

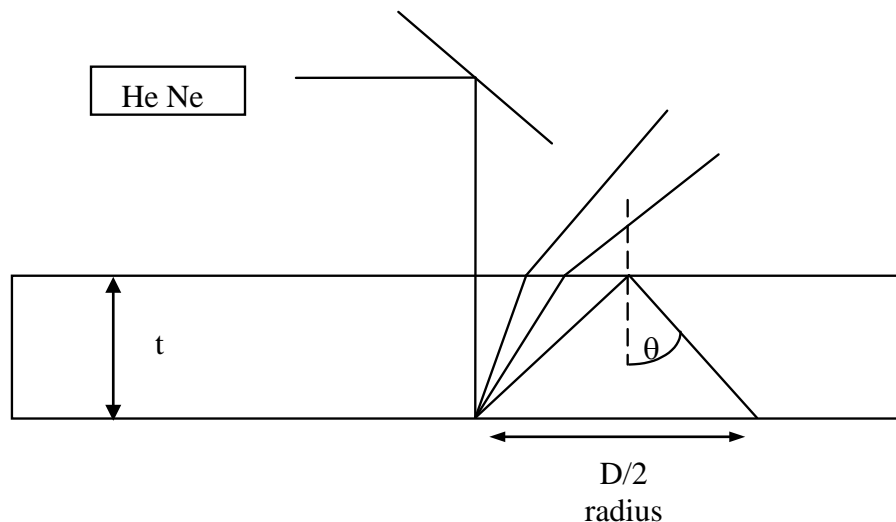
Computer Exercise—Simulation of the Pfund Experiment
DUE Monday 9/26/11 at the start of our Lab Lecture

The purpose of this exercise is to use a spreadsheet (Excel) to trace rays of light. We will use the Pfund experiment as a simple example of how to trace rays according to the Law of Reflection, taking Total Internal Reflection (TIR) into account. When finished, your spreadsheet will model the same experiment you did in lab, creating a plot of the dark circle/light area just as you saw using a real glass plate. The effects of plate thickness and refractive index are included in the model.

This homework assumes that you are familiar with the basic use of Excel, but as a quick review: A spreadsheet is a computer program that presents you with a blank workspace, divided up into cells. In turn, these cells are organized as a “matrix” of rows and columns. You can input either text or numbers in any of the cells, using standard “copy, cut, and paste” editing functions. The spreadsheet program has many built-in mathematical and Boolean (logical) operations, allowing you to manipulate numbers and make decisions as needed. The cells in a spreadsheet may be organized in many different ways, according to the preference of the user.

NOTE—In this exercise, the following notation is used:

- θ the angle of incidence that the ray makes with the glass/air interface (the top surface of the plate)
- ϕ the azimuthal angle of the ray (measured CCW from the x-axis).
- x the x-coordinate where the ray strikes the bottom of the glass plate
- y the y-coordinate where the ray strikes the bottom of the glass plate
- r the radial distance from (0,0) to where the ray strikes the bottom of the glass plate
- t the thickness of the glass plate, in mm
- n the refractive index of the glass plate



In this exercise, you will:

- organize each of the following steps in a row-by-row fashion.
- use relative (“A5”) and absolute (“\$A\$5”) cell referencing.
- enter both text and mathematical functions into cells.
- need to remember to convert degrees into radians using $[x*PI()/180]$ where x is an angle in degrees.
- use the random number generator function $[=RAND()]$
- use the logical “IF” statement to check whether or not an angle is greater than, or less than, the critical angle of the glass plate.
- convert ray locations in polar coordinates (r,ϕ) to rectangular coordinates (x,y) .
- use an x-y scatter plot to produce a picture of the distribution of light seen in the glass plate.

Finally, and most importantly, you will use your spreadsheet model to predict the behavior of the dark circle as you change the plate’s thickness and refractive index.

