LAB 2: REFRACTIVE INDEX AND SNELL'S LAW

Measuring the refractive index of a material is one of the most fundamental optical measurements, and one of the most practical. In this lab you will use several different methods for determining index.

THEORY

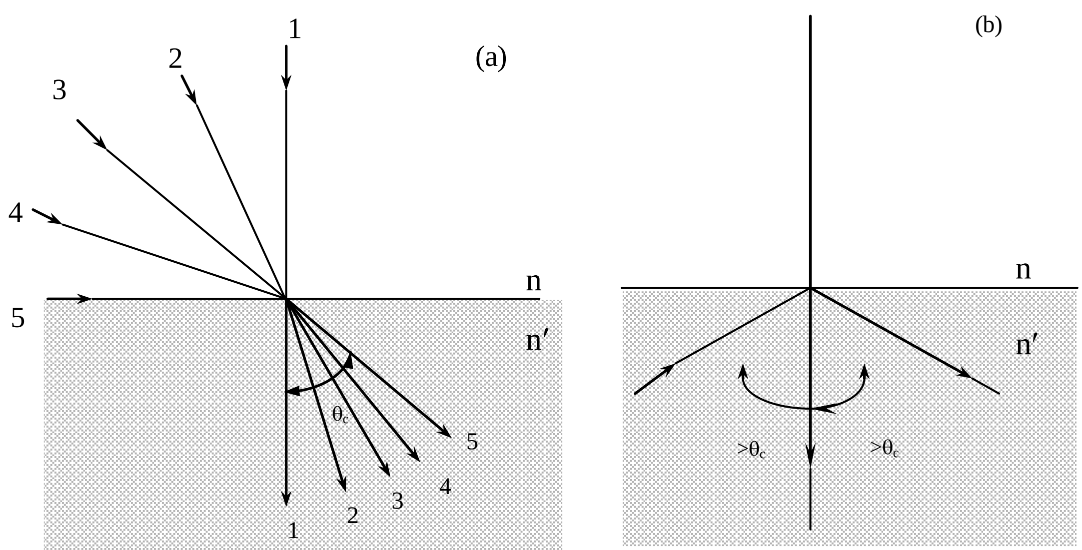
All of these various methods for measuring index rely on two basic concepts: Snell's Law and the Critical Angle. A brief review of these concepts is worthwhile.

Index of refraction, or refractive index, is defined as the ratio of the speed of light in vacuum to the speed of light in the medium: $n = c/v$. A typical value of n for optical glass is 1.52, indicating that light travels 34% slower in glass than it does in vacuum. For purposes of this lab, we will consider air to have an index of 1.000, the same as a vacuum.

Snell's Law describes how a ray of light is bent, or refracted, when crossing a boundary between materials of different indices of refraction. Snell's Law is given as:

$$n \cdot \sin(\theta) = n' \cdot \sin(\theta') \qquad (2.1)$$

where n and n′ are the indices of refraction, and θ and θ′ are the angles that the ray makes with respect to the surface normal. The following diagram shows rays being refracted:



(a) Refraction up to the critical angle. (b) Total Internal Reflection beyond the critical angle.

Figure 2.1. Refraction and Total Internal Reflection.

A ray passing from a material of low index to one of higher index will be refracted across the boundary over all angles of incidence. However, when travelling in the other direction, from high index to low index, an angle is reached in the material of high index for which the refraction angle on the low index side is 90 degrees. This is known as the Critical Angle and is found by setting the refraction angle on the low index side in Snell's Law to 90 degrees:

$$\theta' = \arcsin(n/n') \tag{2.2}$$

For angles of incidence on the high index side greater than the critical angle, the ray is not refracted into the low index side, but is 100% reflected back into the high index material. This is known as Total Internal Reflection, abbreviated as TIR (see Fig. 2.1). We will see an example of TIR in the experiment using the Pfund technique.

