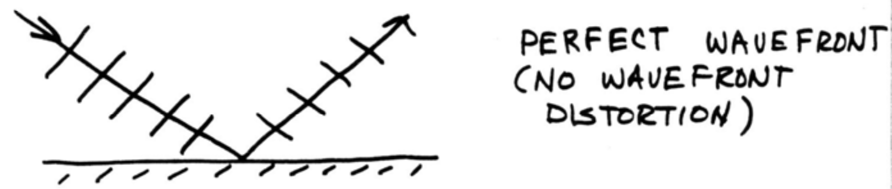
SPECKLE

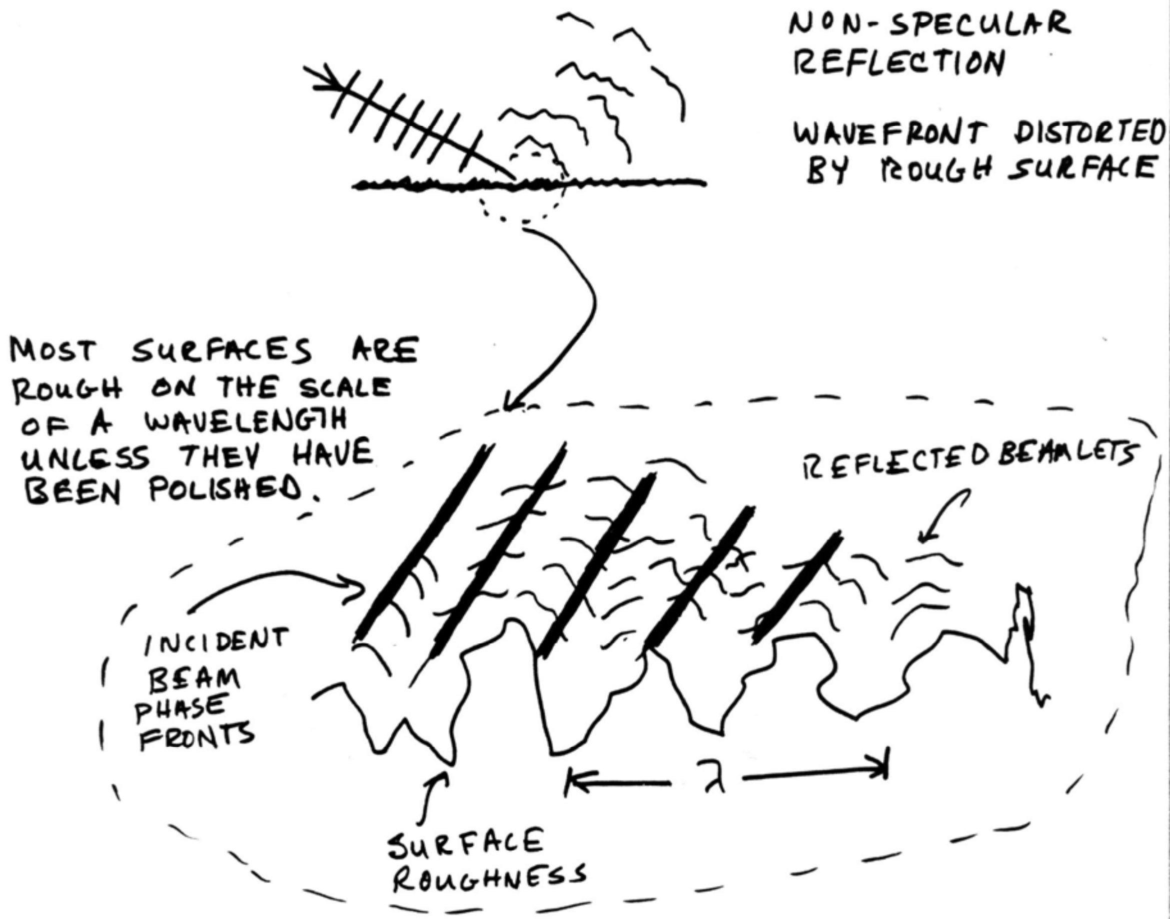
Reference: Hariharan, Optical Interferometry, Ch 14.

