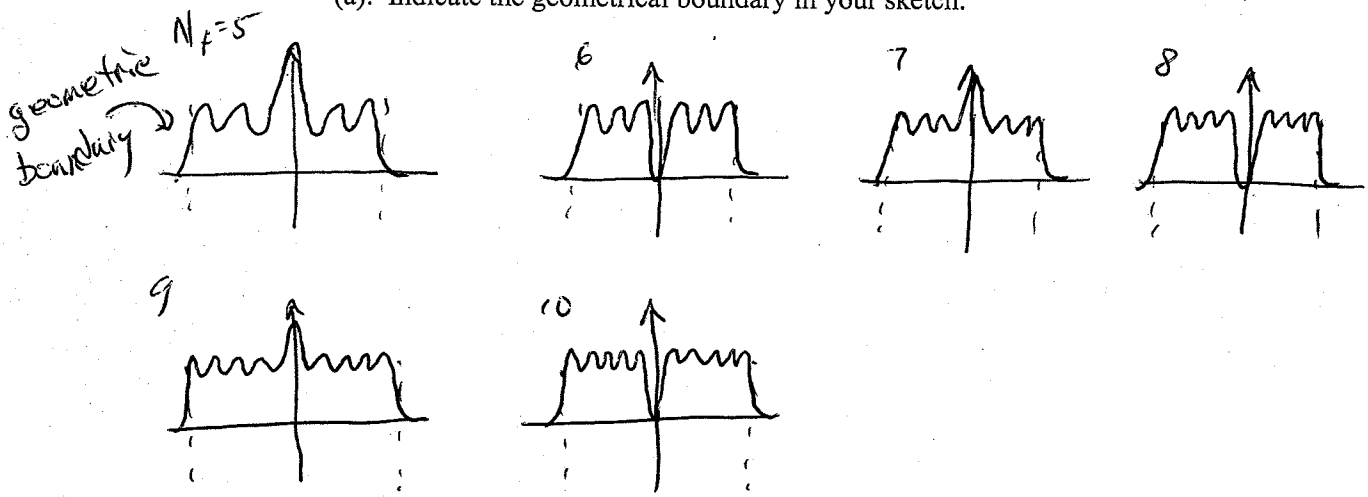


a.) (10 pts.) Sketch the on-axis irradiance from $z = 0.15$ m to $z = 0.3$ m, indicating the positions of odd and even Fresnel numbers.

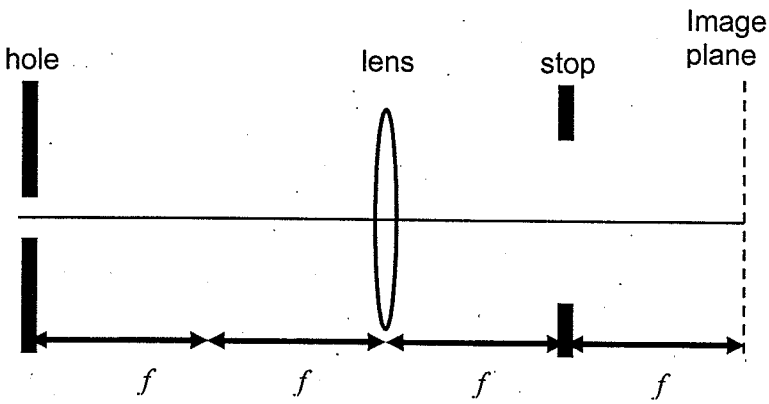
N_f	z_2
5	0.2999
6	0.2499
7	0.2142
8	0.1824
9	0.1666
10	0.1499



b.) (5 pts.) Sketch the transverse irradiance profiles at each integer Fresnel number in part (a). Indicate the geometrical boundary in your sketch.



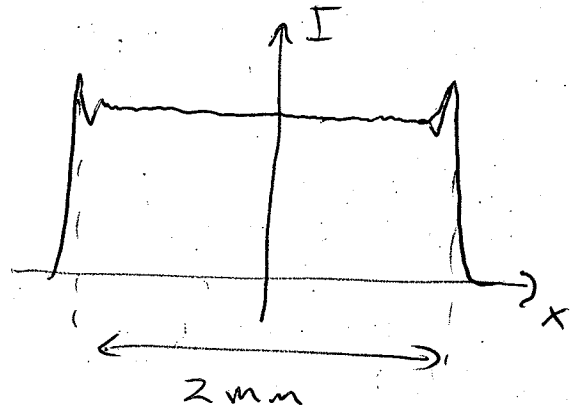
2.) A lens is positioned in back of the hole of Problem (1) as shown below. The focal length of the ideal thin lens is 15 mm. Other parameters are the same as Problem (1).



(The drawing is not to scale)

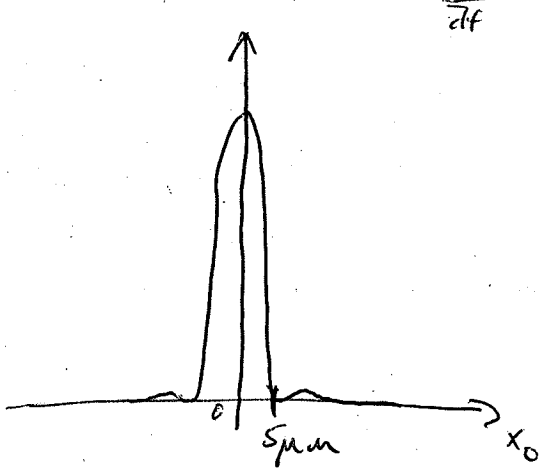
a.) (5 pts) Sketch the irradiance profile at the lens.

$$N_f = \frac{a^2}{\lambda z} = 50$$



b.) (5 pts) Sketch the irradiance profile in the back focal plane at distance f behind the lens in the x direction if the lens diameter is much greater than the irradiance boundary in part (a). Label the width of any primary features and their positions. (Hint: Treat this problem like a converging wave incident on an aperture, where the width of the irradiance in part (a) defines the aperture.)

$$I \sim \left| \mathcal{F}\left\{ \mathrm{rect}\left(\frac{x}{d}\right) \right\}_{\frac{x_0}{\lambda f}} \right|^2 \sim \mathrm{sinc}^2\left(\frac{d x_0}{\lambda f}\right)$$



distance to first zero

$$x_0 = \frac{\lambda f}{d} = \frac{0.667\times10^{-6} \cdot 15\times10^{-3}}{2\times10^{-3}}$$

$$= 5\times10^{-6}$$

3.) The hole of problem (2) is filled with a simple cosine grating that has an amplitude transmission function $t(x) = 1 + \cos\left(2\pi \frac{\alpha_0}{\lambda} x\right)$. $\frac{\alpha_0}{\lambda} = \nu_{x0} = \frac{1}{5} \times 10^6 m^{-1}$. That is, the grating period is 5 μm. Other parameters are identical to those for problem (2).

a.) (5 pts) Find an expression for the angular spectrum in the x direction of the transmitted light immediately behind the grating. You simplify the problem by assuming a one-dimensional geometry with the transmission of the aperture represented by a rect function.

$$\nu_{x0} = \frac{\alpha_0}{\lambda}$$

$$A_{z=0}(\nu_x) = \mathcal{F}\left\{ \mathrm{rect}\left(\frac{x}{d}\right)\left[1 + \cos\left(2\pi \frac{\alpha_0}{\lambda} x\right)\right]\right\}_{\nu_x}$$

$$= d\,\mathrm{sinc}(d\nu_x) * \left[\delta(\nu_x) + \tfrac{1}{2}\delta(\nu_x - \nu_{x0}) + \tfrac{1}{2}\delta(\nu_x + \nu_{x0})\right]$$

b.) (5 pts) Find an expression for the angular spectrum in the x direction of the light incident onto the lens.

$$A(\nu_x, z) = A_{z=0}(\nu_x)\, e^{jk\gamma z}$$

$$= d e^{jk\gamma z}\left[\mathrm{sinc}(d\nu_x) * \left[\delta(\nu_x) + \tfrac{1}{2}\delta(\nu_x - \nu_{x0}) + \tfrac{1}{2}\delta(\nu_x + \nu_{x0})\right]\right]$$

c.) (5 pts) Accurately graph the amplitude and phase of (b) in the regions of non-zero amplitude. There should be three such regions (-1 order, zero order and +1 order). Limit the range of ν_x to +/- 4000 m^{-1} around each maximum amplitude. Notice that the phases around the +/- 1 orders are well represented by straight lines.

