Answer Key.

1.) Consider a simple optical system with a circular exit pupil in the rear focal plane, where λ = 550 nm, m_T = -1 and NA = 0.1. For the cases of aberration listed below, calculate the point-source response by taking the Fourier transform of the field in the pupil, with the appropriate scaling factors. You will need to write a computer program to perform the calculation. (You can download and use hankel_calculation_at_rear_focus505R.m from the website as a starting point, if you like.) Then, find the percentage of encircled energy contained in the point-source response as a function of radius in the image plane. The functional form for aberration in the stop is $\exp\left\{j2\pi W\left(\frac{x_s}{a},\frac{y_s}{a}\right)\right\}$, where (x_s,y_s) are coordinates in the exit pupil and a is the exit pupil radius. For each case, graph the spot profile and the encircled energy. Graph your encircled energy results as a percentage of total power in the focal plane. Determine the radius at 80% encircled energy. If you have trouble with access to a computer for this calculation, contact the instructor for help. Show your work.

a.) (2.5 pts) No aberration: $W\left(\frac{x_s}{a},\frac{y_s}{a}\right)=0$.

80% EE @ 2.33μm

b.) (2.5 pts.) 1 wave of defocus: $W\left(\frac{x_s}{a},\frac{y_s}{a}\right)=\left(\frac{x_s}{a}\right)^2+\left(\frac{y_s}{a}\right)^2$.

80% EE @ 9.88

c.) (2.5 pts.) 1 wave of spherical: $W\left(\frac{x_s}{a},\frac{y_s}{a}\right)=\left[\left(\frac{x_s}{a}\right)^2+\left(\frac{y_s}{a}\right)^2\right]^2$.

80% EE @ 13.52μm

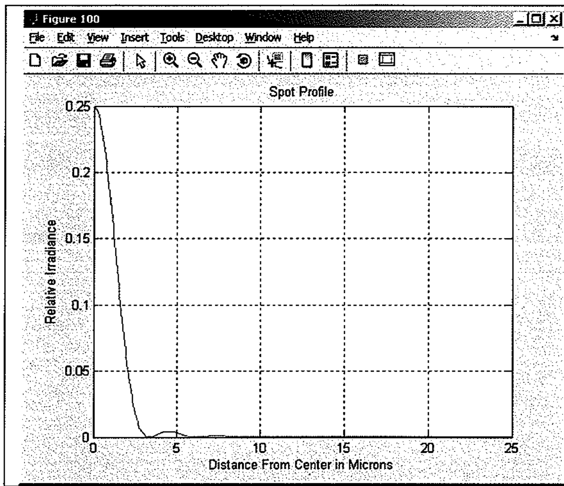
d.) (2.5 pts.) 1 wave of spherical minus 1 wave of defocus:

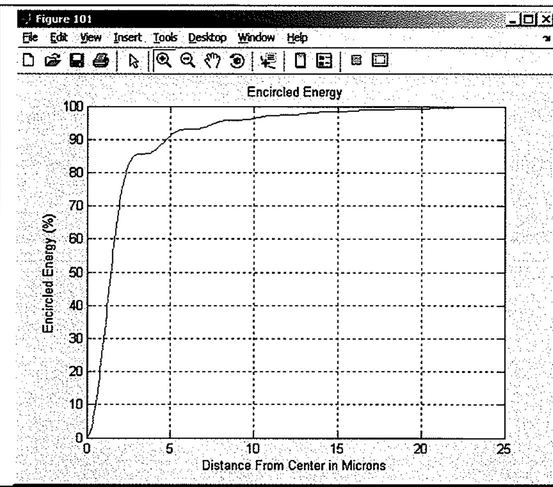
$$W\left(\frac{x_s}{a},\frac{y_s}{a}\right)=\left[\left(\frac{x_s}{a}\right)^2+\left(\frac{y_s}{a}\right)^2\right]^2-\left[\left(\frac{x_s}{a}\right)^2+\left(\frac{y_s}{a}\right)^2\right].$$

