CHAPTER 3

- The one dimensional wave equation in free space is:

$$\frac{\partial^2 U_y}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 U_y}{\partial t^2} = 0$$

- The real solution to this equation is known as a 1-D traveling wave. For a wave traveling in the +x direction, the solution is given by: $U(x,t) = A_0 \cos(kx - \omega t + \phi)$ where $k = \frac{\omega}{c}$.
- The field resulting from several waves is determined by a linear superposition of the constituent waves. For the combination of several waves of the same frequency but with different amplitudes & phases:

$$U(x,t) = A_0 e^{j\phi_0} e^{j(kx-\omega t)} = \sum_{n=1}^{N} A_n e^{j\phi_n} e^{j(kx-\omega t)} \Rightarrow |U(x,t)|^2 = A_0^2 = \sum_{n=1}^{N} A_n^2 + \sum_{n=1}^{N}\sum_{m=1, m<n}^{N} 2A_nA_m \cos(\phi_n - \phi_m)$$

If phases are random over time, then we have incoherent addition $\Rightarrow |U(x,t)|^2 = NA^2$ (assuming $A_n = A$)

If phases are constant over time (i.e. $\phi_n = \phi$), then we have coherent addition $\Rightarrow |U(x,t)|^2 = \left(\sum_{n=1}^{N} A_n\right)^2 = N^2A^2$ (assuming $A_n = A$)

- For two waves traveling with the same amplitude, but different frequencies, we observe beats:

$$\mathrm{Re}\{U(x,t)\} = 2A \underbrace{\cos(k_{HF}x - \omega_{HF}t + \alpha)}_{\text{high frequency term}} \underbrace{\cos(k_m x - \omega_m t + \beta)}_{\text{modulation term}}$$

where: $\omega_{HF} = \frac{1}{2}(\omega_a + \omega_b)$ $\quad k_{HF} = \frac{1}{2}(k_a + k_b)$; $\omega_m = \frac{1}{2}(\omega_a - \omega_b)$ $\quad k_m = \frac{1}{2}(k_a - k_b)$; $\alpha = \frac{1}{2}(\phi_a + \phi_b)$ $\quad \beta = \frac{1}{2}(\phi_a - \phi_b)$

Additional properties of beat terms: $\lambda_{eq} = \frac{\lambda_a \lambda_b}{|\lambda_a - \lambda_b|} = \frac{\lambda_m}{2}$; $V_m = \frac{\omega_m}{k_m} = \frac{c(\lambda_b - \lambda_a)}{\lambda_b n_a - \lambda_a n_b}$; $V_{HF} = \frac{c(\lambda_b + \lambda_a)}{\lambda_b n_a + \lambda_a n_b}$

- For two waves with the same amplitude, traveling in opposite directions, we observe standing waves:

$k_a = k_b = k \Rightarrow k_{HF} = \frac{k}{2}$, $k_m = 0$; $\omega_a = -\omega_b \Rightarrow \omega_{HF} = \frac{\omega}{2}$, $\omega_m = 0$; $U(x,t) = 2A\cos(kx + \alpha)\cos(-\omega t + \beta)$

Additional properties of standing waves: $\lambda_{eq} = \frac{\lambda}{2}$; $V_m = \infty$; $V_{HF} = 0$

- The 3D Helmholtz equation and its solution (a harmonic plane wave traveling in the $\hat{k}$ direction) in free space are:

$$\nabla^2 \vec{U} - \frac{1}{c^2}\frac{\partial^2 \vec{U}}{\partial t^2} = 0 \;;\; \vec{U}(\vec{r},t) = \vec{U}_0 e^{j(\vec{k}\cdot\vec{r} - \omega t)} = |U_0|\hat{a}e^{j(\vec{k}\cdot\vec{r} - \omega t)} = A_0\hat{a}e^{j(\vec{k}\cdot\vec{r} - \omega t)}$$

The solution to the Helmholtz equation in spherical coordinates is:

$$\vec{U}(r,t) = \frac{\vec{U}_0}{r} e^{j(kr - \omega t)} \quad \text{where } r = \sqrt{x^2 + y^2 + z^2} \;;\; \psi = kr - \omega t$$

