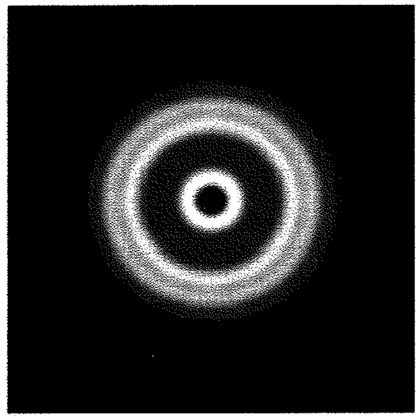
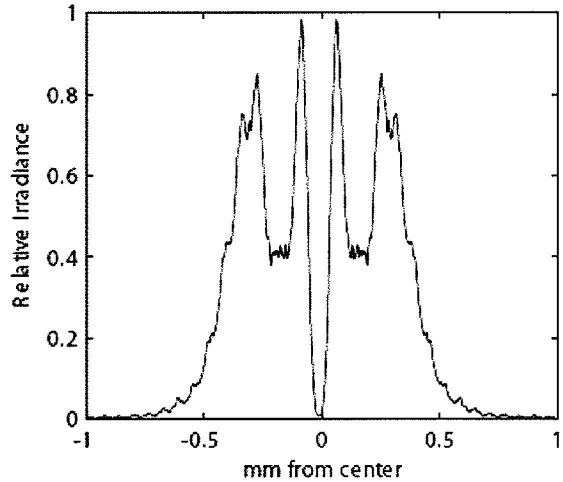
1.) (10 pts) The diffraction pattern below results from a $\lambda = 532$ nm collimated laser beam passing through a 1.0 mm diameter round hole. What is the distance between the aperture and the plane of observation? State any assumptions that you make.





$$N_f = 4 = \frac{a^2}{\lambda L} = \frac{a^2}{\lambda z_0}$$

$$z_s \to \infty \Rightarrow z_0 = L$$

$$z_0 = \frac{a^2}{\lambda 4} = \frac{(0.5\times10^{-3})^2}{0.532\times10^{-6}\times 4} = \boxed{0.1175\,m}$$

2.) (15 pts) Describe the three free-space point spread function (fsPSF) techniques, when it is appropriate to use them, and any limitations implied in their use.

Assume plane-to-plane propagation.

1) Huygens-

$$h(P_0;P_1) = \frac{\sqrt{1+(kr_{01})^2}}{2\pi r_{01}^2}\,\gamma_z\, e^{j\left[kr_{01} - \tan^{-1}(kr_{01})\right]}$$

this fsPSF can be used directly after the aperture and throughout all of the observation space. There are no approximations or limitations associated with its use.

2) Fresnel - parabolic wavelets

$$h(P_0;P_1) = -\frac{j}{\lambda} e^{jkz_0} e^{j\frac{k}{2z_0}(x_0^2+y_0^2)} e^{-j\frac{k}{z_0}(x_0x_1+y_0y_1)} e^{j\frac{k}{2z_0}(x_1^2+y_1^2)}$$

$$= -\frac{j}{\lambda} e^{jkz_0} e^{j\frac{k}{2z_0}\left[(x_0-x_1)^2+(y_0-y_1)^2\right]}$$

This fsPSF is strictly useful only when

$$z_0 \gg \sqrt[3]{\frac{\pi}{4}}\,\lambda\left[\left(\frac{x_0-x_1}{\lambda}\right)^2+\left(\frac{y_0-y_1}{\lambda}\right)^2\right]^{2/3}_{max},$$

but in practice can be used at much closer distances due to the principle of stationary phase.