80% EE @ 5.10μm

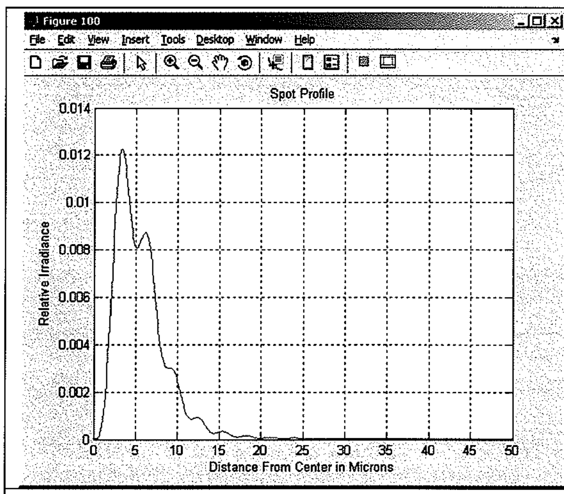
505R Final Exam Spring 2008

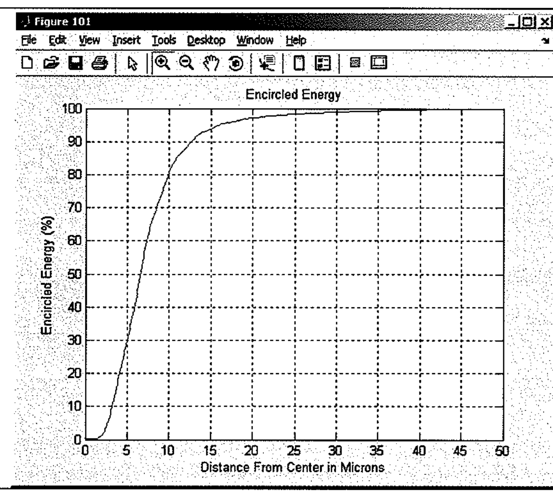
1.)



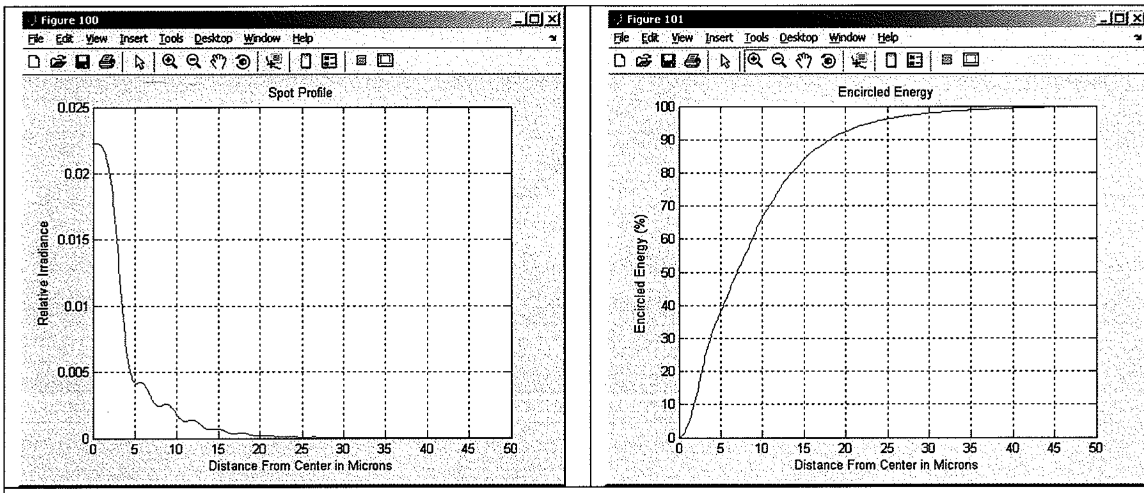


No Aberration. 80% EE radius = 2.33μm.