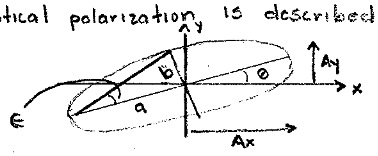
- A field can be polarized in several orientations. Consider an EM plane wave traveling in the $+z$ direction such that the field oscillates perpendicular to z. The field is then given by:

$$\vec{U}(z,t) = \vec{U}_0 e^{j(kz - \omega t)} = \left[A_x\hat{x} + A_y e^{j\phi}\hat{y}\right] e^{j(kz - \omega t)} \quad \boxed{\text{EQ 1}}$$

- Two useful mathematical formalisms used to describe the polarization of a field are Jones and Stokes vectors. Jones vectors are only applicable to a fully polarized field. Stokes vectors are applicable to partially polarized fields.
- The table below summarizes polarization properties of the field given by $\boxed{\text{EQ 1}}$:

POLARIZATION	A_x	A_y	ϕ	JONES	STOKES	NOTES
linear - horizontal	A_0	0	0	$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$	
linear - vertical	0	A_0	0	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}$	
linear - +45°	A_0	A_0	0	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$	
linear - -45°	A_0	$-A_0$	0	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$	$\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}$	
LHC	A_0	A_0	$\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\j\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$	rotates: CW w/z, CCW w/t
RHC	A_0	A_0	$-\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-j\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$	rotates: CCW w/z, CW w/t

- Elliptical polarization is described by $\boxed{\text{EQ 1}}$. The ellipticity (ϵ) is defined as follows:



$\tan\theta = \frac{A_y}{A_x}$

$\tan\epsilon = \pm\frac{b}{a}$; $\epsilon > 0 \Rightarrow$ RHC , $\epsilon < 0 \Rightarrow$ LHC

- Optical elements can affect the state of polarization of a field passing through them. An example of such an element is a uniaxial crystal. Properties of uniaxial crystals:

  $N_x = n_x + jK_x$, $N_y = n_y + jK_y$ $\Rightarrow$ Retardation plate: $n_x \neq n_y$, $K_x \approx K_y \approx 0$; Polarizer plate: $n_x \approx n_y$, $K_x \neq K_y$

  $n_x = n_o$ = ordinary axis (slow); $n_y = n_e$ = extraordinary axis (fast); $n_z = n_o$

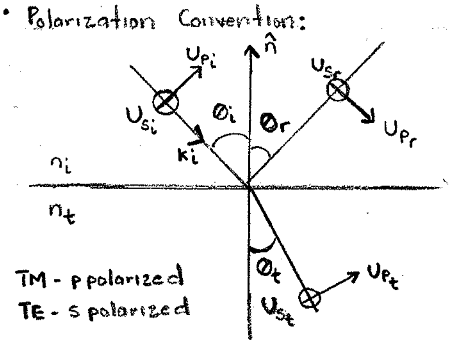
  $n_e > n_o \Rightarrow$ pos crystal; $n_e < n_o \Rightarrow$ neg crystal

  A quarter wave plate is a uniaxial crystal where: $n_o = n_x = 1.544$, $n_e = n_y = 1.553$, $K_x = K_y = 0$
- We can use Jones and Muller matricies to describe the polarization effect of optical elements on a field. Jones representations can be used if the impinging field is fully polarized. Muller matrices are used if the impinging field is described by a Stokes vector (i.e. partially polarized). The table below summarizes polarization properties for several optical elements:

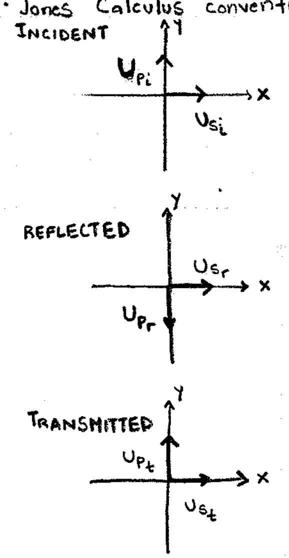
MATRIX	Horizontal linear polarizer	Vertical linear polarizer	Linear Polarizer @+45°	Linear Polarizer @-45°	QWP - fast axis vertical	QWP - fast axis horizontal	Homogeneous Circular Polarizer - right	Homogeneous Circular polarizer - left
JONES	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}0&0\\0&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$	$e^{j\frac{\pi}{4}}\begin{pmatrix}1&0\\0&-j\end{pmatrix}$	$e^{j\frac{\pi}{4}}\begin{pmatrix}1&0\\0&j\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&j\\-j&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&-j\\j&1\end{pmatrix}$
MULLER	$\frac{1}{2}\begin{pmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&0&1&0\\0&0&0&0\\1&0&1&0\\0&0&0&0\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&0&0&-1\\0&0&0&0\\0&0&0&0\\-1&0&0&1\end{pmatrix}$

- For a general retarder, $M = e^{j\psi}\begin{pmatrix}1&0\\0&e^{j\phi}\end{pmatrix}$. So, $M_{HWP} = e^{j\psi}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$
- For a mirror: $M_M = \begin{pmatrix}r_s&0\\0&-r_p\end{pmatrix}$. If the mirror is ideal then $r_s = r_p = 1 \Rightarrow M_M = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$
- If an optical element is rotated we must apply a rotation matrix to the matrix of the element to project into the coordinate space of the rotated element, then project back to the original reference frame using the adjoint of the rotation matrix:

  example: rotated QWP $\Rightarrow M_{QWP}^{R} = M_R^{\dagger} M_{QWP} M_R$ where $M_R = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$, $M_R^{\dagger} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$
- Cascaded elements can be represented by algebraically multiplying component matrices into a system matrix, M_{SYS}.
- Polarization Convention:



- Jones Calculus convention (looking into wave)



$$M_r = \begin{pmatrix}r_s&0\\0&-r_p\end{pmatrix}$$

$$M_t = \begin{pmatrix}t_s&0\\0&t_p\end{pmatrix}$$

- Fresnel's Equations

$$r_s = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$

$$r_p = \frac{-n_t\cos\theta_i + n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t}$$

$$t_s = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$

$$t_p = \frac{2n_i\cos\theta_i}{n_t\cos\theta_i + n_i\cos\theta_t}$$

- Degree of polarization: $V_p = \frac{I_p}{I_p + I_u} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$

CHAPTER 4

- When 2 plane waves add linearly in space, only like polarization components contribute to wave combination effects. For example:

$$Re\{[\vec{U}_1(\vec{r},t)+\vec{U}_2(\vec{r},t)]_y\} = \{A_{01}\, a_{y1} \exp(j(\vec{k}_1\cdot\vec{r}-\omega_1 t+\phi_{1y})) + A_{02}\, a_{y2} \exp(j(\vec{k}_2\cdot\vec{r}-\omega_2 t+\phi_{2y}))\}\,\hat{y}$$
$$= 2A_y \cos(\vec{K}_{HF}\cdot\vec{r}-\omega_{HF}t+\alpha)\cos(\vec{K}_m\cdot\vec{r}-\omega_m t) \quad [\text{for the case, } A_{01}a_{y1}=A_{02}a_{y2}=A_y]$$

- Consider two plane waves of the same frequency traveling in a non dispersive media: $\omega_m = 0$, $\omega_{HF} = \omega$

$$Re\{[\vec{U}_1(\vec{r},t)+\vec{U}_2(\vec{r},t)]_y\} = 2A_y\cos(\vec{K}_{HF}\cdot\vec{r}-\omega t+\alpha)\cos(\vec{K}_m\cdot\vec{r}-\omega_m t+\beta)\,\hat{y}$$

Note that $K_{HF} \perp K_m$. The power flow is along $\vec{K}_{HF}$. See picture →

FIG 1

- When measuring the field on a detector, we are actually measuring the time-averaged irradiance given by:

$I(\vec{r}) = \langle|\vec{U}(\vec{r},t)|^2\rangle_t$; For 2 plane waves of the same frequency: $I(\vec{r}) = A_y^2(1+\cos(\vec{k}_1-\vec{k}_2)\cdot\vec{r})$

From this result: $(\vec{k}_1-\vec{k}_2)\cdot\vec{r} = 2\pi m \longrightarrow$ bright fringe (constructive interference)

$(\vec{k}_1-\vec{k}_2)\cdot\vec{r} = 2\pi(m+\frac{1}{2}) \longrightarrow$ dark fringe (destructive interferense)

- Fringe spacing is given by $\Lambda = \frac{2\pi}{|\vec{k}_1-\vec{k}_2|}$. The minimum possible fringe spacing occurs when waves are traveling in opposite directions (i.e. $\vec{k}_1 = \frac{2\pi}{\lambda}(z)$, $\vec{k}_2 = \frac{2\pi}{\lambda}(-z)$) leading to the result: $\Lambda_{min} = \frac{\lambda}{2}$ (for $\lambda_1=\lambda_2$)
- An important metric for fringe patterns is the visibility, defined as:

$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$; For the case of interference of two unequal amplitude plane waves $(\theta_1=\theta_2=0)$:

$$I(\vec{r}) \propto A_1^2 + A_2^2 + 2A_1A_2\cos((\vec{k}_1-\vec{k}_2)\cdot\vec{r}) \propto I_1+I_2+2\sqrt{I_1I_2}\cos((\vec{k}_1-\vec{k}_2)\cdot\vec{r}) \Rightarrow V = \frac{2\sqrt{I_1I_2}}{I_1+I_2}$$

- If the two interferring waves have different states of polarization, then:

$$I(\vec{r}) = |U_1(\vec{r},t)+U_2(\vec{r},t)|^2 = I_1+I_2+2\sqrt{I_1I_2}\,Re\{\hat{a}_1\cdot\hat{a}_2\}\cos((\vec{k}_1-\vec{k}_2)\cdot\vec{r}) \Rightarrow V = \frac{2\sqrt{I_1I_2}}{I_1+I_2}Re\{\hat{a}_1\cdot\hat{a}_2\}$$

From this result we note that polarization affects the visibility of fringes.

Consider the case of two s-polarized waves of the same amplitude: $Re\{\hat{a}_1\cdot\hat{a}_2\} = 1 \longrightarrow V = 1$

Consider the case of two p-polarized waves of the same amplitude: $Re\{\hat{a}_1\cdot\hat{a}_2\} = \alpha_1\alpha_2+\beta_1\beta_2$ [where: $\hat{a}_1 = \alpha_1\hat{x}+\beta_1\hat{y}$, $\hat{a}_2 = \alpha_2\hat{x}+\beta_2\hat{y}$]

- If the two interfering waves are of the same amplitude, same λ but the "direction dynamics" are reversed, then the orientation of $\vec{K}_{HF}$ & $\vec{K}_m$ switch (see FIG 2 & compare to FIG 1):

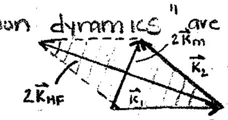


FIG 2

The power flow is still along $\vec{K}_{HF}$. The fringe field is more tightly spaced.

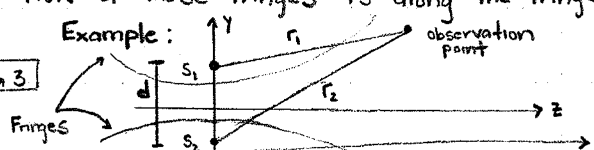
- If the two interfering waves are not of the same wavelength $(\lambda_1\neq\lambda_2)$ then we see the following properties for the resultant fringe field: $\omega_m = \frac{\pi c}{\lambda_{eq}}$, $V_m = \frac{\omega_m}{|K_m|}$, $K_{HF} \not\perp K_m$

Under these conditions, we must have $\Delta t \ll \frac{\Lambda}{V_m} = \frac{\lambda_{eq}}{c} \sim \frac{1}{\Delta\nu}$ to see fringes on our detector.