PFUND'S METHOD

Although this technique is only accurate to about 0.5%, it is inexpensive, fast, and can measure liquids. The output of a HeNe laser is normally incident on a plane parallel plate. Paint on the back of the plate scatters the light diffusely. Some of the scattered light is totally internally reflected (TIR) at the top surface forming a ring ("doughnut") of light on the back. The diameter of the dark inner circle (D) is related to the thickness of the plate and the refractive index (see Fig. 2.2) by:

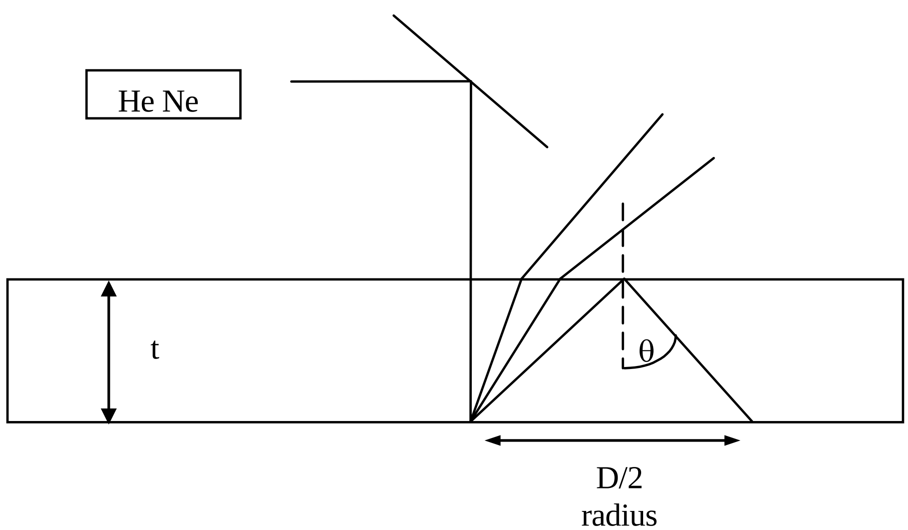


Figure 2.2. Experimental setup for the Pfund Method.

$$n_g = \frac{\sqrt{D^2 + 16t^2}}{D} \tag{2.3}$$

DETAILED EXPLANATION

The beam enters normal to the plate and strikes the rear painted surface. This is the bright central dot that you see. The painted surface acts as a diffuse reflector, scattering rays upwards at all angles within the hemisphere. Over the range $0° \leq \theta < \theta_c$, these rays refract out of the glass/air interface, and leave the plate. Because they travel outward over a wide range of angles, your eye receives only a small percentage of them, causing this region of the plate to appear dark. Now, over the range $\theta_c \leq \theta < 90°$, all of these rays experience TIR at the glass/air interface. They are reflected back to points on the rear surface of the glass beyond the dark circular boundary where they scatter diffusely. Because these rays have undergone TIR, their intensity is large and the scattered rays coming out of the plate from this region are bright enough to see (just like the central spot).

Start with the thin plate. Measure the plate thickness (t) using calipers. (This is most accurately done at the corner of the plate having some of the paint removed. Shine the laser beam onto the plate and observe the dark inner circle bordered by a bright red ring of light. To help in understanding where all of the rays of scattered light go, place a white piece of paper with a small hole in it on top of the plate. Observe the distribution of light on the paper. Move the paper a small distance above the plate and observe.

Measure the diameter of the dark inner circle (D) with the calipers. Some error will be introduced due to parallax, but center the calipers on the circle as best you can. Take a number of readings and average for better accuracy. Calculate the refractive index of the plate. Repeat for the thick plate.

Use the thin plate with the plastic ring glued onto it, for the following part. Put a THIN layer of water in the well, shine the laser beam onto the plate, and notice the two rings. SLOWLY add water and observe that the inner ring grows in diameter, eventually becoming larger than the other stationary ring. (If the "moving" ring starts out larger in diameter than the stationary ring, you started with too much water. Empty the well and try again.) Measure the diameter of the appropriate (!) circle, and calculate the index of water using the formula:

$$n_s = \frac{n_g D}{\sqrt{D^2 + 16t^2}} \tag{2.4}$$

where n_s = Index of the liquid sample (the water)

n_g = Index of the glass plate

* Show with a ray diagram why the thinner plate produced a smaller diameter circle than did the thicker plate (for the first part of the experiment using no water). Use side-by-side drawings similar to Fig. 2.2 showing two different plate thicknesses.

* Repeat with a series of drawings (3 or 4) showing how varying the thickness of the water layer caused one ring to move past the other one. Show the stationary ring also.

* When measuring the index of water you saw two rings. Which one did you use for your calculation? Why?

* What happens to the light that is not totally internally reflected? Why do you not see it? How could (or did) you prove your answer?