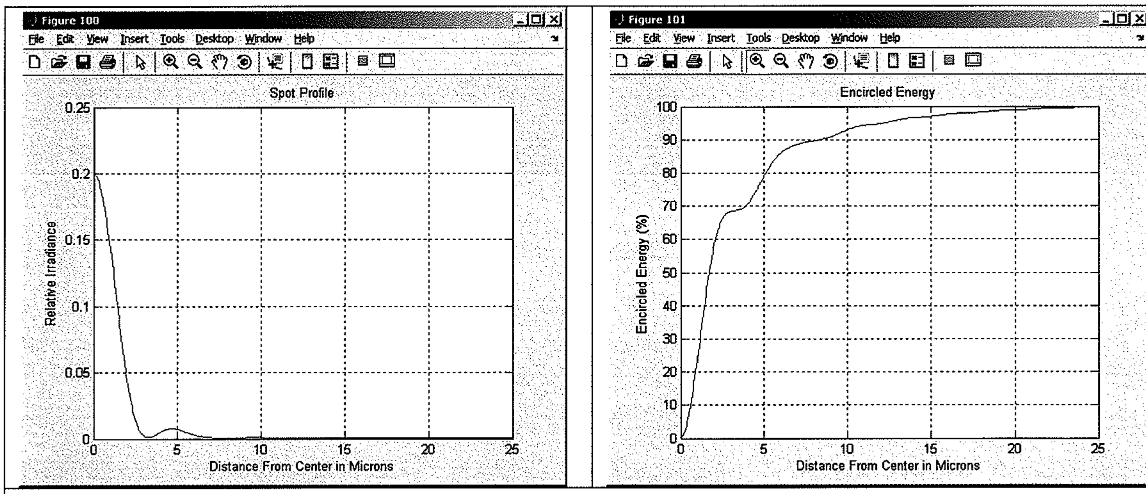




1 Wave Defocus. 80% EE radius = 9.88μm.



1 Wave Spherical. 80% EE radius = 13.52μm.

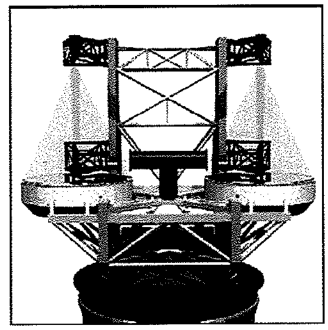


1 Wave Spherical Minus 1 Wave Defocus. 80% EE radius = 5.10μm.

```
%parameters
r                    = 5e-3;  %radius of pupil
z0                   = 50e-3;   %distance from pupil to image
rho_0_vec            = linspace(0,25e-6,500);  %vector of observation-plane radii
wavelength           = 550e-9;
k                    = 2*pi/wavelength;

U_0                  = [];

bessel_arg_vec       = 2*pi*rho_0_vec*r/wavelength/z0;
W                    = @(rho) 0*rho.^2 ;  %aberration as a function of rho, the↙
normalized pupil radius

disp('Working on calculation')
for ii  = 1:length(rho_0_vec)

    rho_0            = rho_0_vec(ii);
    if rho_0 == 0
        rho_0 = 1e-10;
    end
    bessel_arg       = bessel_arg_vec(ii);
    mybessel         = @(rhoprime) rhoprime.*exp(j*2*pi*W(rhoprime)).*besselj(0,↙
bessel_arg*rhoprime);
    field_value      = quad(mybessel,0,1);
    U_0              = [U_0 field_value];

end

I_0                  = abs(U_0).^2;

figure(100);plot(rho_0_vec*1e6,I_0);grid
title('Spot Profile')
ylabel('Relative Irradiance')
xlabel('Distance From Center in Microns')

EE                   = cumsum(I_0.*rho_0_vec);
EE                   = 100*EE/max(EE)

figure(101);plot(rho_0_vec*1e6,EE);grid
title('Encircled Energy')
ylabel('Encircled Energy (%)')
xlabel('Distance From Center in Microns')

disp('Calculation complete')
```

```
freq_vec         = freq_sampling*(0:(length(xvec)/2-1))/1000;

%only plot out to 250lp/mm
plot_indx        = find(freq_vec <= 250);

figure(10);plot(freq_vec(plot_indx),MTFx(plot_indx)/MTFx(1))
title('X-direction LBT MTF Profile')
xlabel('Spatial Freq (lp/mm)')
ylabel('Relative Visibility (Contrast)')
grid
figure(11);plot(freq_vec(plot_indx),MTFy(plot_indx)/MTFy(1))
title('Y-direction LBT MTF Profile')
xlabel('Spatial Freq (lp/m)')
ylabel('Relative Visibility (Contrast)')
grid
```

2.) The Large Binocular Telescope (LBT) sits on top of Mt. Graham in southern Arizona. Some of its features are:



- Number of Primary Mirrors: 2
- Primary Spacing: 14.417 meter center-to-center
- Primary Physical Diameter: 8.417 meter

a.) (5 pts.) For the sake of calculation, assume that both mirrors act as if they are part of one larger mirror and that they focus to a common point a distance of 100 m behind the mirrors. If the wavelength of observation is 1 μm, sketch the MTF of the system in the $(\xi,0)$ and $(0,\eta)$ profiles.