Laser beam reflection from surfaces:

Smooth surface:



Rough surface:



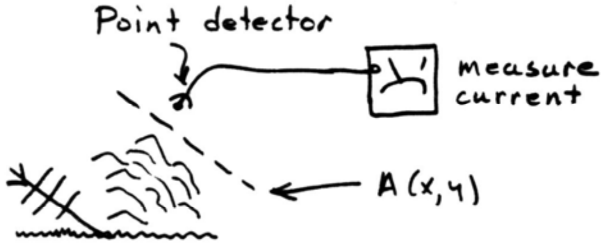
REFLECTED BEAMLETS ACT LIKE THEY COME FROM A COLLECTION OF TINY MIRRORS ON THE SURFACE.

We assume that the reflected wavefront is the sum of the complex amplitudes of the diffracted waves from the microscopic mirrors, that is

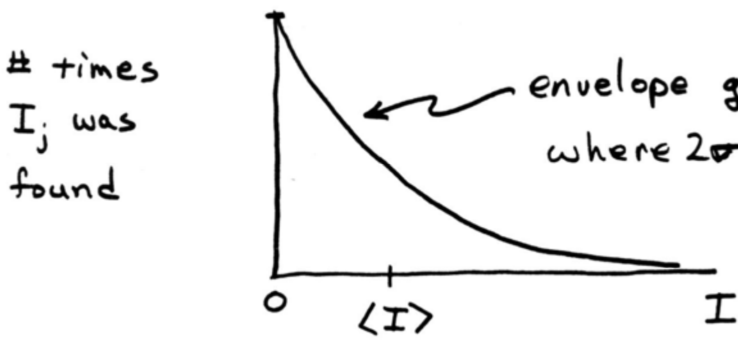
$$A(x,y) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} |A_m| e^{-j\phi_m} \quad , \; N \text{ elemets (mirrors)}.$$

If we assume that the amplitude and phase of the components are statistically independent and the range of phase shifts is equally likely between $\phi_m = 0$ to 2π, the real and imaginary parts of the complex amplitude obey gaussian statistics.

Gaussian statistics ⇒ The average amplitude of $A(x,y)$ is zero. Phase has uniform circular distribution.



If we take a point detector and randomly sample the irradiance in plane $A(x,y)$, we can plot a histogram of the result and get



envelope goes as $e^{-I/2\sigma^2}$, where $2\sigma^2 = \langle I \rangle$ = average irradiance.

How big are the speckles?

can look at autocorrelation of the intensity in the speckle pattern. The result is

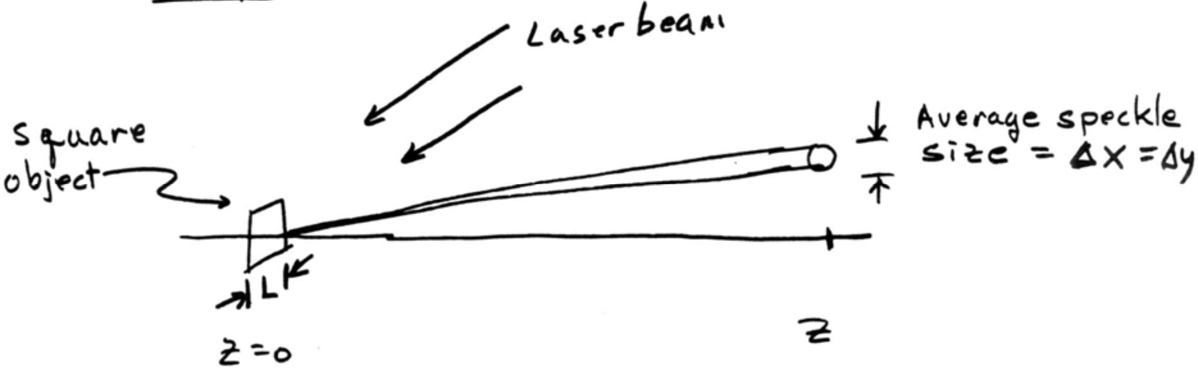
$$R_I(x_2,y_2) = \langle I \rangle^2 \left\{ 1 + \frac{\iint_{-\infty}^{\infty} |A(x_1,y_1)|^2 e^{j\frac{k}{z}(x_1x_2+y_1y_2)} dx_1 dy_1}{\iint_{-\infty}^{\infty} |A(x_1,y_1)|^2 dx_1 dy_1} \right\}$$

where (x_1, y_1) are the coordinates of the plane just after the rough surface and (x_2, y_2) are the coordinates of the plane of observation. Notice that this is just a normalized Fourier transform of the size of the illumated surface.