- The Weiner experiment showed that optical detectors only measure fringe fields created by the E-field of the wave (not the B-field).
- If we interfere two spherical waves, we observe hyperboloidal fringe surfaces. The power flow of these fringes is along the fringe lines.

Example:

FIG 3



$$U_1 = \frac{1}{r_1}e^{j(kr_1-\omega t)}, \quad U_2 = \frac{1}{r_2}e^{j(kr_2-\omega t)} \Big\} \; I(x,y,z) \propto \frac{1}{r_1^2}+\frac{1}{r_2^2}+2\frac{1}{r_1r_2}\cos(k(r_1-r_2))$$

So, for bright fringes: $2m\pi = k(r_1-r_2) = \frac{2\pi}{\lambda}\left(\sqrt{x^2+(y-\frac{d}{2})^2+z^2}-\sqrt{x^2+(y+\frac{d}{2})^2+z^2}\right)$

This result leads to the equation that describes the shape of the fringe pattern for 2 interfering spherical waves. The fringe pattern is given by the hyperpoidal equation:

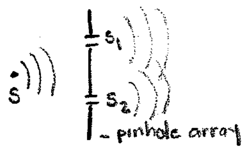
$$\frac{y^2}{y_m^2} - \frac{x^2+z^2}{b^2} = 1 \quad \text{where } y_m = -\frac{1}{2}m\lambda,\; b^2 = \left(\frac{d}{2}\right)^2 - y_m^2$$

Increasing the separation of the sources produces more fringes. The plane of zero OPD_0 is midway between the sources (due to the symmetry). Beyond a source the maximum OPD_0 is reached. Some important properties of the fringe pattern for this setup are listed below:

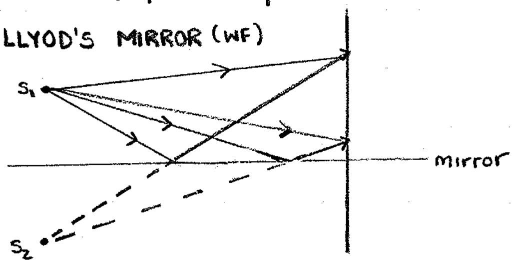
$$m = -\frac{y_m}{\lambda/2n} \;;\; m^* = \max(OPD_0) = \frac{dn}{\lambda} \;;\; OPD_0 = n(r_1-r_2) = m\lambda \;\; [\text{for bright fringes}]$$

- A useful approximation (paraxial) for OPD_0 is: $OPD_0 = n(r_1-r_2) \approx -nd\sin\theta = -nd\frac{y}{z_0}$ [For $y_0 \ll z_0$, $z_0 \gg d$]
- For the fringe pattern from the interference of two spherical waves as oriented in FIG 3, the xz planar intercept creates a concentric circle fringe pattern where the fringe radius is given by: $r_{m'} = y_0\sqrt{\frac{2m'}{m^*}}$ where $m' = m^*-m = \frac{2n}{\lambda}\left(\frac{d}{2}-y_m\right)$. The yz planar intercept creates a pattern of approximately straight line fringes at $z=z_0$ where the spacing is given by: $\Delta y = \frac{\lambda z_0}{nd}$

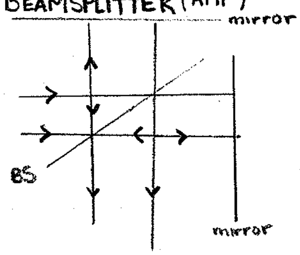
- The interference of two spherical waves created from two polychromatic sources results in a reduction in fringe visibility (compared to the case of monochromatic sources). We observe good contrast at near zero OPD ($m=0$) and fringe washout for larger m.
- Interference of a plane wave and a spherical wave creates an fringe pattern of concentric circles. The fringe radius is given by: $r_m = \sqrt{2\lambda m z_0 / n}$ (for large z_0)
- Interference of a plane wave and a cylindrical wave results in a cylindrical shell-like fringe pattern. The fringe height is given by $y_m = \sqrt{2\lambda m z_0 / n}$
- Several optical devices can be used to create interfering waves. These devices can be put into two categories: division of amplitude (multiple source images) and division of wavefront (multiple beams derived from source). A few common devices are listed below.
- YOUNG'S DOUBLE PINHOLE (WF)



