LAB 9: THICK LENSES I

The concepts of thin and thick lenses are often confusing to the student of optics. The words inherently seem to imply something about the actual thickness of a lens, automatically placing it into one of two categories. The temptation is to consider lenses that look "thin" to be thin, and lenses that appear "thick" to be thick lenses. In fact, either term may be applied to a given lens. A more general distinction is needed.

In order to discuss thin or thick lenses, we must understand the concept of principal planes. First consider a single refracting spherical surface, shown in Fig. 9.1. In the paraxial limit, we may replace the curved surface with a plane surface having the same refracting properties. The location of this plane is *defined* to be at the intersection of the rays in image and object space, and is known as the **principal plane**. This plane is found to be tangent to the vertex of the curved surface and normal to the optical axis. Any ray striking the principal plane will be refracted *at this plane* according to the equations describing paraxial imaging.

Now consider two refracting surfaces spaced a distance apart--a lens, in other words, as shown in Fig. 9.2. Each surface has a principal plane associated with it. The location of each of these planes may be found from the definition of a principal plane.

For the first surface, consider rays that leave the **front focal point**, **F**. By definition in the paraxial limit, these rays leave the second surface parallel to each other. The principal plane of the first surface is found where the diverging rays in object space appear to intersect the parallel rays in image space. The refraction that occurs at the first surface may now be thought of as occurring at the front principal plane. In effect, we have replaced the first curved surface with an equivalent refracting plane surface. The point of intersection of this principal plane with the optical axis is called the **front principal point**, **P**.

For the second surface, consider parallel rays that strike the first surface. By definition in the paraxial limit, these rays leave the second surface and converge to the **rear focal point**, **F***. As before, the principal plane of the second surface is located where the parallel rays in object space appear to intersect the converging rays in image space. The point of intersection of this rear principal plane with the optical axis is called the **rear principal point, P***.

Note that in the general case, P and P* may be located anywhere with respect to the vertices of the lens. This is a function of how the lens is shaped, or "bent", as shown in Fig. 9.3. For a biconvex or biconcave lens, P and P* are located symmetrically with respect to the vertices of the lens.

The principal planes are the paraxial refracting surfaces equivalent to the actual curved surfaces of a lens. Through the use of principal planes, one can more easily trace a ray from an object to the front principal plane, and then trace a ray from the rear principal plane to the image. The question yet to be answered is, how does one trace a ray between the principal planes themselves? Figure 9.4 provides the answer. This figure shows a lens with its associated principal planes P and P*. Two rays are shown--one parallel in object space, and the other parallel in image space (each one is refracted at the appropriate principal plane). From the geometry shown, it is apparent that the height of a ray is the same at the front and rear principal planes. In other words, **the principal planes are conjugate planes of positive unit magnification.** Once the ray height at the front principal plane is found, the transfer to the rear principal plane is made with a positive magnification of $M_t = 1$.

In addition to principal points, a lens has associated with it two points of unit angular magnification, called **nodal points**. A ray that enters a lens at angle θ with the front nodal point N, leaves the lens at the same angle θ with the rear nodal point N*. The six optical points of interest of a thick lens, namely the front and rear *focal points*, *principal points*, and *nodal points* are called the **cardinal points** of the lens. Figure 9.5 shows a lens, with the cardinal points labeled.

For convenience, the cardinal points are referenced and measured from the vertices of the lens. The vertices are convenient in that they are physically accessible when making measurements. The equations giving the distances between the principal planes and their respective vertices are:

$$\delta \equiv \overline{V_1P} = +n\left(\frac{t}{n'}\right)\cdot\frac{\phi_2}{\phi}$$

$$\delta^* \equiv \overline{V_2P^*} = -n''\left(\frac{t}{n'}\right)\cdot\frac{\phi_1}{\phi} \tag{9.1}$$

This completes the paraxial description of a lens. If the actual thickness of the lens is not neglected, we treat the lens as being a thick lens. In the paraxial limit, this thick lens may be replaced with two principal planes separated from the vertices and each other by some calculated amount. If the thickness of the lens is neglected, t = 0 in the above equations, and the lens may be replaced by two principal planes that coincide at the center of the lens.