(see plots)

b.) (5 pts.) Ideally, what are the approximate dimensions of the smallest object that the telescope can resolve in terms of milli-arc-seconds? State any assumptions that you make.

x ~ 9.0 mas

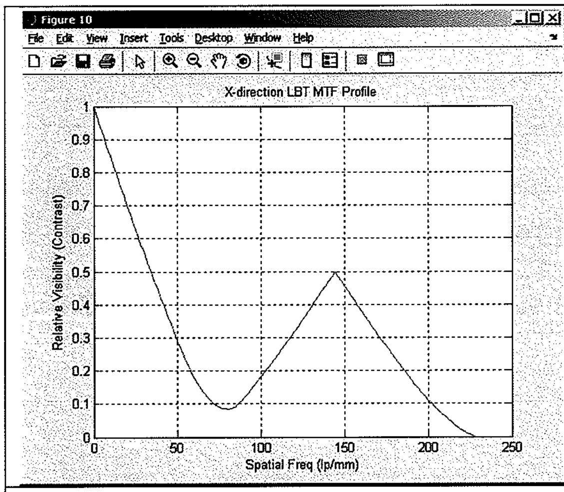
y ~ 24.6 mas

resolution ~ $\frac{\lambda}{D_{x,y}}$

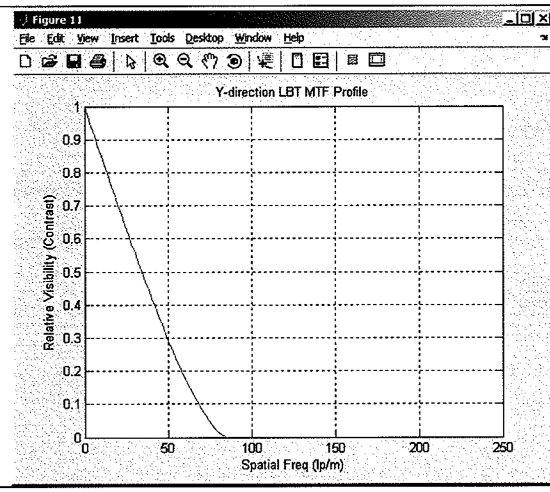
D_x = 22.8m

D_y = 8.4m

2.)



X MTF (Cuttoff = 228 lp/mm)



Y MTF (Cuttoff = 84 lp/mm)

```
%Brute force calculaiton of Problem 2 S2008 Final Exam

xvec              = linspace(-50,50,500);
yvec              = xvec;

[X Y]             = meshgrid(xvec,yvec);
R                 = (X.*X + Y.*Y).^(0.5);

segment_indx      = find(R<=8.417/2);
Seg_mat           = repmat(0,size(X));
Seg_mat(segment_indx)    = repmat(1,size(segment_indx));

%shift value
sampling          = xvec(2)-xvec(1);
shift             = 14.417/2/sampling;

%make left segment
L_seg             = circshift(Seg_mat,[0 round(-shift)]);

%make right segment
R_seg             = circshift(Seg_mat,[0 round(shift)]);

%make LBT
LBT               = L_seg + R_seg;


MTFx              = [];

disp('Working on x profile')
for ii = 0:(length(xvec)/2-1);

    MTFx          = [MTFx sum(sum(circshift(LBT,[0 ii]).*LBT))];

end

MTFy              = [];

disp('Working on y profile')
for ii = 0:(length(yvec)/2-1);

    MTFy          = [MTFy sum(sum(circshift(LBT,[ii 0]).*LBT))];

end

%frequency scale
D                 = 8.417+14.417;
f                 = 100;
NA                = D/2/f;
lambda            = 1e-6;
Incoherent_cutoff    = 2*NA/lambda;  %lines per meter
freq_sampling     = sampling/f/lambda;
```

3.) A GaP substrate ($n = 3.493$) is used with a $\lambda = 532$ nm laser beam.

a.) (5 pts.) What are the refractive index and thickness t of an ideal single-layer anti-reflective coating for normal incidence?

$$n = \sqrt{3.493} = 1.869$$

$$t = \frac{\lambda}{4n} = 71.16\ nm$$

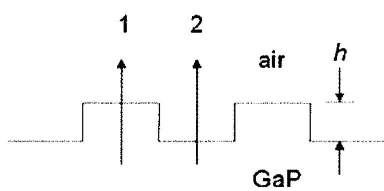
b.) (5 pts.) Some possible materials to use for the coating are cryolyte (1.35), LiF (1.37), MgF2 (1.39), ThF4 (1.52), CeF3 (1.62), PbF2 (1.73), ZnS (2.3), ZnSe (2.55), Si (3.50), Ge (4.20), Te (4.80), PbTe (5.50), SiO2 (1.48), Al2O3 (1.60), MgO (1.72), Y2O3 (1.82), ScO3 (1.86), SiO (1.95), HfO2 (1.98), ZrO2 (2.10), CeO2 (2.20), NbO5 (2.20), Ta2O5 (2.10) and TiO2 (2.45). Alternatively, you can search the web for an appropriate material with an index close to your ideal value. Whatever material you choose, make sure that it has a low absorption value at the 532 nm wavelength. What is the actual reflection given the material that you chose, assuming that you use a quarter-wave layer of the material?