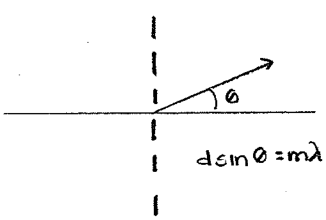
- LLYOD'S MIRROR (WF)



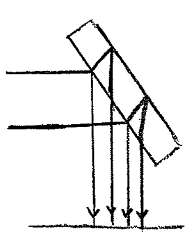
- FRESNEL MIRROR (WF) & FRESNEL BIPRISM
  (See notes for diagrams)
- BEAMSPLITTER (AMP)



- DIFFRACTION GRATING (AMP)



- POLARIZATION CRYSTALS (WF & AMP)
  (see notes for diagrams)
- PLANE PARALLEL PLATE (WF & AMP)



CHAPTERS 5 & 6

- A coherence function is a second-order average of the statistical properties of a coherent field. A useful property that can be derived from the coherence function is the ability of a field has to create interference patterns.
- There are two types of coherence: spatial and temporal. The behavior of visibility as a function of source size is known as spatial coherence (size $\uparrow$, V $\downarrow$). The behavior of visibility as a function of wavelength distribution is known as temporal coherence.
- The mutual coherence function is the temporal coherence of the electric field at 2 positions in space with respect to time. It is given by: $\Gamma(\vec{r}_1,\vec{r}_2,t_1,t_2) = \langle U(\vec{r}_1, t+t_1)U^*(\vec{r}_2, t+t_2)\rangle = \langle U(\vec{r}_1,t)U(\vec{r}_2,t+\tau)\rangle = \Gamma_{12}(\tau)$ where $\tau = t_2 - t_1$ for stationery fields.
- The irradiance from a young's double pinhole interferometer with a two wavelength point source is effectively the sum of the irradiance pattern created by the individual wavelengths:

  $$\frac{1}{2}\langle |U_{total}|^2\rangle = A^2(2+\cos(\omega_a\tau)+\cos(\omega_b\tau)) = A^2(2+\cos(K_a OPD_0)+\cos(K_b OPD_0)) \approx A^2\left(2+2\cos\left(\frac{2\pi}{\bar{\lambda}} OPD_0\right)\cos\left(\frac{\pi}{\lambda_{eq}} OPD_0\right)\right)$$

  where $\bar{\lambda} = \frac{\lambda_a+\lambda_b}{2}$, $\lambda_{eq} = \frac{\lambda_a\lambda_b}{|\lambda_a - \lambda_b|}$

  The cosine terms produce fringes of spacing Λ_a & Λ_b resulting in an overall fringe spacing of $\Lambda = \frac{z_0\bar{\lambda}}{d}$

  The visibility is a smooth function that decays to zero at $y_0 = \frac{z_0\lambda_{eq}}{d}$.
- The coherence length is defined as the OPD_0 corresponding to the distance between the maximum fringe contrast ($y_0=0$) and the first zero in the fringe visibility: $L_c = \frac{\lambda_{eq}}{2} \approx \frac{\bar{\lambda}^2}{2\Delta\lambda}$
- As the wavelength distribution of the source widens, visibility along the observation plane decreases.
- For a source with a continuous wavelength distribution, we must consider the power spectrum of the source.

  $$I(\tau;\nu)\,d\nu = \tfrac{1}{2}cn\epsilon_0 A^2\left[K_1+K_2+2\sqrt{K_1K_2}\cos(2\pi\nu\tau)\right]g(\nu)\,d\nu \Rightarrow I(\tau) = \tfrac{1}{2}cn\epsilon_0A^2\left[K_1+K_2+2\sqrt{K_1K_2}\,m_{12}(\tau)\cos(\phi_{12}(\tau))\right]$$

  where $m_{12}(\tau) = |\mathcal{F}\{g(\nu)\,step(\nu)\}|$, $\phi_{12}(\tau) = \arg(\mathcal{F}\{g(\nu)\,step(\nu)\})$