Finally, a system of lenses is treated the same as a thick lens. The intersections of object and image space rays define the location of the *system* principal planes. Equations used to locate the principal planes of a single lens are also used for a system of lenses. The result is that the entire system of lenses may be replaced with just two equivalent refracting plane surfaces.

The difficulty in measuring the power of thick lenses is that the locations of the principal planes are unknown with respect to the physical landmarks of the lens (i.e., the vertices). This lab looks at two different methods for locating the cardinal points of thick lenses.

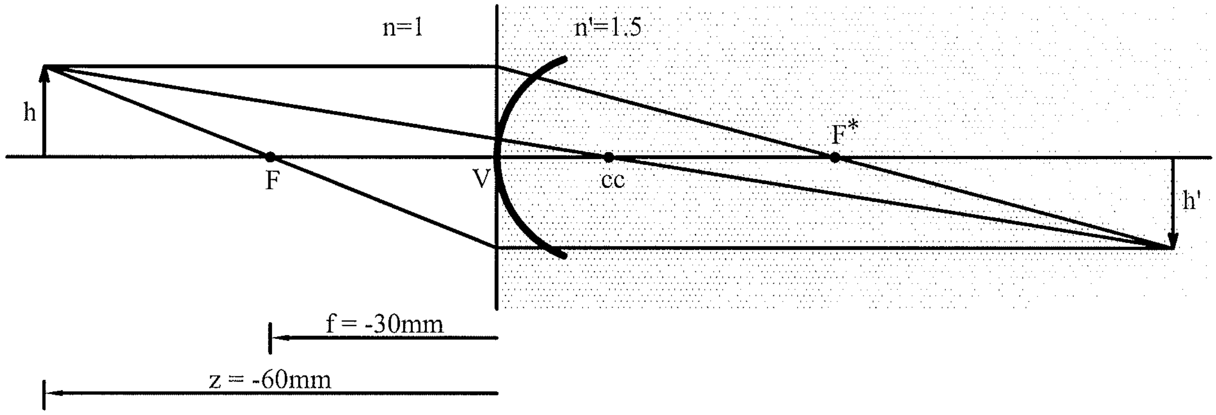


Figure 9.1. Spherical refracting surface.

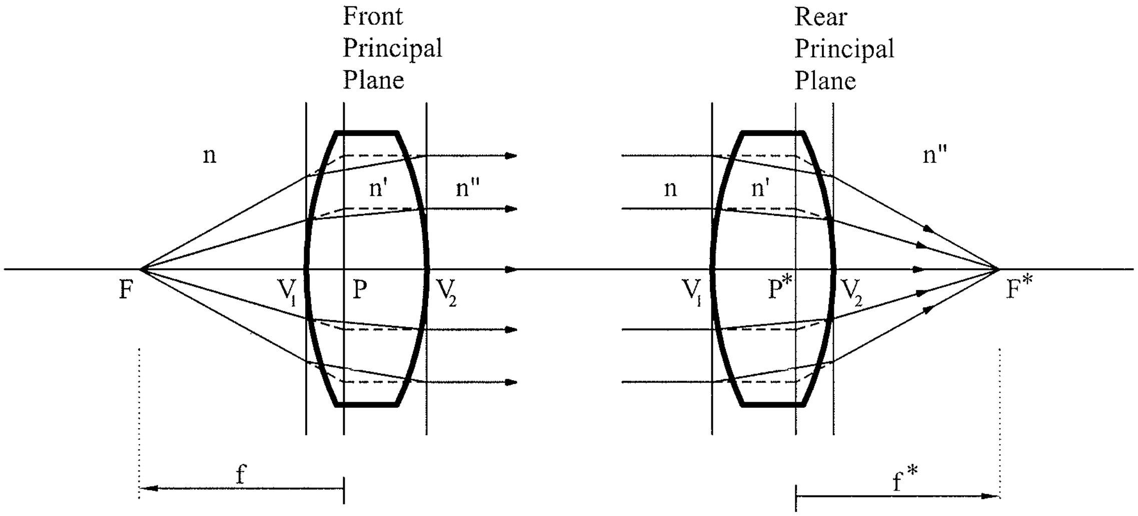
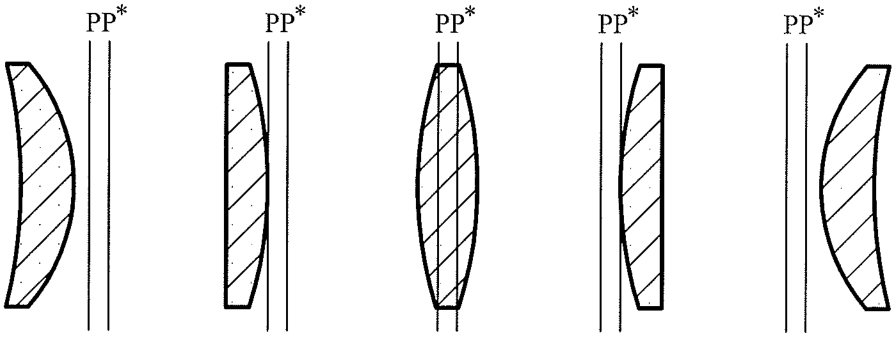