(Use an additional page)

d.) (5 pts) Find the coefficients for the approximation of the phase of $H(\gamma; z) = e^{jkz\sqrt{1-(\lambda\nu_x)^2}}$ as expressed below around $\nu_x = \nu_{x0} = 2\text{x}10^5\ m^{-1}$ with $z = 30$ mm and $\lambda = 0.667$ μm.(Hint: Remember that the Taylor series approximation of function $f(x)$ around $x = a$ is given by

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots.)$$

$$kz\sqrt{1-(\lambda\nu_x)^2} \approx 2\pi z\left[c_0 \mp c_1(\nu_x \pm \nu_{x0}) + c_2(\nu_x \pm \nu_{x0})^2\right] \qquad (\lambda\nu_{x0}) = 0.1334$$

for c_0: $c_0 = \frac{1}{\lambda}\sqrt{1-(\lambda\nu_{x0})^2} = 1.4859\times10^{-6}$

for c_1: $c_1 = \frac{\partial}{\partial\nu_x}\left[\frac{1}{\lambda}\sqrt{1-(\lambda\nu_x)^2}\right] = \frac{-(\lambda\nu_x)}{\sqrt{1-(\lambda\nu_x)^2}} = -0.1346$

for c_2: $c_2 = \frac{\partial^2}{\partial\nu_x^2}\left[\frac{1}{\lambda}\sqrt{1-(\lambda\nu_x)^2}\right] = \frac{\lambda}{\left[1-(\lambda\nu_x)^2\right]^{3/2}} = 6.8521\times10^{-7}$

e.) (5 pts) Plot the difference between the true phase $kz\sqrt{1-(\lambda\nu_x)^2}$ and the linear approximation $2\pi z\left[c_0 + c_1(\nu_x \pm \nu_{x0})\right]$ in regions of the +/- 1st orders, and compare the residual phase with the phase of the zero order found in part (c). Notice that this phase distribution causes the diffraction effects observed in Problem 2(a).

(Use an additional page)

f.) (5 pts) Show through mathematical development or reasoning that the field incident onto the lens can be approximated by

$M(x) + \frac{1}{2}M(x+c_1z)e^{j\phi}e^{-j2\pi\nu_{x0}x} + \frac{1}{2}M(x-c_1z)e^{j\phi}e^{j2\pi\nu_{x0}x}$, where $M(x)$ is the field that produces the irradiance pattern of Problem 2(a).

$\phi = 2\pi z c_0$

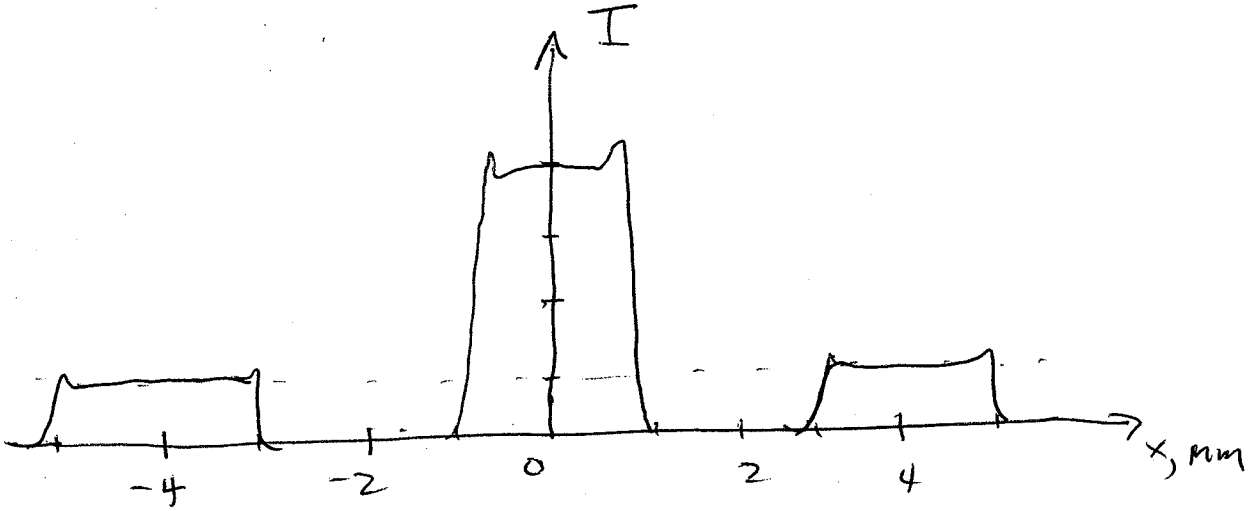
$$A(\nu_x;z) \approx d\,e^{j\gamma z}\,\mathrm{sinc}(d\nu_x) + \frac{d}{2}e^{j2\pi z c_0}e^{j2\pi z c_2(\nu_x-\nu_{x0})^2}\,\mathrm{sinc}\left(d[\nu_x-\nu_{x0}]\right)e^{j2\pi z c_1(\nu_x-\nu_{x0})} + \frac{d}{2}e^{j2\pi z c_0}e^{j2\pi z c_2(\nu_x+\nu_{x0})^2}\,\mathrm{sinc}\left(d[\nu_x+\nu_{x0}]\right)e^{-j2\pi z c_1(\nu_x+\nu_{x0})}$$

$$\mathcal{F}_1^{-1}\{A(\nu_x;z)\} = \mathcal{F}_1^{-1}\{d\,e^{jk\gamma z}\,\mathrm{sinc}(d\nu_x)\} + \frac{1}{2}\left[\mathcal{F}_1^{-1}\{d\,e^{j2\pi z c_2\nu_x^2}\,\mathrm{sinc}(d\nu_x)\} * \delta(x+c_1z)\right]e^{-j2\pi\nu_{x0}x}e^{j\phi} + \frac{1}{2}\left[\mathcal{F}_1^{-1}\{d\,e^{j2\pi z c_2\nu_x^2}\,\mathrm{sinc}(d\nu_x)\} * \delta(x-c_1z)\right]e^{j2\pi\nu_{x0}x}e^{j\phi}$$

$$= M(x) + \frac{1}{2}M(x-c_1z)e^{j2\pi\nu_{x0}x}e^{j\phi} + \frac{1}{2}M(x+c_1z)e^{-j2\pi\nu_{x0}x}e^{j\phi}$$

where $M(x) = \mathcal{F}_1^{-1}\{d\,e^{jk\gamma z}\,\mathrm{sinc}(d\nu_x)\}$

g.) (5 pts) Sketch the irradiance profile at the lens in the x direction.



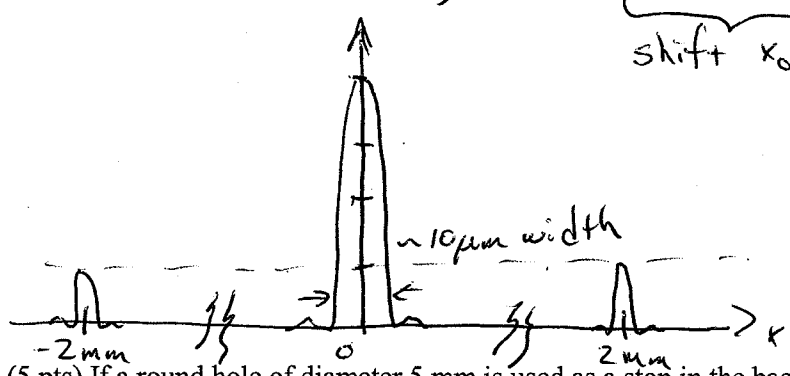
h.) (5 pts) Sketch the irradiance profile in the back focal plane at distance f behind the lens in the x direction if the lens diameter is much greater than the irradiance in part (g). Label the width of any primary features and their positions.

$$U(x_0) \sim \mathcal{F}\{U_{lens}(x_1)\}_{x_0/\lambda f} = A(\nu_x; z)$$

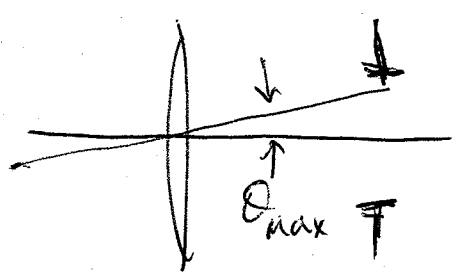
$$\nu_y = \frac{x_0}{\lambda f}$$

$$\therefore I(x_0) \propto \mathrm{sinc}^2\left(\frac{dx}{\lambda f}\right) + \frac{1}{4}\mathrm{sinc}^2\underbrace{\left[d\left(\frac{x_0}{\lambda f} - \nu_{x0}\right)\right]}_{\text{shift } x_0 - \frac{\lambda f}{T}} + \frac{1}{4}\mathrm{sinc}^2\underbrace{\left[d\left(\frac{x_0}{\lambda f} + \nu_{x0}\right)\right]}_{\text{shift } x_0 + \frac{\lambda f}{T}}$$

$$\frac{\lambda f}{T} = \frac{0.667\times10^{-6}\times15\times10^{-3}}{5\times10^{-6}} = 2\text{mm}$$



i.) (5 pts) If a round hole of diameter 5 mm is used as a stop in the back focal plane, what is the minimum grating period that can be imaged by the lens?