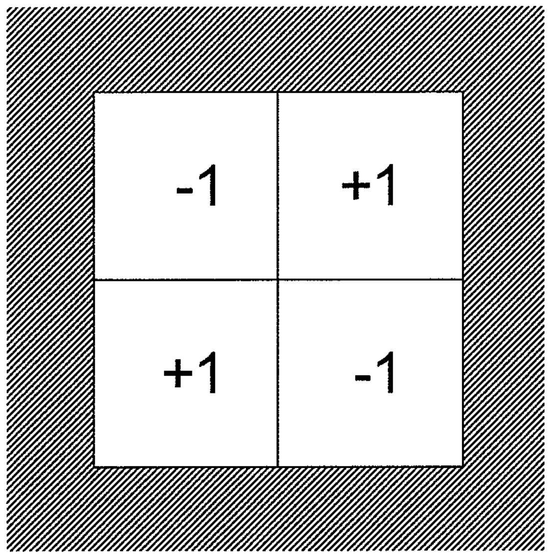
3) Fraunhofer - Planar wavelets

$$h(P_0;P_1) = -\frac{j}{\lambda} e^{jkz_0} e^{j\frac{k}{2z_0}(x_0^2+y_0^2)} e^{-j\frac{k}{z_0}(x_0x_1+y_0y_1)}$$

strictly useful when $z_0 \gg \frac{k_0}{2}(x_1^2+y_1^2)_{max}$.

Also can estimate useful distance by Fresnel number, where $L \gg \frac{a^2}{\lambda}$.

3.) (20 pts.) The square aperture below is placed in an otherwise opaque screen. The aperture is divided into four equal-area squares, where the amplitude transmission of each square is shown. The width of each square is 0.5 mm. (The shading represents part of the opaque screen.) What is the value of the Fraunhofer pattern at $(x_0, y_0) = (0,0)$ and a distance of 2 m if the aperture is illuminated with a collimated, on axis plane wave of amplitude A with $\lambda = 500$ nm?



Babinet's Principle:

A	B
C	D

$$U_A(x_1,y_1) = -\,\mathrm{rect}\left(\frac{x_1+\Delta}{2a}\right)\mathrm{rect}\left(\frac{y_1-\Delta}{2a}\right)$$
$$U_B(x_1,y_1) = \mathrm{rect}\left(\frac{x_1-\Delta}{2a}\right)\mathrm{rect}\left(\frac{y_1-\Delta}{2a}\right)$$
$$U_C(x_1,y_1) = \mathrm{rect}\left(\frac{x_1+\Delta}{2a}\right)\mathrm{rect}\left(\frac{y_1+\Delta}{2a}\right)$$
$$U_D(x_1,y_1) = -\mathrm{rect}\left(\frac{x_1-\Delta}{2a}\right)\mathrm{rect}\left(\frac{y_1+\Delta}{2a}\right)$$
$$U_A(x_0,y_0) = -\,C\,\mathrm{sinc}(2a\xi)\,\mathrm{sinc}(2a\eta)\,e^{j2\pi(-\Delta\xi+\Delta\eta)}$$
$$U_B(x_0,y_0) = C\,\mathrm{sinc}(2a\xi)\,\mathrm{sinc}(2a\eta)\,e^{j2\pi(\Delta\xi+\Delta\eta)}$$
$$U_C(x_0,y_0) = C\,\mathrm{sinc}(2a\xi)\,\mathrm{sinc}(2a\eta)\,e^{j2\pi(-\Delta\xi-\Delta\eta)}$$
$$U_D(x_0,y_0) = -C\,\mathrm{sinc}(2a\xi)\,\mathrm{sinc}(2a\eta)\,e^{j2\pi(\Delta\xi-\Delta\eta)}$$
$$C = -\frac{j(2a)^2}{\lambda z_0}e^{jkz_0}e^{j\frac{k\rho_0^2}{2z_0}} \qquad \xi = \frac{x_0}{\lambda z_0} \quad \eta = \frac{y_0}{\lambda z_0}$$

Evaluated at $(x_0,y_0)=(0,0)$:

$U_A = -C(0,0)$

$U_B = C(0,0)$

$U_C = C(0,0)$

$U_D = -C(0,0)$

$\boxed{U_{TOTAL}(0,0) = 0}$

(could also recognize by inspection from the central ordinate theorem.)