Figure 9.2. Principal planes--location.



"Bending" a lens.

Figure 9.3. Principal planes--shape factor.

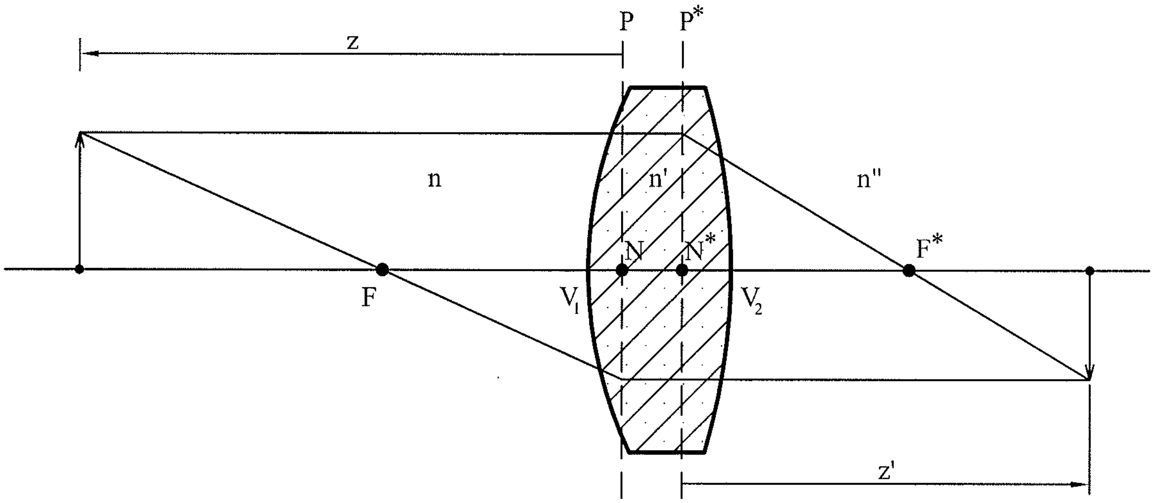


Figure 9.4. Principal planes--transverse magnification.