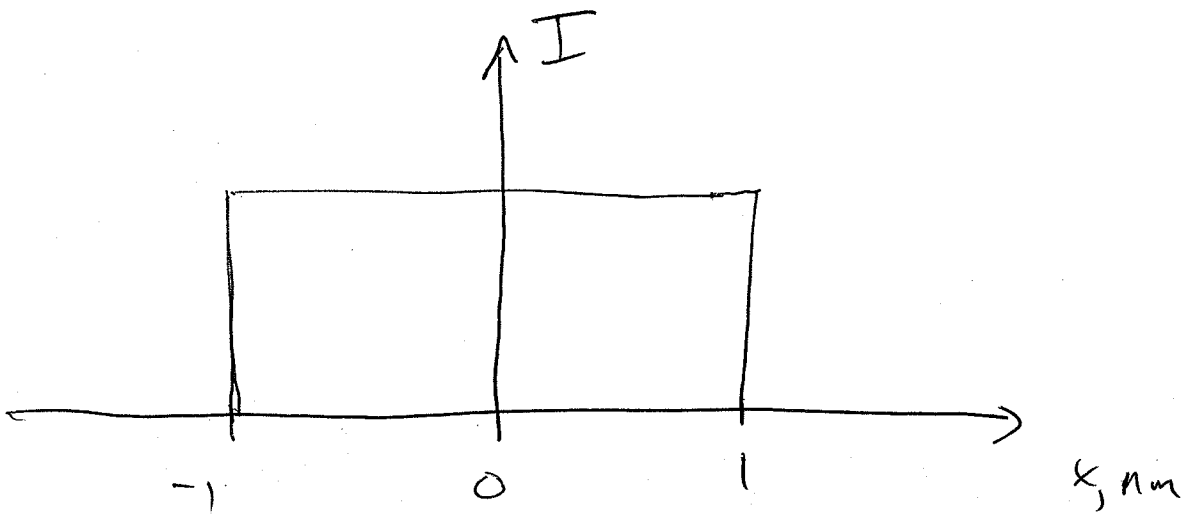


$$\theta_{max} = \tan^{-1}\left(\frac{2.5}{15}\right) = 9.462^\circ$$

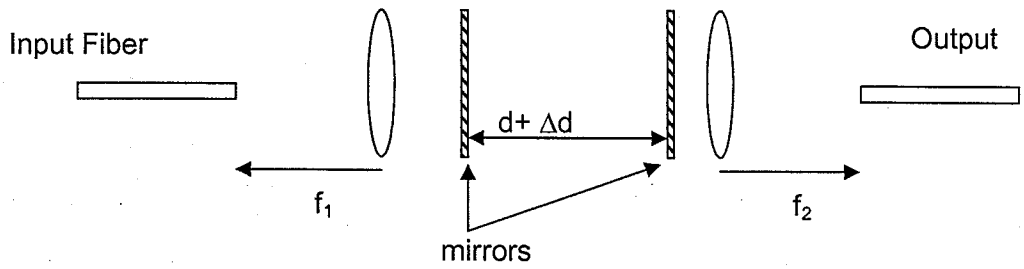
$$\Rightarrow (\alpha_0)_{max} = \sin\theta_{max} = 0.1644$$

$$\Rightarrow T_{min} = \frac{0.667\,\mu m}{0.1644} = 4.06\,\mu m$$

j.) (5 pts) For grating periods less than the minimum period found in part (i), describe the irradiance in the image plane by making a simple sketch of the irradiance in the x direction.



4.) Design a wavelength filter for a fiber communications network as shown below. Light from the input fiber is collimated and passed through a Fabret-Perot resonator. The transmitted light is collected by the output lens and coupled into the output fiber. By changing the distance between the mirrors, one wavelength band is selected at a time. The bands are centered around a wavelength of 1550 nm, they each have a bandwidth of 10 GHz, they are separated by 100 GHz, and there are 15 bands.



(a) (5 pts) What should be the nominal distance *d* and the distance Δ*d* in order to change from one band to the next?

$$\Delta\nu_{FSR} = 1.4\,THz = 1.4\times10^{12}\,Hz = \frac{c}{2nd}$$

$$d_{min} = \frac{c}{2\Delta\nu_{FSR}} = \frac{3\times10^{8}\,m/s}{2\cdot 1.4\times10^{12}\,1/s} = 1.07\times10^{-4} = \boxed{0.107\,mm}$$

$$\frac{\Delta d}{d} = \frac{\Delta\nu}{\nu} \Rightarrow \Delta d = \frac{\Delta\nu d}{c/\bar{\lambda}} = \frac{\Delta\nu d\lambda}{c} = \frac{100\times10^{9}\times1.07\times10^{-4}\times1.55\times10^{-6}}{3\times10^{8}} = \boxed{5.53\times10^{-9}\,m}$$

(b) (5 pts) What is the proper value for the reflectance of the mirrors?

$$\mathcal{F} = 15 \doteq \frac{\pi\sqrt{R}}{1-R} \Rightarrow R \sim 0.81$$

5.

a) (5 pts) Determine the refractive index and thickness of a thin film to be deposited on a glass surface (n_g=1.54.). Such that no normally incident light of wavelength 633nm is reflected.

$$n_1 = \sqrt{1.54} = 1.241$$

$$d = \lambda/4/n_1 = 127.52 \text{ nm}$$

b) (5 pts) Plot reflectance vs wavelength from 400nm to 1000nm at zero degrees and 45 degrees angle of incidence. For the 45 degrees angle of incidence, plot both *s* and *p* polarization responses. Assume that the indices of refraction for the film and the glass are not a function of wavelength.

$$r = \frac{-M_{21} + M_{11}\gamma_0 - M_{22}\gamma_s + M_{12}\gamma_0\gamma_s}{M_{21} + M_{11}\gamma_0 + M_{22}\gamma_s + M_{12}\gamma_0\gamma_s}$$

$M_{11} = \cos k_0 h$ $\quad k_0 h = \frac{2\pi}{\lambda} n_1 d \cos\theta_1$

$M_{12} = -i \sin k_0 h / \gamma_1$ $\quad n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_s \sin\theta_s$

$M_{21} = -\gamma_1 i \sin k_0 h$ $\quad n_0 = 1 \quad n_1 = 1.241$

$M_{22} = \cos k_0 h = M_{11}$ $\quad n_s = 1.54 \quad d = 127.52 \times 10^{-9}$

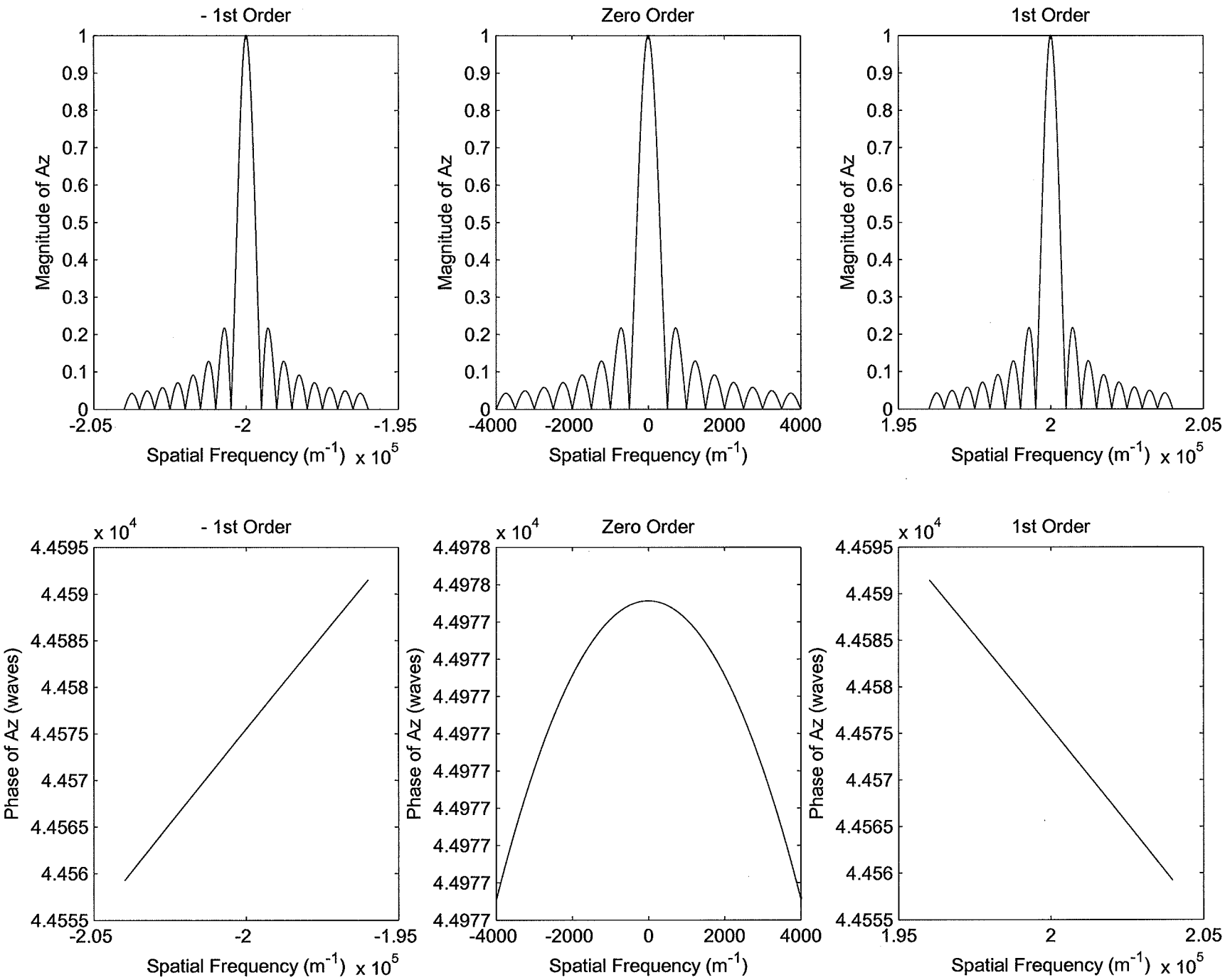
$\gamma_{0s} = \sqrt{\frac{\epsilon_0}{\mu_0}} n_0 \cos\theta_0$ $\quad \gamma_{0p} = \sqrt{\frac{\epsilon_0}{\mu_0}} n_0 / \cos\theta_0$