  This leads to the result: $V(\tau) = \frac{2\sqrt{K_1K_2}}{K_1+K_2}m_{12}(\tau)$

  $\beta_{12}(\tau) = \arg(\mathcal{F}\{f(\nu)\})$

  We simplify this result further by assuming $g(\nu)\,step(\nu) = f(\nu-\bar{\nu}) \rightarrow I(\tau) = \frac{1}{2}cn\epsilon_0A^2\left[K_1+K_2+2\sqrt{K_1K_2}\,m_{12}(\tau)\cos(2\pi\bar{\nu}\tau+\beta_{12}(\tau))\right]$
- Consider a rectangular power spectrum. Then, $I(\tau) = \frac{1}{2}cn\epsilon_0A^2\left[1+\left|\mathrm{sinc}\left(\frac{\Delta\nu d y_0}{c z_0}\right)\right|\left(\cos\left(2\pi\frac{\bar{\nu} d y_0}{c z_0}\right)+\beta_{12}\left(\frac{\Delta\nu d y_0}{c z_0}\right)\right)\right]$
- Coherence time is defined as $t_c = \frac{L_c}{c} = \frac{1}{\Delta\nu}$
- Consider a Twymann Green interferometer w/ Gaussian power spectrum. Then, $g(\nu)\,step(\nu) = \frac{1}{\Delta\nu}\,gaus\left(\frac{\nu-\bar{\nu}}{\Delta\nu}\right) \rightarrow V(\tau) = gaus(\Delta\nu\tau)$

  We can than replace $\tau \rightarrow \frac{OPD_0}{c} = \frac{2}{c}|d_1-d_2|$ to determine the maximum mirror spacing.
- Additional properties of spatial coherence: fringe visibility is not a function of location in observation space.
- Visibility is inversely proportional to source size.
- For continuous source functions we must consider the radiant exitence of the source $m_R(x_s,y_s)$.

  This results in an irradiance of: $I(\tau) = I_0\left(K_1+K_2+2\sqrt{K_1K_2}\,\mu_{12}\left(\frac{d}{\lambda z_s}\right)\cos\left(2\pi\bar{\nu}\tau+\beta_{12}\left(\frac{d}{\lambda z_s}\right)\right)\right)$

  Where, $\mu_{12}\left(\frac{d}{\lambda z_s}\right) = \left|\mathcal{F}^{-1}_{d/\lambda z_s}\left\{\int_{-\infty}^{\infty} m_R(x_s,y_s)\,dx_s\right\}\right|$, $\beta_{12}\left(\frac{d}{\lambda z_s}\right) = \arg\left(\mathcal{F}^{-1}_{d/\lambda z_s}\left\{\int_{-\infty}^{\infty} m_R(x_s,y_s)\,dx_s\right\}\right)$

  If we define $\theta_{ph} = \frac{d}{z_s}$ then, $I(\tau) = I_0\left(K_1+K_2+2\sqrt{K_1K_2}\,\mu_{12}\left(\frac{\theta_{ph}}{\lambda}\right)\cos\left(2\pi\bar{\nu}\tau+\beta_{12}\left(\frac{\theta_{ph}}{\lambda}\right)\right)\right)$