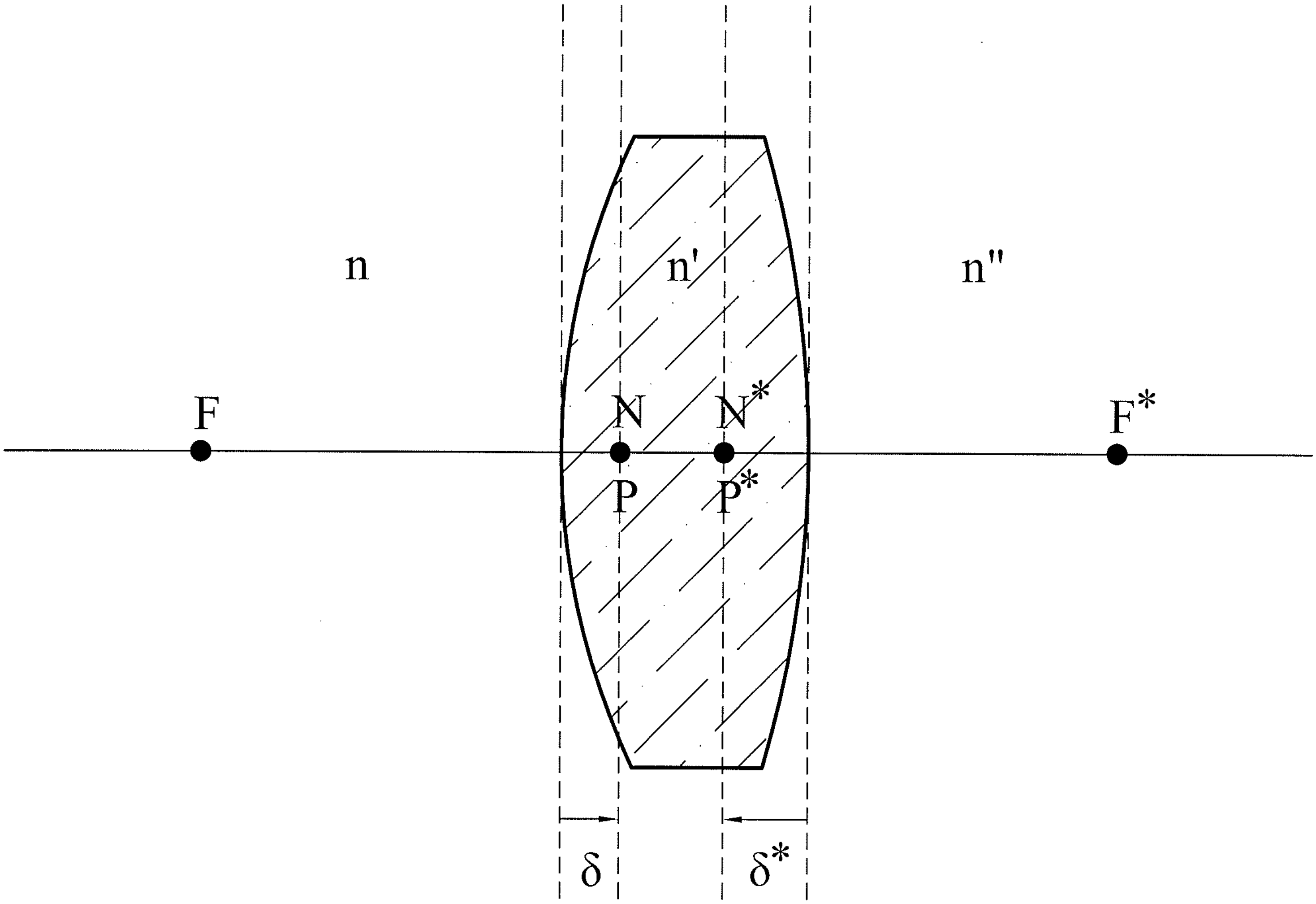


Figure 9.5. Cardinal points of a thick lens.

EXPERIMENT 9-2: THE NODAL SLIDE

The nodal slide is an instrument used for locating and measuring the cardinal points of a lens or a system of lenses. It allows F, F*, P, P*, N, and N* to be located with respect to the vertices of the lens or lens system. This is important, as the vertices are the only physical landmarks of a lens accessible for alignment and measurement. The cardinal points themselves are usually points in space or in the interior of a lens that are difficult or impossible to align to. Refer back to Fig. 9.5 which shows a lens and the nomenclature used for this lab.

If the indices of object and image space are identical, the nodal points and principal points of a lens coincide (P and N are coincident, and P* and N* are coincident). Assume that a lens or system of lenses is illuminated with a collimated beam of light. **When the lens or system of lenses is rotated about the secondary nodal point N*, the image appears stationary.** *Even though the lens rotates, the image does not.* At this point, N* and P* are known to be located over the axis of rotation. Measurement of the distance between the rear vertex and the image gives the back focal distance (BFD). Next, the lens is translated so the rear vertex V_2 is over the axis of rotation. The distance the lens is translated is a direct measure of δ^*. At this point, the positions of P*, N*, and F* are known with respect to the rear vertex. The lens is turned around end-for-end and the process repeated to find the other three cardinal points: P, N, and F.