$\gamma_{ss} = \sqrt{\frac{\epsilon_0}{\mu_0}} n_s \cos\theta_s$ $\quad \gamma_{sp} = \sqrt{\frac{\epsilon_0}{\mu_0}} n_s / \cos\theta_s$

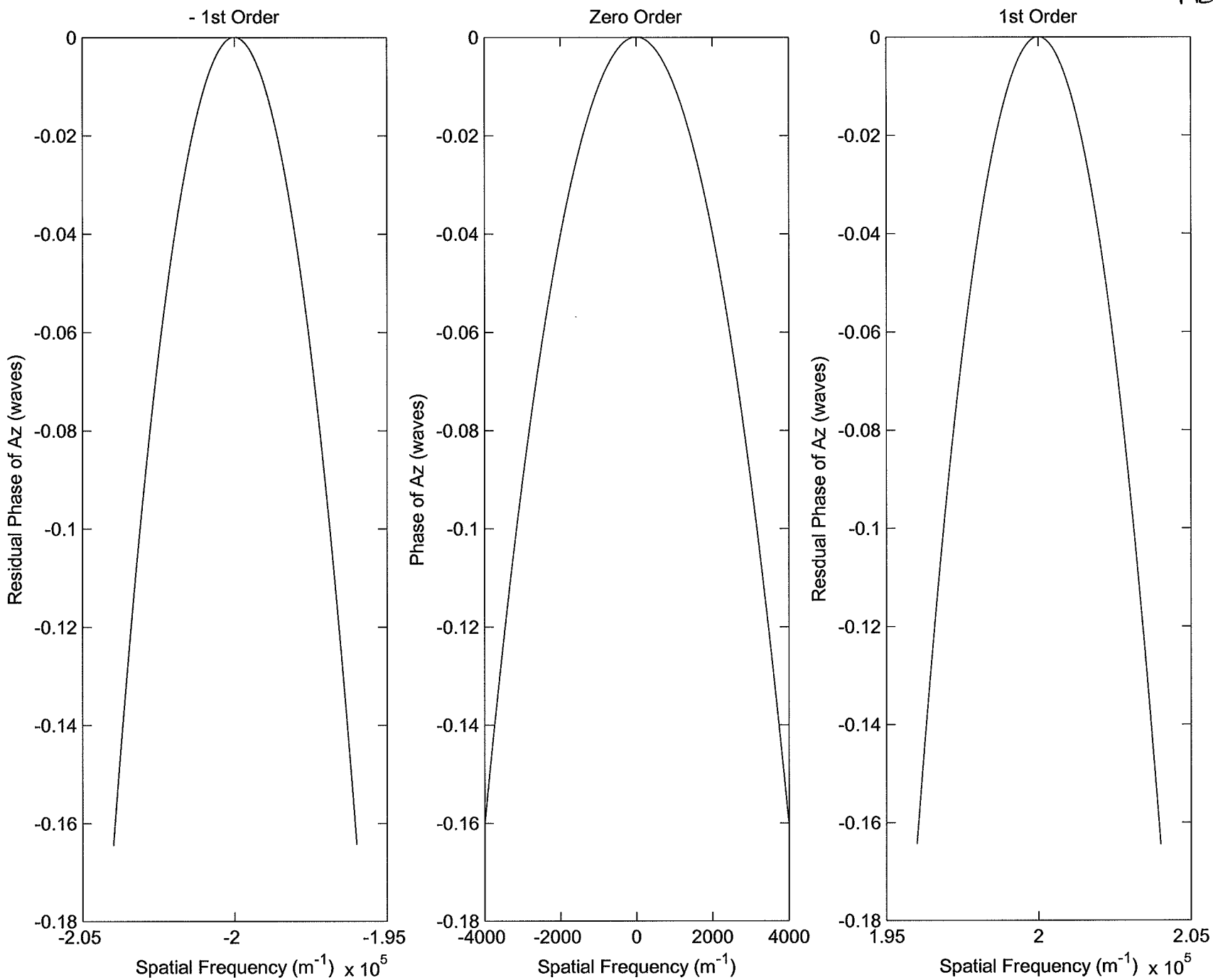
$\gamma_{1s} = \sqrt{\frac{\epsilon_0}{\mu_0}} n_1 \cos\theta_1$ $\quad \gamma_{1p} = \sqrt{\frac{\epsilon_0}{\mu_0}} n_1 / \cos\theta_1$

(see attached plots and matlab mfile for results).

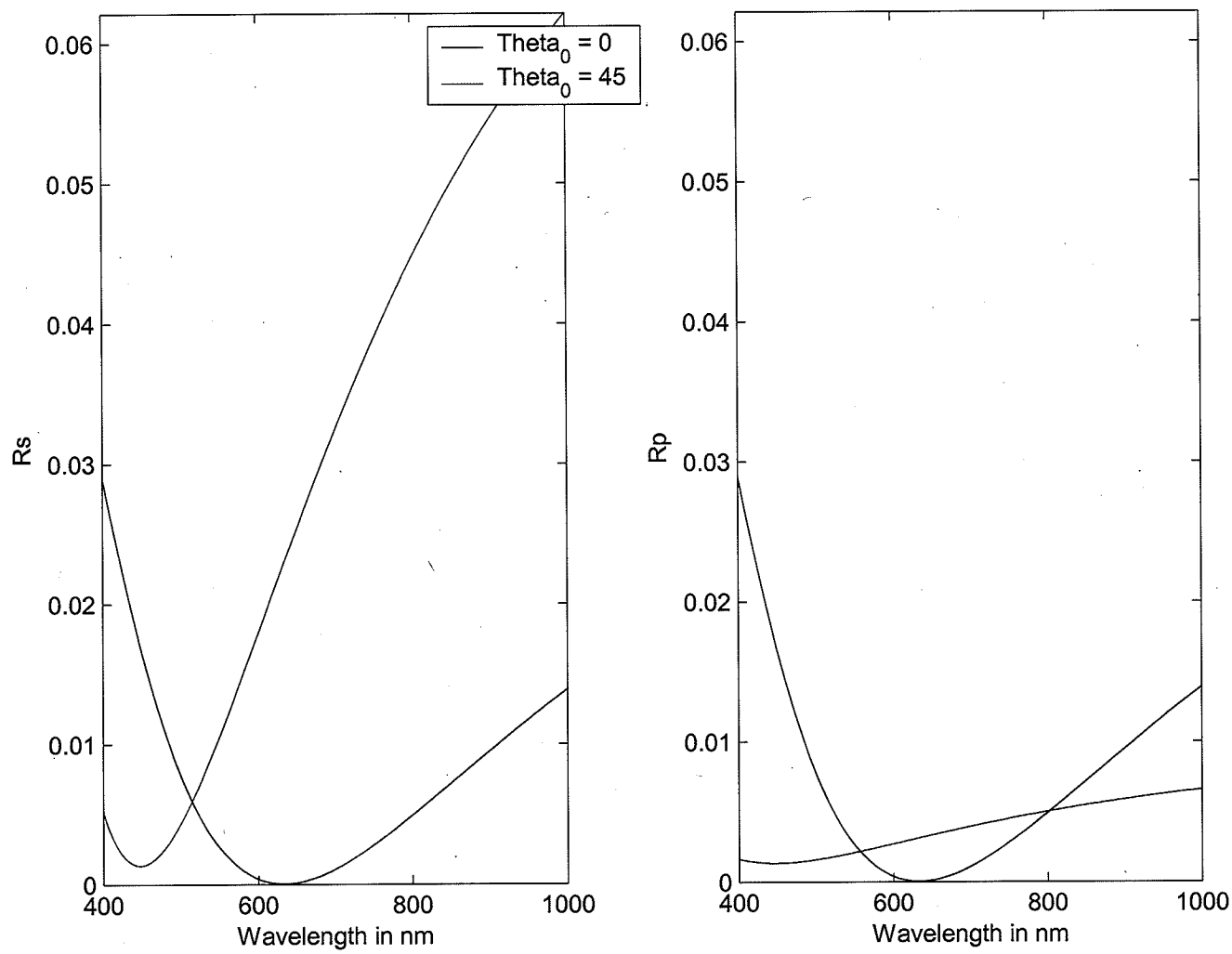
TEST 2 PROB 3C



TEST
PROB 3E


- 1st Order
Zero Order
1st Order
Residual Phase of Az (waves)
Phase of Az (waves)
Resdual Phase of Az (waves)
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-2.05
-2
-1.95
-4000
-2000
0
2000
4000
1.95
2
2.05
Spatial Frequency (m-1) x 10^5
Spatial Frequency (m-1)
Spatial Frequency (m-1) x 10^5

PROBLEM 5(b).