4.) A Fresnel zone plate with primary focal length 200mm and diameter 10mm is illuminated with a collimated, on-axis λ= 500 nm laser beam and an irradiance of $1 W/cm^2$. The even Fresnel zones in the plate are blocked in order to make the lens.

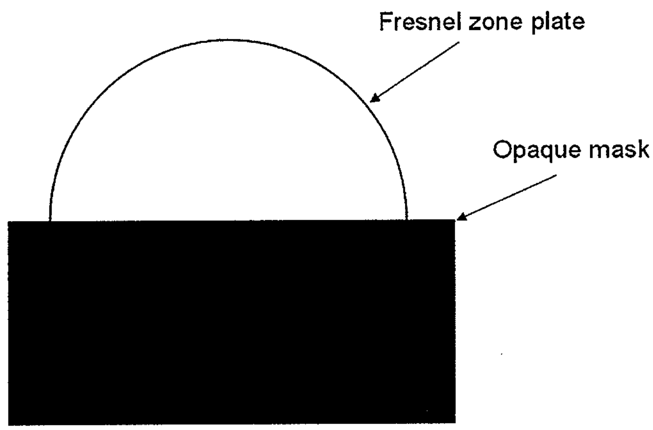
a.) (5 pts.) What is the width of the outmost Fresnel zone?

$$N_f = \frac{a^2}{\lambda f_1} = \frac{(5\times10^{-3})^2}{(500\times10^{-9})(200\times10^{-3})} = 250$$

$$\rho_{249} = \sqrt{249\times500\times10^{-9}\times200\times10^{-3}} \approx 0.00499$$

width $\approx 0.00500 - 0.00499 = 0.00001\,m = \boxed{10\,\mu m}$

An opaque mask is placed over ½ of the zone plate, as shown below.



b.) (5 pts.) What is the on-axis irradiance at the primary focus?

$$I(P) = \frac{1}{4}\cdot 4 I_\infty(P) N_{FZ}^2 = 1 W/cm^2 \cdot (125)^2 = \boxed{15,625\ W/cm^2}$$

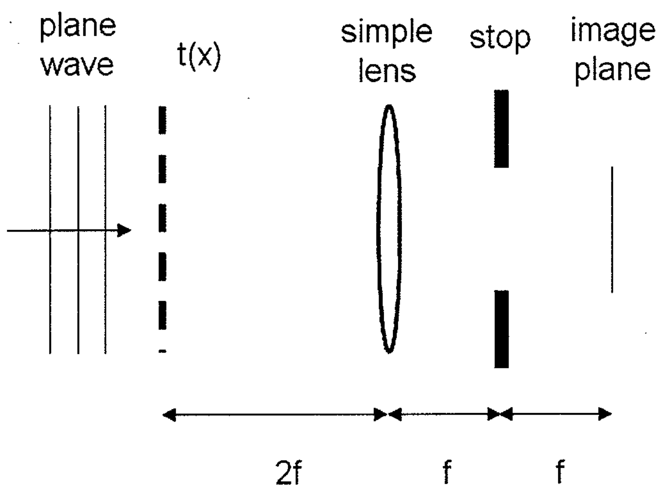
c.) (5 pts.) What is the location of the $+3^{rd}$ order focus?

$$f_{+3} = \frac{f_1}{3} = \frac{200\times10^{-3}}{3} = \boxed{66.7\ mm}$$

d.) (5 pts.) What is the on-axis irradiance at the $+3^{rd}$ order focus?