(Scandium Oxide) ScO_3 is best, but hard to find absorption data. Is ok @ 532. $R \approx 0.0023\%$

(Yttrium Oxide) Y_2O_3 is OK @ 532nm.

$$R = \left[\frac{n_0 n_s - n_1^2}{n_0 n_s + n_1^2}\right]^2 = \left[\frac{3.493 - (1.82)^2}{3.493 + (1.82)^2}\right]^2 = 7.0 \times 10^{-4} = 0.07\%$$

c.) (5 pts.) A diffraction grating is etched into one surface of the substrate, as shown below. What is the difference in OPL between paths 1 and 2, in terms of the height h of the step?

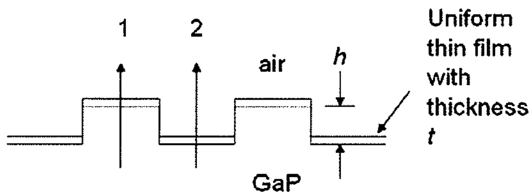


$OPL_1 = hn$

$OPL_2 = h$

$OPD = OPL_1 - OPL_2 = (n-1)h$

d.) (5 pts.) Assume that a conformal AR coating is made on top of the diffraction grating, as shown below. What is the difference in OPL between paths 1 and 2, in terms of the height h of the step and any other relevant parameters? Assume only a single pass through the coating.



$OPL_1 = hn + tn_{AR}$

$OPL_2 = h + tn_{AR}$

$OPD = OPL_1 - OPL_2 = (n-1)h$

SAME AS (c)

4.) A Young's double slit interferometer (YDSI) is placed a distance z_s from a quasimonochromatic source. A slit of width w is placed in front of the source, where $\bar{\lambda}$ = 650 nm and w = 1 mm. Distance between the slits is d = 0.5 mm. The source slit is oriented parallel to the YDSI slits.

a.) (5 pts.) What is the minimum distance z_s for which fringes are visible in the observation plane of the YDSI?

assume $V_{min} \sim 0.2$.

$V(z_s) = \left|\mathrm{sinc}\left(\frac{wd}{\lambda z_s}\right)\right|$

at V_{min}, $\frac{wd}{\lambda z_s} = 0.82625$.

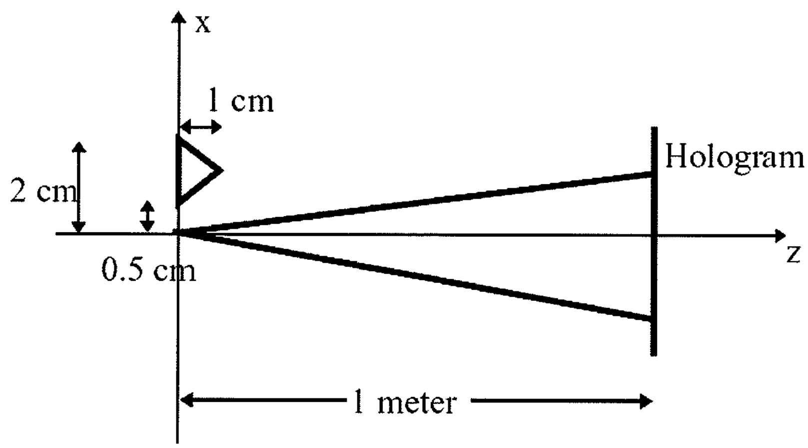
$\therefore z_s = \frac{wd}{\lambda (0.82625)} = \frac{10^{-3} \times 0.5\times 10^{-3}}{650\times 10^{-9} \times 0.82625}$