A drawing of the nodal slide used in this lab appears in Fig. 9.7. A rotation stage slides along the optical rail on a carrier that may be locked down at any position. The rotation may also be fixed with a locking screw, located on the side of the stage. Note that the center of rotation is coincident with the marker line on the carrier. On top of the rotation stage is fixed a precision slide stage, with a lens holder on the movable part of the stage. The position of the lens on the slide is read with a vernier scale, having a resolution of 1/20 mm, or 50 microns.

This arrangement of stages allows any part of the lens to be positioned over the center of rotation of the rotary base. It is this feature of the nodal slide that allows the cardinal points of a lens to be measured, in particular the nodal points themselves, N and N*. Note that the vernier scale is not referenced to any particular point. The readings taken will be referenced to the vertices and cardinal points of the lens under test.

The detailed use of the nodal slide is now outlined. Follow the steps given to determine the cardinal points of the lenses provided.

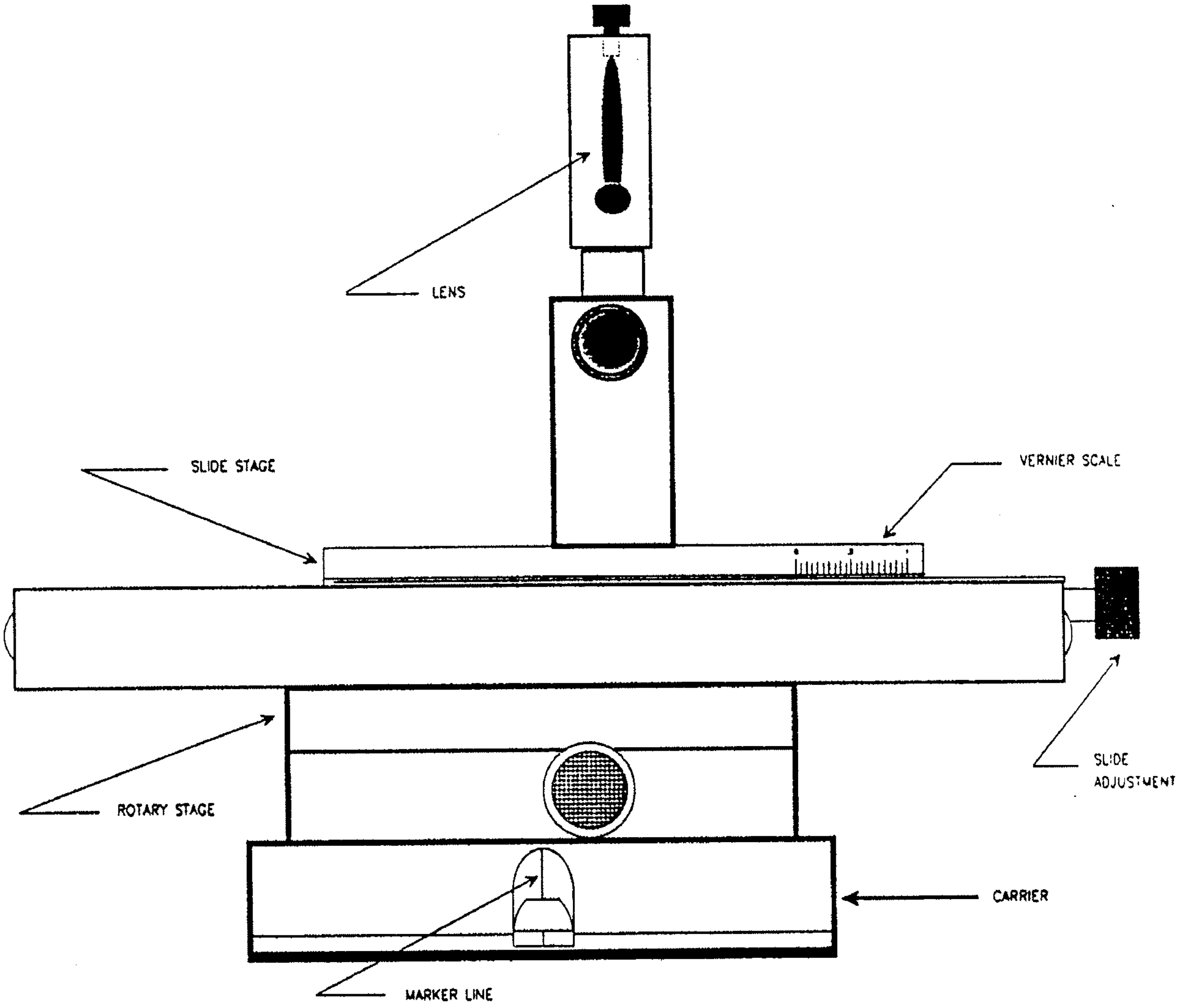
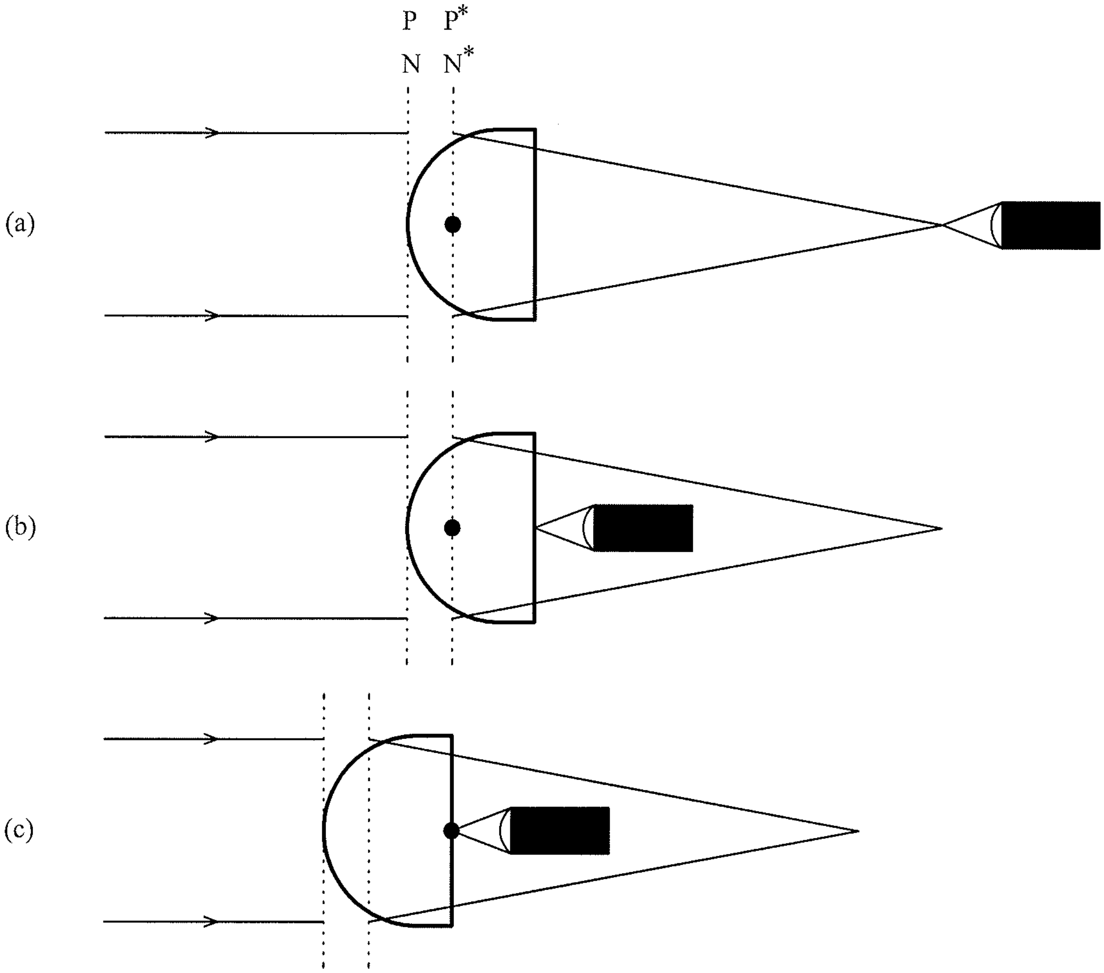


Figure 9.6. Nodal slide--mechanical layout.