$$R_I(x_2,y_2) = \langle I \rangle^2 \left\{ 1 + \frac{\mathcal{F}\{|A(x_1,y_1)|^2\}}{\mathcal{F}\{|A(x_1,y_1)|^2\}_{x_2=0,\ y_2=0}} \right\} \quad \xi = \frac{x_2}{\lambda z} \quad \eta = \frac{y_2}{\lambda z}$$

So we can find the average size of the speckles by Fourier transforming the illuminated object.

Example:

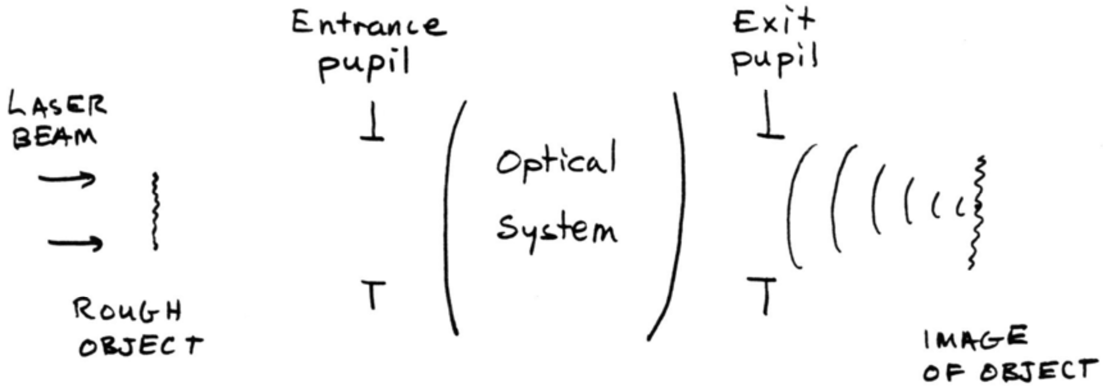


$$R_I(x_2,y_2) = \langle I \rangle^2 \left\{ 1 + \operatorname{sinc}\left(\frac{Lx_2}{\lambda z}\right) \operatorname{sinc}\left(\frac{Ly_2}{\lambda z}\right) \right\}$$

$\Delta x \approx$ distance to first zero in autocorrelation

$$\approx \boxed{\frac{\lambda z}{L}}$$

Speckle in optical systems:



The rough object illuminated by the laser beam forms speckle in the entrance pupil.

The exit pupil is the image of the entrance pupil, so the speckle pattern is seen in the exit pupil.

The speckle pattern in the exit pupil is random amplitude and phase perturbations of the electric field. This is the same as the pattern in the plane just after the object. Therefore, we can treat the field in the exit pupil as originating from an object of the same size.

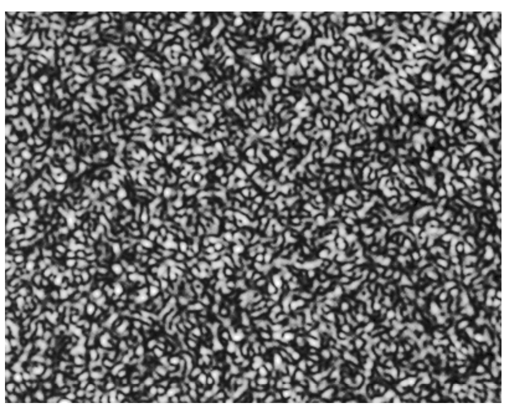
For a circular exit pupil of radius ρ, the average size of the speckles in the image is