$= \boxed{0.9310\ m}$

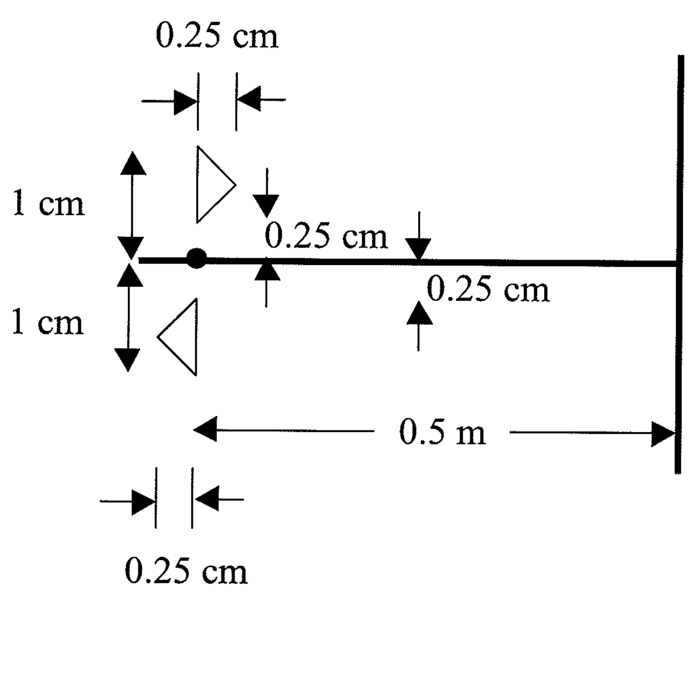
b.) (5 pts.) With z_s positioned as in part (a), what differences in the fringe pattern are observed if the number of slits is increased to five slits, where neighboring slits are spaced by d? Justify your answer.

Only brightness increases and visibility reduces. The fringe shape does not change, because there is not sufficient spatial coherence to observe multiple-slit effects.

5.) (15 pts.) A hologram is made by interfering a point source and a triangular shaped object as shown below. The hologram is reconstructed using a point source 0.5 m from the hologram. Give the x and z coordinates of the primary and conjugate images. Give a sketch to show what the images look like.



Note that the longitudinal magnification goes as the square of the lateral magnification.

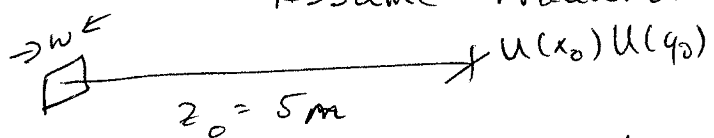


6.) (20 pts.) Optical Coherence Tomography (OCT) is very similar to the Vertical Scanning Interferometer (VSI) described in class. Use a UA search engine or the web to find a scientific paper that discusses an OCT system. Analyze the system presented in the paper in terms of the coherence notes from this class. Justify or refute the author's claims for parameters like vertical resolution and sensitivity, based on equations and concepts from the notes. Attach a photocopy of the paper to your exam. The reference does not need to be from a refereed journal, but it needs to be more than a trade magazine. Contact me if you have a question about your reference.

7.) A 1cm by 1cm square beam from a $\lambda = 405$ nm laser illuminates a rough surface.

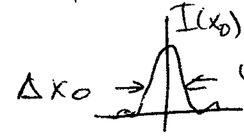
a.) (5 pts.) Determine the average speckle size at a distance of 5 m from the surface.

Assume Fraunhofer ⇒



$\lambda = 405\text{nm}$

$U(x_0) \sim \text{sinc}\left(\frac{x_0 w}{\lambda z_0}\right)$



width $\sim \frac{\lambda z}{w}$

$= \frac{0.405\times10^{-6}\times 5}{10^{-2}}$

$= 2\times10^{-4} = \boxed{0.2\text{ mm}}$

b.) (5 pts.) If the surfaced is imaged with a high-quality (negligible aberrations) lens system, where $m_T = -1/10$ and f/# = 10, find the average speckle size in the image.

$f/\# = 10 \Rightarrow NA = \frac{1}{2\cdot 10} = 0.05$

$\Delta x_0 \sim \frac{\lambda}{NA} = \frac{0.405\times10^{-6}}{0.05} \approx 8\times10^{-6} = \boxed{8\mu m}$

circular No m m.

4.94 μm 5.435 μm

c.) (5 pts.) Speckle is produced when a rough object is illuminated with a laser beam. Does this illumination condition change the PSF of the optical system used to image the object onto a detector? Explain.

NO. A single point on the object is imaged through the system with a simple PSF. The random phases between points on the object cause the speckle.