● Locates axis of rotation of the Nodal Slide

Figure 9.7. Use of the nodal slide.

(1) Form an Image with the Lens

Start with knob of nodal slide towards microscope. Mount a lens in the holder on top of the nodal slide. Position the light source so the diffuse pinhole aperture is on the optical axis of the lens. Lock down the position of the source. Position the microscope at the opposite end of the rail and lock it down on one of the rail markings. Form an image of the source in the microscope by moving the entire nodal slide along the optical rail. Adjust the height of the microscope until the image is in the center of the cross hairs. Lock down the position of the nodal slide along the rail. Record the location of the microscope.

(2) Position P* and N* Over the Axis of Rotation

While looking through the microscope at the focused image, rotate the nodal slide 10-20 degrees in either direction. Notice that at this point the image will also rotate. Move the slide stage in one direction, refocus the image by sliding the entire nodal slide along the optical rail, and again rotate the nodal slide. If the image rotation appears less, repeat this process until the image appears stationary with rotation of the nodal slide. If the image rotation appeared greater after the first attempt, move the slide in the other direction and repeat the process until the image appears stationary. At this point, P* and N* are over the axis of rotation of the nodal slide. Figure 9.8 (a) shows the situation. Record the reading of the vernier scale on the nodal slide.

(3) Measure the Back Focal Distance (BFD)

Move the microscope forward until it is focused on the rear vertex of the lens (closest to the image). A small amount of dust on the lens surface makes this easy to do. Record the location of the microscope. Calculate the **back focal distance (BFD)** as the difference in locations of the microscope in steps (1) and (3).

(4) Position the Lens Vertex over the Axis of Rotation

Rotate the nodal slide 10-20 degrees. Observe the image of the lens surface. Move the slide stage a small distance, refocus the microscope on the vertex, and rotate the nodal slide. Observe the image of the lens surface. If the image rotates, continue this process until the image appears stationary. At this point the lens vertex is over the axis of rotation of the nodal slide. Figure 9.8 (c) shows the situation. Record the reading of the vernier scale on the nodal slide. Calculate the distance δ* as the difference in vernier scale readings (4)-(2):

$$\delta^* = (4)-(2)$$

ROTATE THE NODAL SLIDE 180° TO MEASURE THE OTHER SIDE OF THE LENS.

DO NOT ROTATE JUST THE LENS OR ITS HOLDER

(5) Move the microscope back to the opposite end of the rail from the source and record the location of the microscope. Refocus the image of the light source by sliding the entire nodal slide.

(6) Position P and N over the Axis of Rotation

Repeat the procedure in step (2) above to position P and N over the axis of rotation of the nodal slide. Record the reading of the vernier scale on the nodal slide. Calculate $\overline{PP^*}$ as the absolute value of the difference in readings in steps (2) and (6):

$$\overline{PP^*} = |(2) - (6)|$$

Record the location of the microscope.

(7) Measure the Front Focal Distance (FFD)

Move the microscope until it is focused on the back surface of the lens (actually the front vertex now that the lens has been rotated 180°). Record the location of the microscope. Calculate the front focal distance (FFD) as the difference in locations of the microscope in steps (5) and (6).

(8) Position the Lens Vertex over the Axis of Rotation

Repeat the procedure in step (4) above to position the lens vertex over the axis of rotation of the nodal slide. Record the reading of the vernier scale on the nodal slide. Calculate δ as the difference in readings in steps (6) and (8):

$$\delta = (6) - (8)$$

(9) In addition, you also have enough data to calculate the thickness of the lens. Calculate t, the lens thickness, as the absolute difference in readings of the nodal slide vernier scale in steps (4) and (8):

$$t = |(\text{Location of } V_2 \text{ in step 4}) - (\text{Location of } V_1 \text{ in step 8})|$$

CALCULATE AND REPORT THE LOCATION OF THE CARDINAL POINTS IN RELATION TO THE VERTICES OF THE LENS.