Theta$_0$ = 0
Theta$_0$ = 45
Rs
Rp
0.06
0.05
0.04
0.03
0.02
0.01
0
400
600
800
1000
Wavelength in nm

```
%solution to TEST2, Problem 5, Fall 2004
%AR coating on glass
e0              = 8.85e-12;
mu0             = pi*4e-7;

n0              = 1;
ns              = 1.54;
lambda_design   = 633e-9;

%Part a
%design thickness d for normal incidence at 633 nm.
n1  = sqrt(ns);
d   = lambda_design/4/n1;
disp(['***** AR Deisgn for lambda = ' num2str(lambda_design*1e9) ' nm'])
disp(['      refractive index = ' num2str(n1) ' thickness = ' num2str(d*1e9) ' nm'])

%now calculate reflectance versus wavelength at 0 degrees and 45 degrees
%for s and p
lambdavec       = linspace(400e-9,1000e-9,100);
k0              = 2*pi./lambdavec;

r_s_vec         = [];
r_p_vec         = [];

theta0vec       = [0 45]*pi/180;

for theta0 = theta0vec

    k0h         = k0*d*n1*cos(theta0);
    theta1      = asin(n0/n1*sin(theta0));
    gamma1s     = sqrt(e0/mu0)*n1*cos(theta1);
    gamma1p     = sqrt(e0/mu0)*n1/cos(theta1);
    m11         = cos(k0h);
    m12s        = -i*sin(k0h)/gamma1s;
    m12p        = -i*sin(k0h)/gamma1p;
    m21s        = -gamma1s*i*sin(k0h);
    m21p        = -gamma1p*i*sin(k0h);
    m22         = m11;

    gamma0s     = sqrt(e0/mu0)*n0*cos(theta0);
    gamma0p     = sqrt(e0/mu0)*n0/cos(theta0);

    thetas      = asin(n0/ns*sin(theta0));

    gammass     = sqrt(e0/mu0)*ns*cos(thetas);
    gammasp     = sqrt(e0/mu0)*ns/cos(thetas);

    numrs       = -m21s+m11*gamma0s-m22*gammass+m12s*gamma0s*gammass;
    numrp       = -m21p+m11*gamma0p-m22*gammasp+m12p*gamma0p*gammasp;

    denos       = m21s+m11*gamma0s+m22*gammass+m12s*gamma0s*gammass;
    denop       = m21p+m11*gamma0p+m22*gammasp+m12p*gamma0p*gammasp;

    r_s         = numrs./denos;
    r_p         = numrp./denop;

    r_s_vec     = [r_s_vec {r_s}];
```

```
    r_p_vec     = [r_p_vec {r_p}];

end

%Plot results
figure(3);
maxR        = max(abs(cell2mat([r_s_vec r_p_vec]).^2));
subplot(121),plot(lambdavec*1e9,abs(cell2mat(r_s_vec(:))).^2);axis([400 1000 0 maxR])
xlabel('Wavelength in nm')
ylabel('Rs')
Mvec        = [];
for ii = 1:length(theta0vec)
    Mvec    = [Mvec {['Theta_0 = ' num2str(theta0vec(ii)*180/pi)]}];
end
M           = strvcat(Mvec);
legend(M);
subplot(122),plot(lambdavec*1e9,abs(cell2mat(r_p_vec(:))).^2);axis([400 1000 0 maxR])
xlabel('Wavelength in nm')
ylabel('Rp')
```

OPTI 505 TEST #2 KEY SPRING 2005

1.) A glass (n = 1.514) window is used in an underwater experiment. The water has a refractive index of n_{water} = 1.33. The wavelength of interest is λ = 550 nm.

a.) (5pts) How much light is reflected from the water/glass interface at normal incidence?

$n_o = 1.33 \quad n_s = 1.514$

$$R = \left(\frac{n_s - n_o}{n_s + n_o}\right)^2 = 0.0041 \quad \text{or} \quad \boxed{0.41\%}$$

b.) (5 pts) What are the thickness and refractive index of an appropriate thin-film anti-reflection coating for the water/glass interface of the window?

$$n = \sqrt{1.33 \cdot 1.51} = 1.419$$

$$d = \frac{\lambda}{4n} = \frac{0.550\,\mu m}{4 \cdot 1.419} = 0.0969\,\mu m \text{ or } \boxed{96.9\text{ nm}}$$

2.) In a particular FECO interferometer, the separation of the test and reference surfaces is 2750 nm. If the test piece is perfectly flat, one wavelength passed through the system is 500 nm.

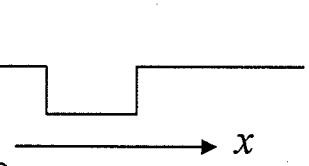
a.) (5 pts) What is the next longest wavelength mode passed through the system for a flat sample?

$$m = 2d/\lambda_m = \frac{2 \cdot 2750}{500} = \frac{5500}{500} = 11$$

$$\lambda_{m-1} = \frac{2d}{m-1} = \frac{5500}{10} = \boxed{550\text{nm}}$$

b.) (10 pts) Draw the output (wavelength on the horizontal axis and x dimension on the vertical axis) of the interferometer versus wavelength if the 50 nm deep trench shown below is in the test piece. Show the mode nominally at 500 nm and the next-longest wavelength mode. Label the critical wavelengths.

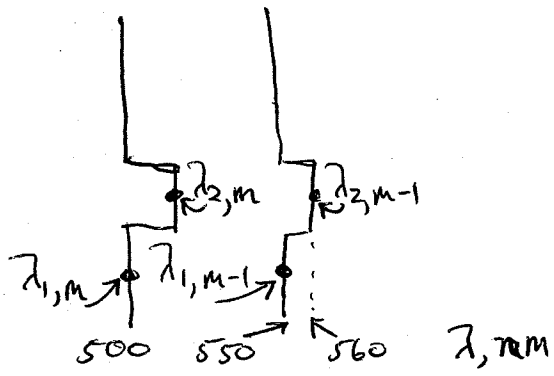
$$\Delta d = \frac{\lambda_{1,m}}{\lambda_{1,m-1} - \lambda_{1,m}}\left(\frac{\lambda_{2,m-1} - \lambda_{1,m-1}}{2}\right)$$



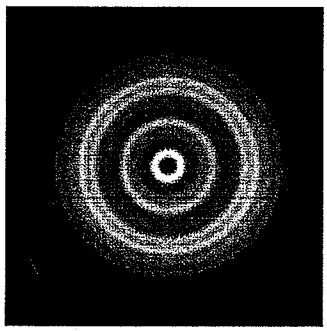
$$50 = \frac{500}{50}\left(\frac{\lambda_{2,m-1} - 550}{2}\right) \Rightarrow \lambda_{2,m-1} = 560\text{nm}$$

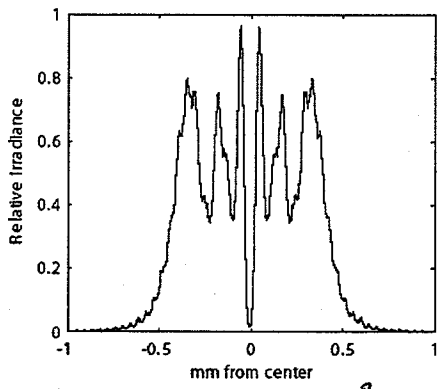
$\lambda_{1,m-1} = 550\text{nm}$

$\lambda_{1,m} = 500\text{nm}$



3.) (10pts) The following pattern is observed in the output of a 632.8 nm wavelength laser beam. Assume that the pattern results from illuminating a 1.0 mm diameter circular hole with a collimated and uniform laser beam. How far is the hole from the observation screen?