$$\Delta x = \Delta y = \boxed{0.61 \frac{\lambda f}{\rho}}$$

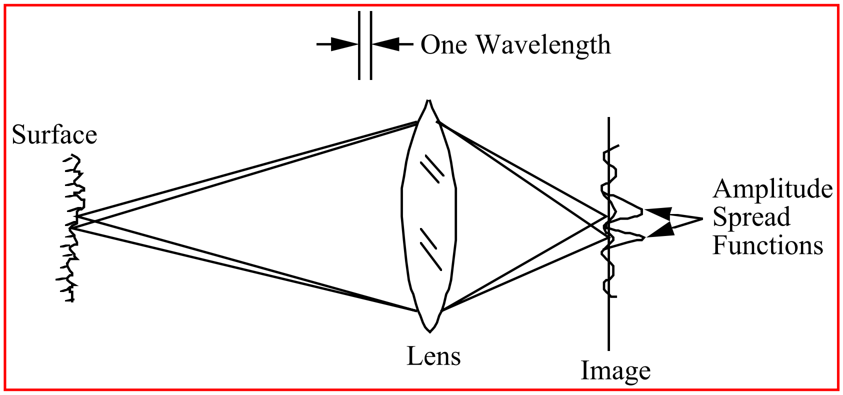
Chapter 18
Speckle Interferometry

- Basic Phenomena
- Applications
  - –Out-of-Plane Surface Vibration
  - –In-Plane Displacement
  - –In-Plane Vibration
  - –Stellar Speckle Interferometry
- Electronic Speckle Pattern Interferometry

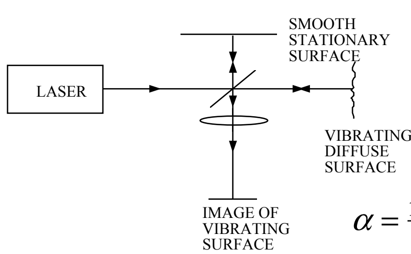
Speckle Pattern Produced by Illuminating a Rough Surface with Laser Radiation



Physical Origin of Speckle for an Imaging System



Experimental Setup for Measuring Out-of-Plane Surface Vibration



$$\alpha = \frac{\text{reference beam}}{\text{object beam}}$$

Surface Height $z = z_o + D\sin\omega t$

Speckle Contrast is $C = \frac{\left[1 + 2\alpha J_o^2(4\pi D/\lambda)\right]^{1/2}}{1+\alpha}$

Speckle Contrast Reduction Due to Out-of-Plane Vibration

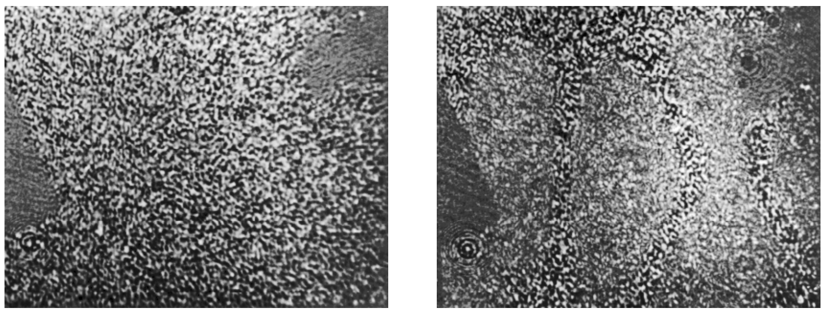
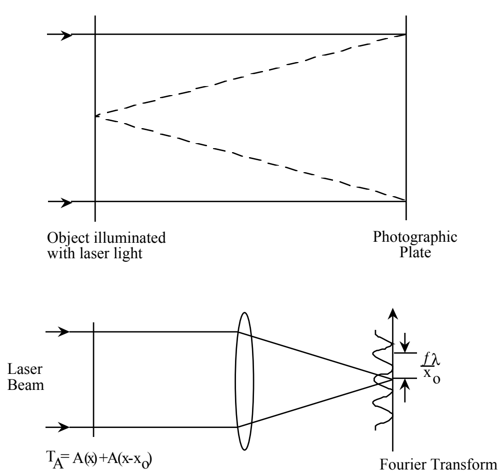