(Same as f_1) $\boxed{15,625\ W/cm^2}$

5.) A grating with amplitude transmittance $t(x) = 0.5 + 0.5\cos(2\pi\xi_0 x)$ is illuminated by an on-axis monochromatic plane wave with $\lambda = 500$ nm. The grating is imaged with a simple lens as shown below, where f = 50 mm. The diameter of the stop is 2 mm. You may assume that the illuminated portion of the grating is very large with respect to the period of the grating. State any other assumptions that you make.



a.) (5 pts.) Calculate the angular spectrum of the light transmitted through the grating.

$$A_{z=0}(\xi,\eta) = [0.5\delta(\xi) + 0.25\delta(\xi-\xi_0) + 0.25\delta(\xi+\xi_0)]\delta(\eta)$$

b.) (5 pts.) How many plane waves are contained in the angular spectrum of part (a)?

3

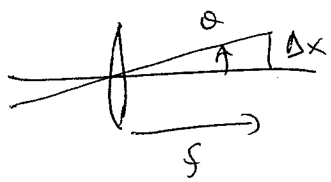
c.) (5 pts.) Derive an expression for the base Talbot distance between the grating and the lens in terms of the grating frequency ξ_0.

$$z_{base} = \frac{1}{2\lambda\xi_0^2} = \frac{1}{2\cdot 500\times10^{-9}\xi_0^2} = \frac{10^6}{\xi_0^2}$$

$(\xi_0 \text{ in } m^{-1})$

5.) (Continued)

d.) (5 pts.) For $\xi_0 = 20000\ \mathrm{m}^{-1}$, sketch an irradiance profile along the x axis in the back focal plane of the lens, indicating the locations and approximate separations of the diffraction orders. You may assume that the diffraction orders are separated far enough so as not to interfere with each other. You may normalize irradiance.



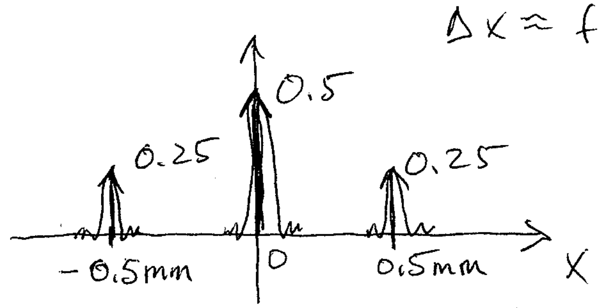
$f = 50\,\mathrm{mm}$

$\Delta x \approx f\theta$ (small θ)

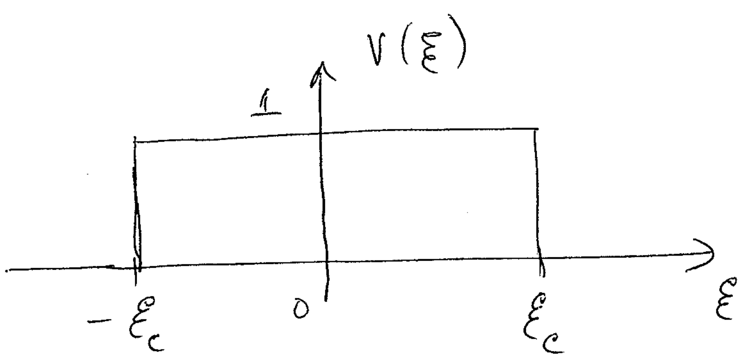
$\theta \approx \lambda \xi_0$

$\Delta x \approx f\lambda\xi_0 = (50\times10^{-3})(500\times10^{-9})(2\times10^{4})$

$= 0.5\,\mathrm{mm}$



e.) (5 pts.) Plot the contrast (visibility) of the grating image as a function of ξ_0.



Δx cutoff at $\Delta x = 1\,\mathrm{mm} \Rightarrow \boxed{\xi_c = 40{,}000\ \mathrm{m}^{-1}}$

f.) (5 pts.) What is the cutoff spatial frequency ξ_C, where no contrast is observed in the image? What is the period?

$T = \frac{1}{\xi_c} = \frac{1}{40\times10^{+3}} = 0.025\times10^{-3}$

$= \boxed{25\,\mu m}$