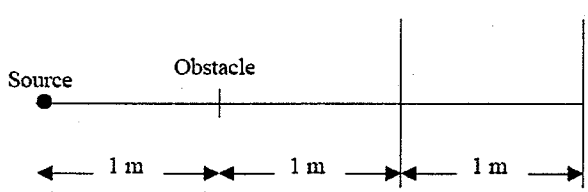
$$N_f = \frac{a^2}{\lambda z} \qquad z = \frac{a^2}{\lambda N_f} = \frac{(0.5\times10^{-3})^2}{(0.6328\times10^{-6})\ 6}$$

$$= 0.0658\,m = \boxed{65.8\ mm}$$

4.) A 1 mm diameter circular obstacle is illuminated with a spherical wave diverging from a point source 1 meter to the left of the circular obstacle. In the plane of the circular obstacle the irradiance of the illuminating beam is 1 watt/cm2. What is the on-axis irradiance

a.) (10 pts) 1 meter to the right of the circular obstacle?

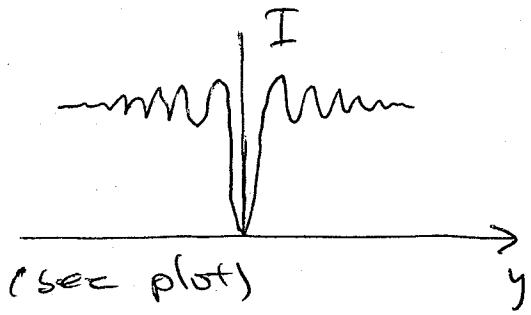
b.) (5 pts) 2 meters to the right of the circular obstacle?

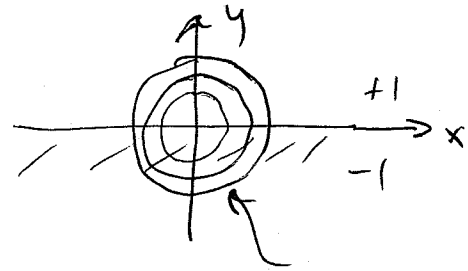


a) $\frac{1}{4} \cdot 1\,W/cm^2 = \boxed{0.25\ W/cm^2}$

b) $\frac{1}{9} \cdot 1\,W/cm^2 = \boxed{0.11\ W/cm^2}$

5.) (10 pts) Sketch the Fresnel diffraction pattern of a knife edge where the knife transmits 100 percent, but introduces a 180 degree phase change for the light transmitted through the knife.





on axis and along KE, phase shift produces zero irradiance.

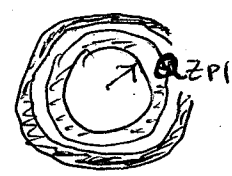
6.) A Fresnel zone plate has a +1 order focal length of 200 mm for a wavelength of 600 nm.

a) (5 pts) What is the diameter of the first Fresnel zone making up the Fresnel Zone plate?

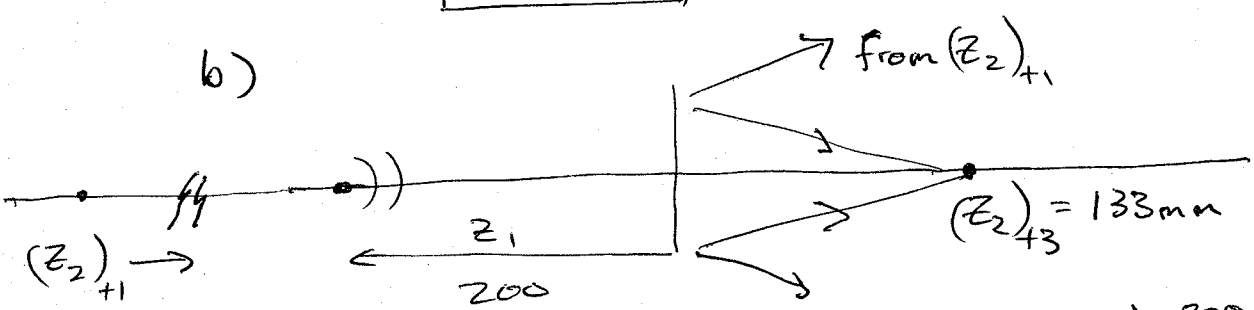
b) (10 pts) The zone plate is illuminated with a spherical wave of wavelength 500 nm coming from a source 200 mm to the left of the plate. Where do the +1 and +3 orders come to focus?

a) $a_{ZP1} = \sqrt{(600\times10^{-9})(20\times10^{-2})}$

$= \boxed{0.346 \text{ mm}}$



b)



$f_{+1} = \frac{a_{ZP1}^2}{\lambda} = \frac{(0.346\times10^{-3})^2}{500\times10^{-9}} = 0.24$ m

$(z_2)_{+1} = \frac{200\cdot240}{200-240} = \boxed{-1{,}200 \text{ mm}}$

$f_{+3} = \frac{a_{ZP1}^2}{3\lambda} = 0.08$ m

$(z_2)_{+3} = \frac{200\cdot80}{200-80} = \boxed{133 \text{ mm}}$

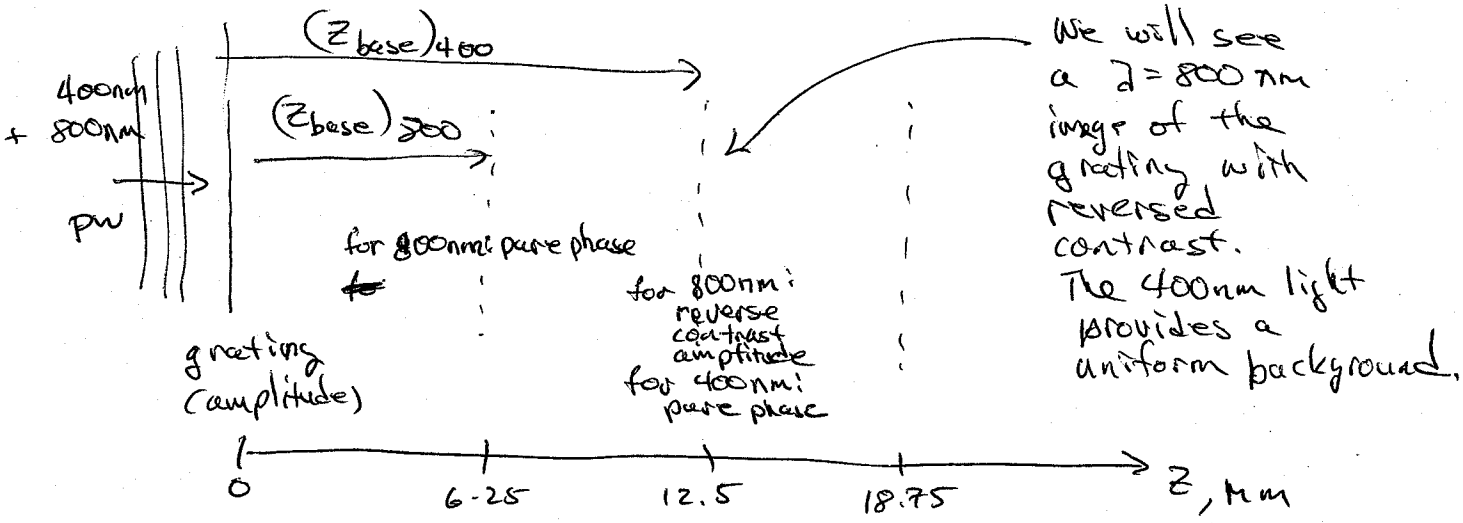
$\frac{1}{f} = \frac{1}{z_1} + \frac{1}{z_2}$ $\quad z_2 = \left(\frac{z_1 - f}{z_1 f}\right)^{-1} = \frac{z_1 f}{z_1 - f}$

7.) (15pts) A 100 μm pitch amplitude grating is illuminated by two collimated laser beams at normal incidence. The laser wavelengths are 400 nm and 800 nm. What observation is made at a distance of 12.5 mm from the grating?

This is a Talbot imaging problem. $\alpha_0 = \xi_0 \lambda = \lambda / T$

$$(z_{base})_{400} = \frac{\lambda}{2\alpha_0^2} = \frac{T^2}{2\lambda} = \frac{(100\times10^{-6})^2}{2\times400\times10^{-9}} = 0.0125\ m = 12.5\ mm$$

$$(z_{base})_{800} = \frac{(100\times10^{-6})^2}{2\times800\times10^{-9}} = 6.25\ mm$$



8.) (10pts) List two advantages and two disadvantages of Vertical Scanning Interferometry compared to Phase Stepping Interferometry.