MICROSCOPE METHOD

An object at the bottom of a pool of water appears closer than it really is, an effect caused by refraction at the water/air boundary Fig. 2.3. Place a pencil or stick in a cup of water and observe the effect.

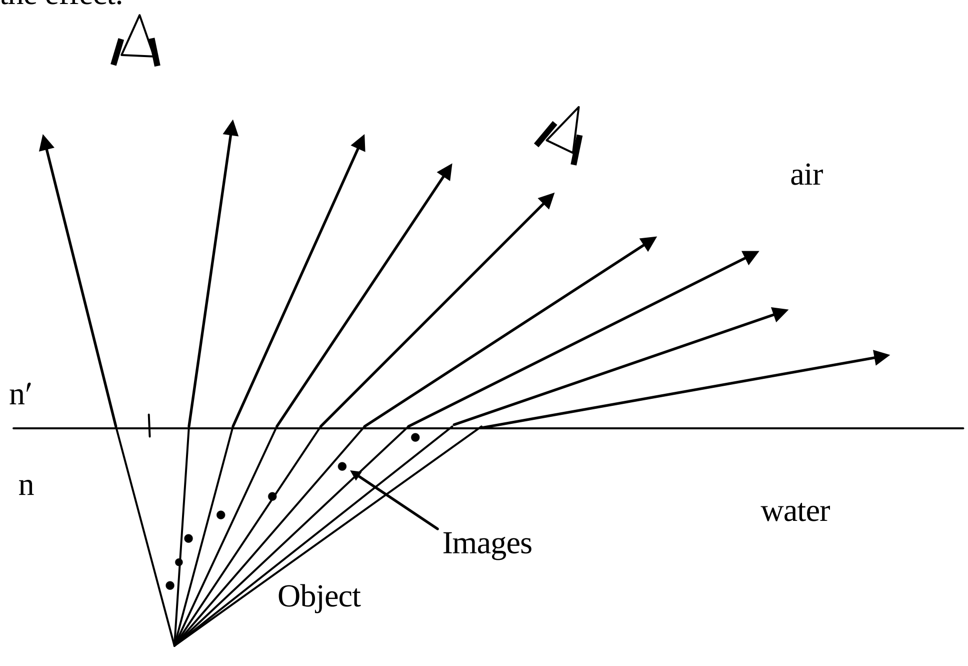


Figure 2.3. Apparent image positions of an object under water.

The microscope method may be more clearly understood by considering the refraction of a single ray out of a glass plate:

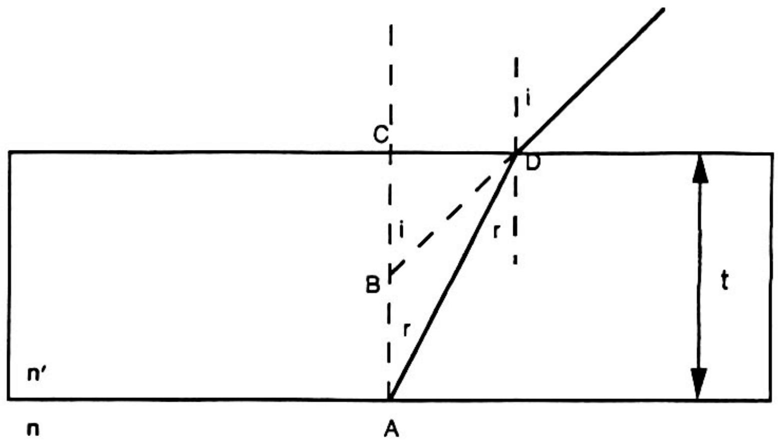


Figure 2.4. Refraction in a single glass plate.

$$n' = \frac{\sin i}{\sin r} \cong \frac{\tan i}{\tan r} \tag{2.5}$$

$$\tan \mathrm{i} = \frac{\mathrm{CD}}{\mathrm{CB}} \quad \tan \mathrm{r} = \frac{\mathrm{CD}}{\mathrm{CA}} \tag{2.6}$$

Thus,

$$\mathrm{n}' = \frac{\mathrm{t}}{\mathrm{CB}} \tag{2.7}$$

Focus a microscope on a scratch or some dust on the front of the plate. This is made easier if you illuminate the front of the plate with a penlight while attempting to focus. When the surface is in focus, note the micrometer reading. Now focus the scope on the back surface, illuminating the back surface with the penlight. Use the new micrometer reading to determine how far the scope traveled (CB). Measure the plate's thickness with the micrometer, and calculate the refractive index.