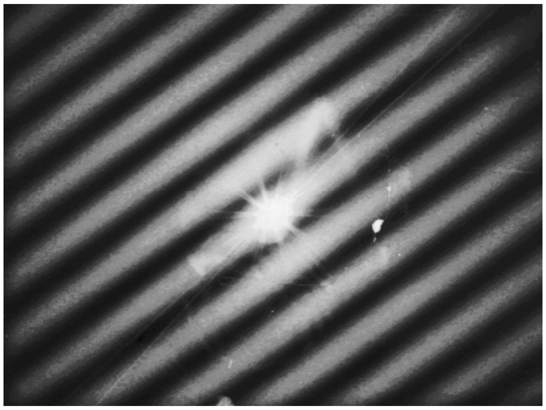
Plate Stationary **Plate Vibrating**

In-Plane Displacement



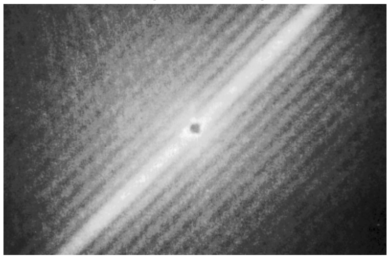
Observing Young's Fringes

Young's Fringes Resulting from In-Plane Displacement



In-Plane Vibration

- Speckle drawn into lines as surface vibrates
- Diffraction pattern gives vibration information



Linear Motion

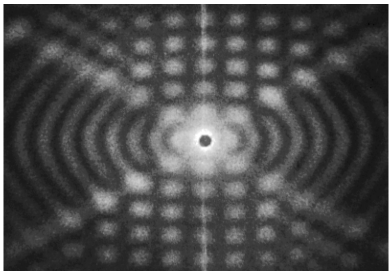


Figure-of-Eight Motion

Stellar Speckle Interferometry

- Atmosphere limits resolution to approximately 1 arc second (10 cm aperture)
- Image of star shows speckles if
  - Exposure time less than period of atmospheric turbulence (1 msec)
  - Spectral bandwidth small (10 nm) so coherence length long
- Speckle size determined by wavelength and telescope diameter (Diffraction-limited resolution)
- Speckles information limited by resolution limit of telescope, not atmospheric turbulence

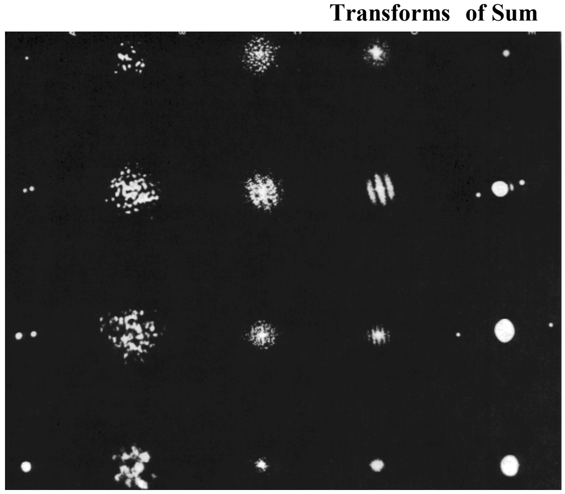
Short Exposure, Narrow Bandwidth, Photograph of Unresolved Star

Stellar Speckle Interferometry Procedure

- Take large number, short exposure, photos of object, where each photo is taken for different realization of atmosphere
- Take Fourier transform of each photo (obtain diffraction pattern)
- Add square modulus of diffraction pattern of all photos
- Take Fourier transform of ensemble average of diffraction patterns
- Result is autocorrelation of diffraction-limited image of object

Stellar Speckle Interferometry Results

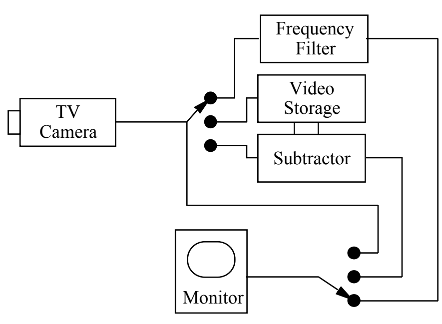
object	Photo	Fourier Transform	Sum of 20 Fourier Transforms	Fourier Transform of Sum



Electronic Speckle Pattern Interferometry (ESPI)

- Use TV system to record speckle instead of film
- Gives real-time measurements
- Minimum speckle size limited by camera resolution
- Can perform computer analysis of speckle data

Block Diagram of Electronic Sequence of ESPI System



Examples of Time-Averaged Vibration Mode Viewing with ESPI