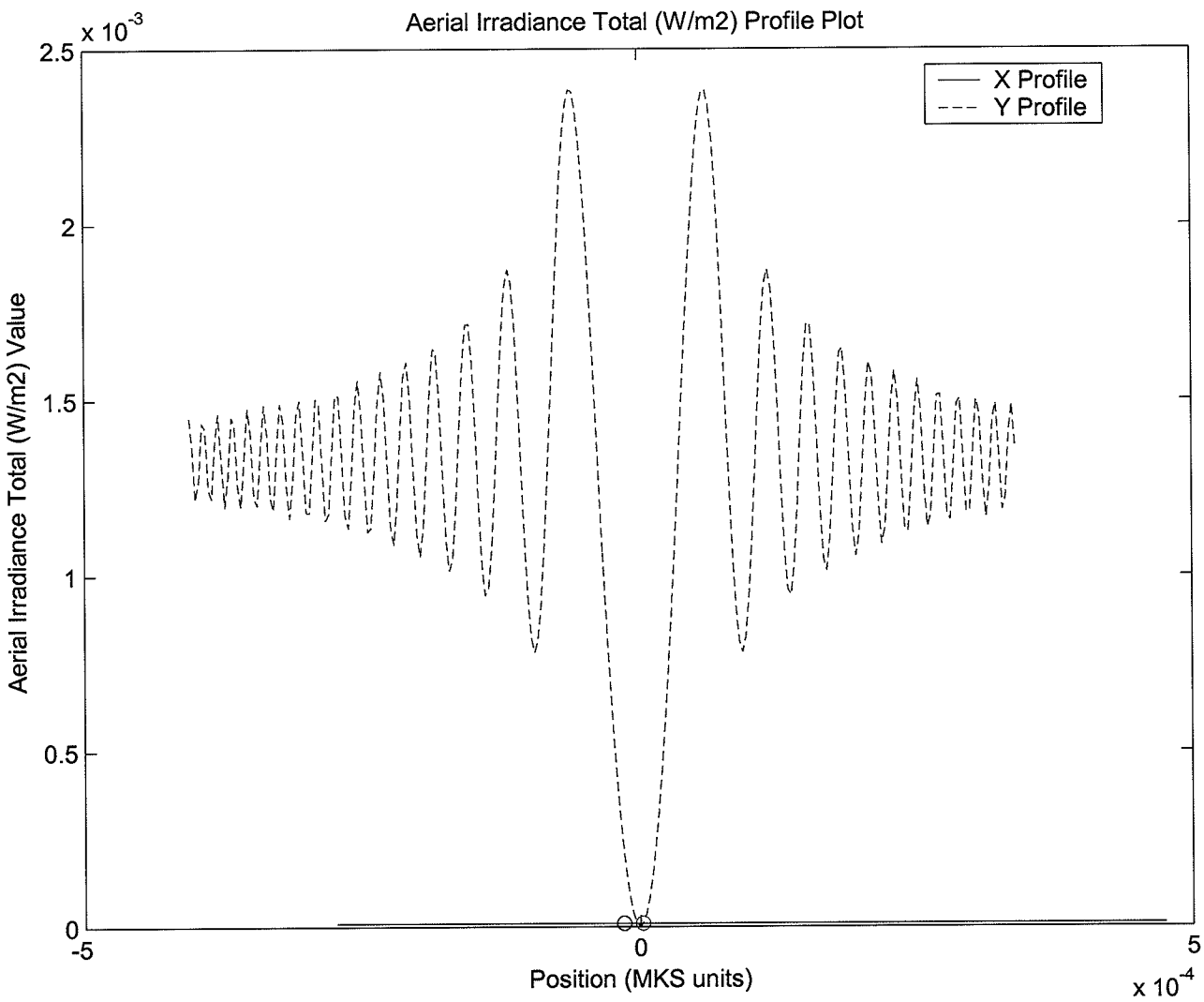
VSI compared to PSI

Advantages

- can operate over larger step height w/o ambiguity
- uses "white" light
- less sensitive to vibration

Disadvantages

- not as accurate
- requires high-contrast fringes
- sensitive to irradiance fluctuations across pupil.
- slower


Aerial Irradiance Total (W/m2) Profile Plot
x 10^-3
2.5
2
1.5
1
0.5
0
Aerial Irradiance Total (W/m2) Value
X Profile
Y Profile
-5
0
5
Position (MKS units)
x 10^-4

KEY

1.) a) (5 pts.) A single-layer anti-reflection coating is desired for a glass (n=1.52) and water (n = 1.33) at λ = 550 nm. What is the ideal material refractive index and thickness for normal incidence?

$$n = \sqrt{(1.52)(1.33)} = 1.42$$

$$t = \frac{\lambda}{4n} = \frac{550\,nm}{4 \cdot 1.42} = 96.8\,nm$$

b.) (5 pts.) If the coating is tilted by 30 degrees, what is the approximate wavelength for minimum reflectance?

$$n' \sin\theta' = n \sin\theta$$

$$n = 1$$

$$n' = 1.42$$

$$\theta = 30^\circ \Rightarrow \theta' = \arcsin\left(\frac{1}{1.42} \sin 30^\circ\right)$$

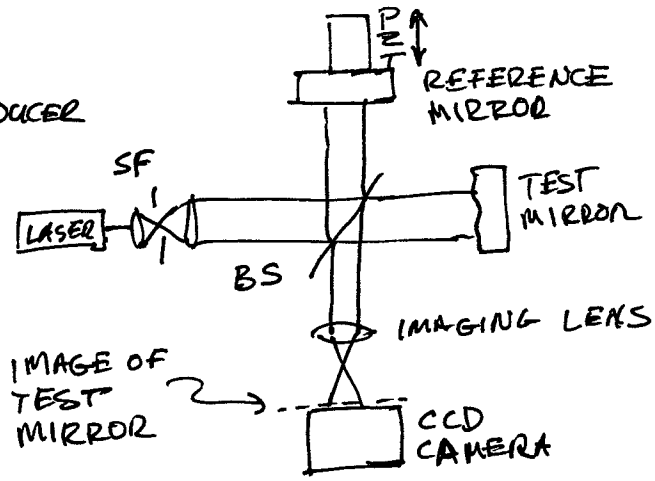
$$= 20.62^\circ$$

$$\lambda' = \lambda \cos\theta' = 550 \cos 20.62^\circ$$

$$= \boxed{514.8\ nm}$$

2.) a.) (5 pts.) Draw a Twyman-Green interferometer that is designed for phase-stepping interferometry.

SF = SPATIAL FILTER
PZT = PIEZO-ELECRIC TRANSDUCER
BS = BEAM SPLITTER



b.) (5 pts.) What is the theoretical minimum number of measurements necessary in phase-shifting interferometry?

3

c.) (5 pts.) List one advantage and one disadvantage of phase shifting interferometry compared to vertical scanning interferometry (coherence probe).

ADVANTAGE: MORE ACCURATE

DISADVANTAGE: AMBIGUOUS MEASUREMENTS FOR STEPS > λ OPD

3.) The major difference in the point-spread function approximations for diffraction can be thought of in terms of the shape each wavelet as it propagates to the observation point. What are the shapes of the wavelets at the observation point for the following cases. (Hint: I am looking for a simple geometric descriptor, like the word 'ellipsoid'.)

a.) (5 pts.) $r_{01} >> \lambda$ and $\gamma_z \approx 1$?

SPHERICAL

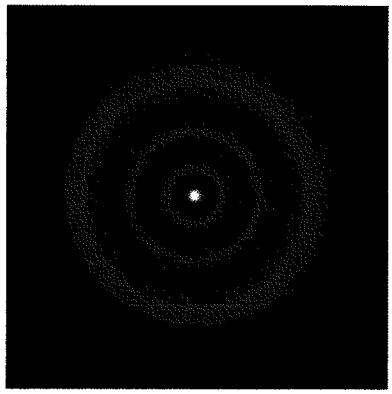
b.) (5 pts.) Fresnel diffraction?

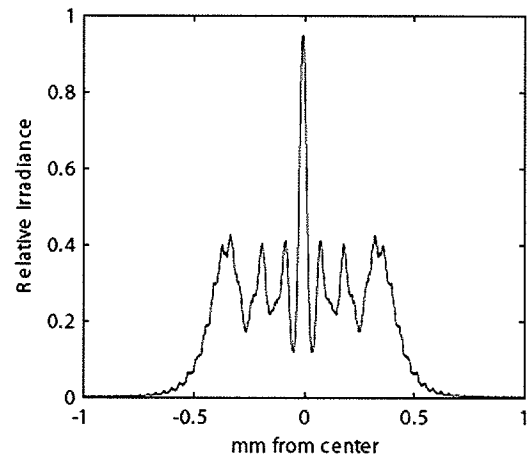
PARABOLA

c.) (5 pts.) Fraunhofer diffraction?

PLANE

4.) (10 pts.) The following pattern is observed in the output of a 458 nm wavelength laser beam. Assume that the pattern results from illuminating a 1.0 mm diameter circular hole with a collimated and uniform laser beam. How far is the hole from the observation screen?