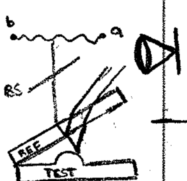
  Note that: $V\left(\frac{\theta_{ph}}{\lambda}\right) = \frac{2\sqrt{K_1K_2}}{K_1+K_2}\,\mu_{12}\left(\frac{\theta_{ph}}{\lambda}\right)$
- The Van Citterdt Zernike Theorem states, if a quasimonochromatic source is a considerable distance from the aperture plane and pinhole separation is small, fringe visibility from an extended source is proportional to the inverse Fourier Transform of the sources spatial distribution. The transform variable is the angular separation of the aperture-plane sampling points divided by the wavelength.
- The coherence area is defined by a region representing separations of points in the aperture area that can interfere with high visibility.
- Fringe localization defines the region of space where interference occurs and fringes with reasonably good contrast are observed.
  - localized everywhere: fringes have high V everywhere in observation space.
  - localized: fringes have high V over some surface in observation space (surface is not necessarily planar).
  - localized at infinity: must use a lens to transform the angular distribution of the fringes to a spatial distribution of fringes on the observation plane w/ high V (Hadinger's fringes).
  - fringes of equal thickness: localized fringes where the fringes correspond to contours of constant thickness between two surfaces.
- Types of fringes created by sources:
  - monochromatic: localized fringes (also called unlocalized)
  - polychromatic: temporal coherence produces localized fringes.
  - quasi-monochromatic extended source: spatial coherence effects V. Depends on geometry of interferometer.
  - quasi-polychromatic extended source: usually analyze temporal & spatial coherence separately.

- Determining fringe localization:
  - determine the fringe period & position in the observation plane given a monochromatic point source.
  - integrate fringe patterns resulting from positioning the point source over the spatial extent of the distributed source, and/or from the separate frequencies in the power spectrum.
  - define regions in the observation space where V is large enough to detect fringes ($V > 0.2$)
- For a plane parallel plate interferometer, we observe Hadinger's Fringes
- The table below summarizes fringe localization for several types of interferometers:

INTERFEROMETER	SOURCE TYPE	FRINGE TYPE
YDPI	MPS PPS	unlocalized localized (near axis)
Plane Parallel Plate W/ fringe lens	MPS QMXS/PXS	unlocalized Hadinger's fringes
Thin Wedge/Film	QMXS	localized @ film
Fizeau	QMXS	fringes of equal thickness
Michelson W/ fringe lens	QMXS	Hadinger's
Michelson - tilted mirrors W/ fringe lens	QMXS	fringes of equal thickness
Twymann-Green	PXS	localized
Lloyd's Mirror	PPS	localized (near axis)
Fresnel's biprism	QMXS	localized

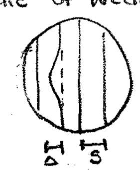
NOTE: SOURCE TYPE LEGEND
MPS = monochromatic point source
PPS = polychromatic point source
QMXS = quasi monochromatic extended source
PXS = polychromatic extended source

- Interferometers:



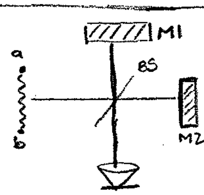
FIZEAU: - fringes are localized at reflective surface of wedge. This surface is imaged to see the fringes.
- often used for optical testing.

FRINGE PATTERN



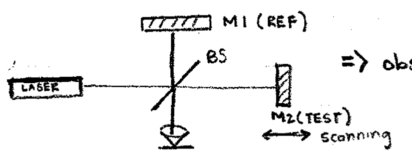
surface height error: $\Delta t = \frac{\lambda \Delta}{2S}$

MICHELSON:



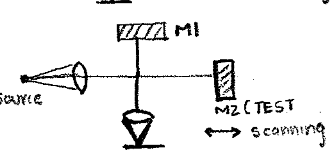
=> observe Hadinger's Fringes

TWYMAN GREEN:



=> observe localized fringes ; $I(OPD_0) \approx \frac{I_0}{2}\left(1+\cos(k\,OPD_0)\right)$

FOURIER TRANSFORM:



IDEA: measure $V\left(\frac{OPD_0}{c}\right)$ by moving M2
inverse fourier transform to get $g(\nu)$

NEWTON'S RINGS: Similar to Fizeau/thin film interferometer, except the object being tested is rotationally symmetric so the resulting fringes of equal thickness become circular contours.

MACH-ZENDER: tests samples in transmission.

LATERAL SHEARING INTERFEROMETER: measures the wavefront slope.