The major source of error in this experiment is the approximation that the sines of angles r and i are equal to their tangents. This is a good approximation, however, for the cone of rays accepted by a typical microscope objective.

* What is the distance CB also called?

* In terms of the microscope's focal distance, what is the thickest plate which can be measured by this method? Assume that the focal distance of a microscope is equal to the distance between the objective and the object. The objective is at the end of the microscope closest to the object.

* If you are lying at the bottom of a swimming pool looking straight up, what is your maximum full Field-of-View (FOV) in the air above the water? In other words, what angular range of objects in the air above can you see? What is the same FOV in the water? Assume the refractive index of water is 1.3.

DEVIATION BY A PLANE PARALLEL PLATE

The deviation of a ray by a plane parallel plate is shown in Fig. 2.5. Set up a simple experiment to measure the deviation of a laser beam, and calculate the refractive index from:

$$n_g = \left[\sin^2 i + \frac{\cos^2 i}{\left(1 - \frac{d}{t \sin i}\right)^2} \right]^{1/2} \qquad (2.8)$$

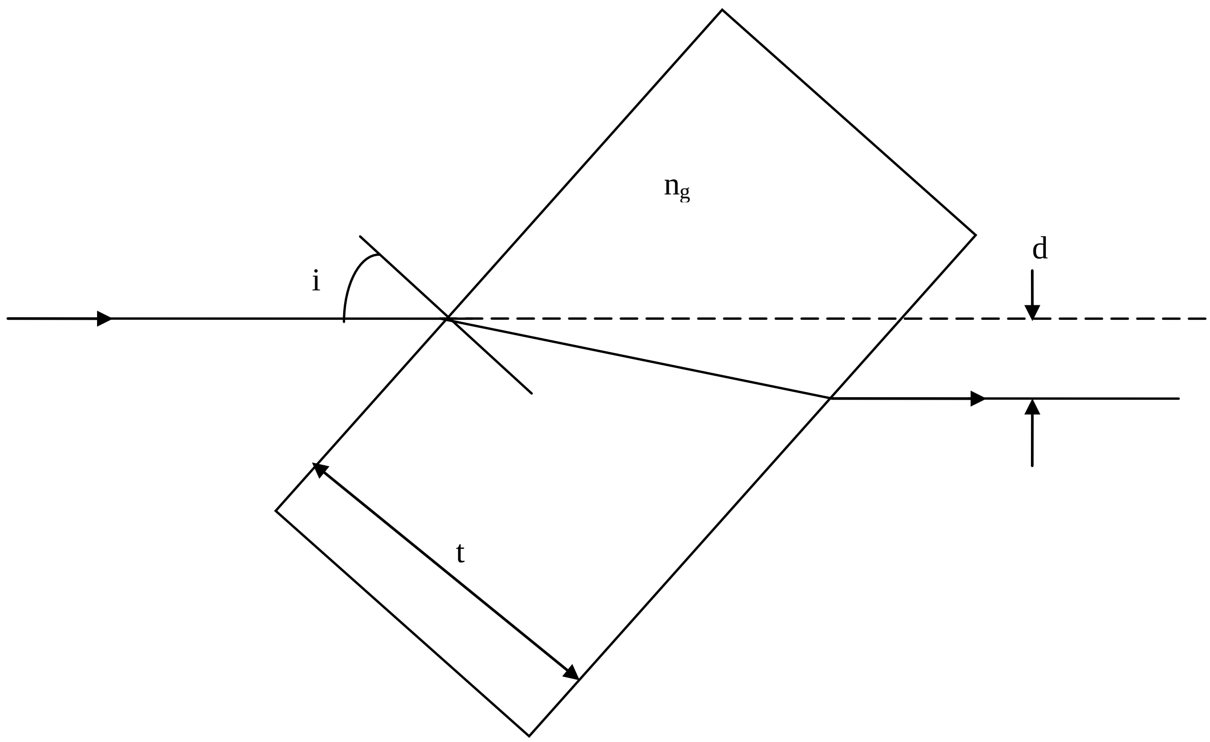


Figure 2.5. Deviation by a plane parallel plate.

∗ Is TIR a potential problem using the set up of Fig. 2.5? Why or why not?