$a = 0.5 \times 10^{-3}\ m$

$N_f = 7$

$\lambda = 458 \times 10^{-9}\ m$

$N_f = \frac{a^2}{\lambda Z_2} \quad (Z_1 \to \infty)$

$\therefore Z_2 = \frac{a^2}{\lambda N_f} = \frac{(0.5 \times 10^{-3})^2}{(458 \times 10^{-9})(7)} = 0.078\ m$

$= 78\ mm$

5.) (15pts) An amplitude grating is illuminated by two collimated laser beams at normal incidence. The transmission function for both wavelengths is $t(x_1) = 1 + \cos(2\pi x_1 / T)$, where T = 100 μm. The laser wavelengths are 400 nm and 800 nm. What irradiance pattern is observed at a distance of 12.5 mm from the grating?

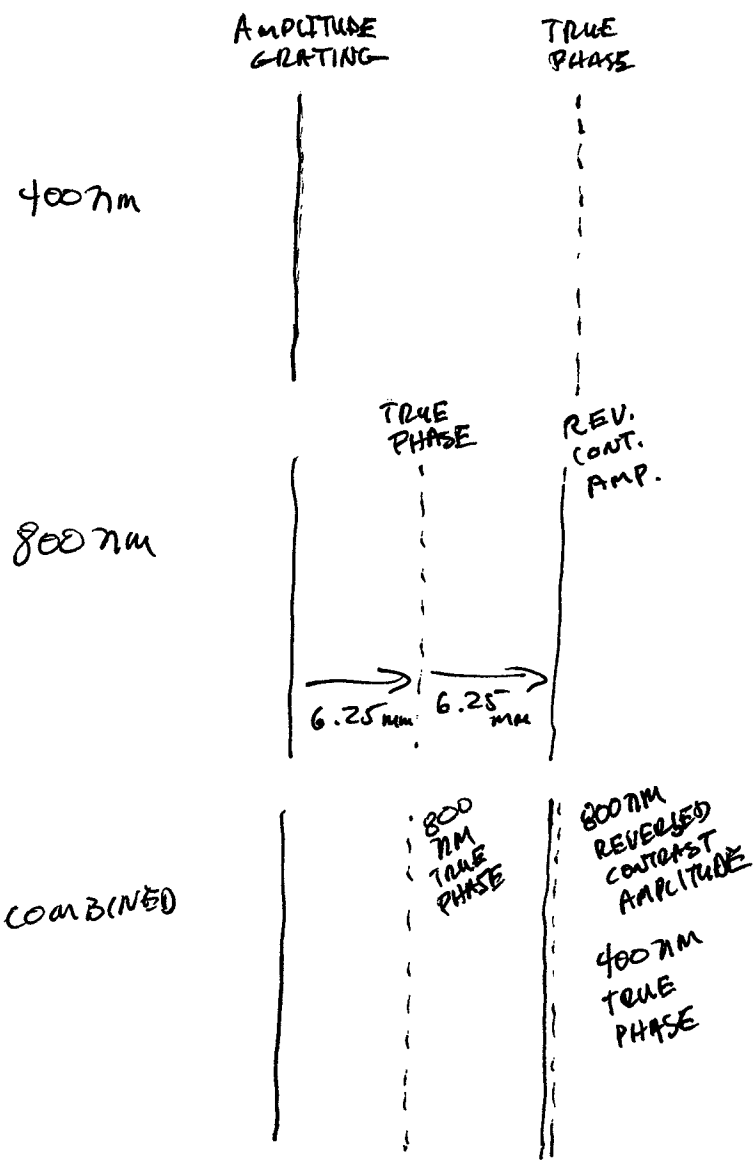
$$\alpha_0 = \lambda \xi_0 = \lambda / T$$

TALBOT IMAGING:

400 nm: $$z_{base} = \frac{\lambda}{2\alpha_0^2} = \frac{T^2 \lambda}{2\lambda^2} = \frac{T^2}{2\lambda} = \frac{(100\times10^{-6})^2}{2\cdot 400\times10^{-9}}$$

$$= 0.0125\ m = 12.5\ mm$$

800 nm: $$z_{base} = \frac{(100\times10^{-6})^2}{2\cdot 800\times10^{-9}} = 0.00625\ m = 6.25\ mm$$



IRRADIANCE IS 800 nm AMPLITUDE COSINE PATTERN WITH UNIFORM 400 nm BACKGROUND.

KEY

6.) A collimated 0.1 W/cm^2 laser beam of wavelength 730 nm illuminates a binary amplitude Fresnel zone plate.

a.) (5 pts.) The primary focus occurs at z = 100 mm. What is the radius of the first open area in the zone plate?

$$f = \frac{a_{ZP1}^2}{\lambda} \Rightarrow a_{ZP1} = \sqrt{\lambda f} = \sqrt{730\times10^{-9}\cdot 100\times10^{-3}}$$

$$= 0.270 \text{ mm}$$

b.) (5pts.) What is the on-axis irradiance at the primary (1st order) focus if there are 50 open zones in the Fresnel zone plate?

$$I(P) = 4N_{FZ}^2 I_\infty(P)$$

$$= 4(50)^2 \cdot 0.1 \text{ W/cm}^2 = 1000 \text{ W/cm}^2$$

c.) (5pts.) What are the maximum on-axis irradiance at the 7th order focus?

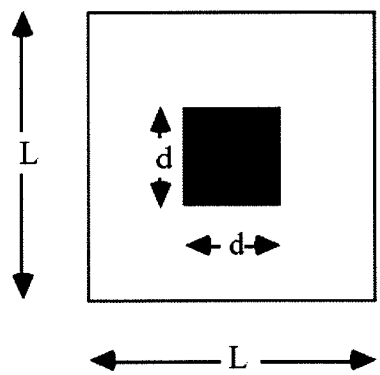
1000 W/cm^2

d.) (5 pts.) If the zone plate is heated, it uniformly expands so that the open and closed areas of the plate increase their diameter. If the plate is heated so that the diameter of the first area increases by 1% , where is the new position of the primary focus?

$$f' = \frac{(0.270\times10^{-3}\cdot 1.01)^2}{730\times10^{-9}}$$

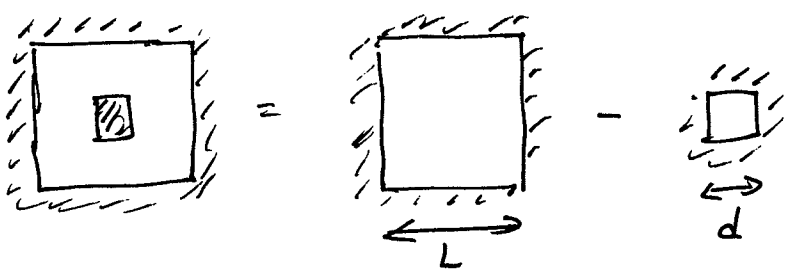
$$= 0.1019 \text{ m} = 102 \text{ mm}$$

7.) (15 pts.) Find an expression for the irradiance distribution in the Fraunhofer diffraction pattern of the aperture shown below. Assume unit-amplitude, 500 nm, normally incident plane-wave illumination with L = 4 mm and d = 2 mm.



opaque outside of square L x L

BABINET'S PRINCIPLE: (APERTURE ALGEBRA)



FRAUNHOFER:

$$|U_0(x_0,y_0)| \propto \frac{1}{\lambda z_0}\left| F_{\frac{y_0}{\lambda z_0}} F_{\frac{x_0}{\lambda z_0}}\left[\mathrm{rect}\left(\frac{x_1}{L},\frac{y_1}{L}\right) - \mathrm{rect}\left(\frac{x_1}{d},\frac{y_1}{d}\right)\right]\right|$$

$$= \frac{1}{\lambda z_0}\left|\left[L^2 \mathrm{sinc}\left(L\frac{x_0}{\lambda z_0}, L\frac{y_0}{\lambda z_0}\right) - d^2 \mathrm{sinc}\left(\frac{d x_0}{\lambda z_0}, \frac{d y_0}{\lambda z_0}\right)\right]\right|$$

$$I(x_0,y_0) \propto |U_0(x_0,y_0)|^2$$

$$\approx \left(\frac{1}{\lambda z_0}\right)^2\left[L^4 \mathrm{sinc}^2\left(\frac{L x_0}{\lambda z_0}, \frac{L y_0}{\lambda z_0}\right) - 2d^2L^2 \mathrm{sinc}\left(\frac{L x_0}{\lambda z_0}, \frac{L y_0}{\lambda z_0}\right)\mathrm{sinc}\left(\frac{d x_0}{\lambda z_0}, \frac{d y_0}{\lambda z_0}\right) + d^4 \mathrm{sinc}^2\left(\frac{d x_0}{\lambda z_0}, \frac{d y_0}{\lambda z_0}\right)\right]$$