The following steps will “lead you through” the construction of this spreadsheet—inputting and manipulating data to trace rays in a simulation of the Pfund experiment. In particular, the spreadsheet will “trace rays” that are diffusely scattered upwards from the laser beam incident on the back of the plate at the $x=0, y=0$ point by:

- (1) generating random angles of scatter (reflection) from $0-90^\circ$, along the positive x-axis.
- (2) using the “IF” statement to check whether or not the ray angles are $>$ or $<$ the critical angle of the glass/air top surface of the plate:
 - * If the ray angle is $<$ the critical angle, that ray is ignored (effectively, it is transmitted out of the top of the plate, “lost” to ever reaching the bottom surface of the plate).
 - * If the ray angle is $>$ the critical angle, that ray is reflected from the top surface of the plate, and:
- (3) The location of where the ray strikes the back surface of the plate (along the positive x-axis) is calculated.
- (4) The ray is “rotated” through a random azimuthal angle ϕ ($0-360^\circ$) to simulate the random scattering into all azimuthal angles around the $x=0, y=0$ laser spot.
- (5) Ray locations at the back of the plate, in terms of (r,ϕ) are converted to rectangular (x,y) coordinates.
- (6) A scatter plot is made of all of these (x,y) points, producing the same pattern of light that you observed in the lab!

Building the Excel Spreadsheet:

- (1) Input t (plate thickness) and n (refractive index).

Enter the value of each in separate cells. Label each constant with text, in an adjacent cell. To use these values as constants, refer to them in following calculations as absolute references, using the \$ sign. For example, if you place the thickness in cell A3, refer to it as \$A\$3 in formulas. Then, as you copy and paste that formula, the reference to cell A3 doesn’t change (i.e. it is an ‘absolute’ cell reference).

- (2) Calculate the critical angle, and store it as a constant value.

Excel inherently works in radians. Make sure you convert to degrees. The constant π is referred to as PI(). For example, to calculate the sin of 45° , you enter “=SIN(45*PI()/180)”

Working downwards in separate columns, do the following:

- (3) Generate 1000 random numbers having values (0-90). These represent (random) angles of scatter upwards (“diffuse reflection”) from the (0,0) incident laser spot. Therefore, these are also the angles of incidence, θ , of the rays at the top (glass-to-air) surface of the plate. Label this column “ θ ”.

Use the “=RAND()*90” function to do this.

- (4) In the next adjacent column, generate 1000 random numbers having values (0-360). These represent (random) angles in azimuth, ϕ (rotation from the positive x-axis, in a CCW sense). Label this column “ ϕ ”.

Use the “=RAND()*360” function to do this.

- (5) In the next adjacent column, reject the values of θ less than the critical angle.

Use the IF statement to check to see if each value of θ is less than the critical angle. For example, suppose that the critical angle is calculated in cell A1, and the particular value of θ that we want to check is in cell C6. The function “=IF(C6<A1,0,C6)” checks to see if the angle in cell C6 is less than the critical angle in cell A1. If it is less than the critical angle, the angle in cell C6 is set to 0. If it is greater than the critical angle, the angle in cell C6 is left unchanged.

- (6) For all values of θ greater than the critical angle, the rays will reflect off of the top surface of the plate, back down into the plate towards the bottom surface. In the next adjacent column, calculate the radial position of where each ray strikes the bottom side of the plate ($x=r, y=0$). Label this column “r”.

(You’re on your own to figure out the appropriate geometry) ☺

- (7) In the next 2 adjacent columns, convert (r, ϕ) to (x, y). Label these columns “x” and “y”. Note that these are the final 2 columns in your spreadsheet. Each of these (x, y) coordinate points represents a location on the bottom surface of the plate where a ray of laser light strikes, after undergoing TIR at the top surface.

(You’re on your own to know how to convert from polar to rectangular coordinates) ☺

- (8) Make a scatter-plot of all the (x,y) coordinate points. The result will look similar to the pattern of light that you observed looking at the actual glass plate (of the same thickness and index).

To make this plot, highlight all of the (x,y) values by left-clicking on the upper-left x cell, and dragging over and down to the last y cell. Leave these cells highlighted, then click on the “Scatter” icon in the Insert\Charts tab. Choose the “Scatter with only Markers” icon (the icon showing no connecting lines, just individual points). An x-y scatter plot (“chart”) will automatically be created, showing the pattern of light seen in the actual Pfund experiment!

(You’re on your own to know how to label and manipulate the plot.) ☺

Turn in the following:

- (a) Run a simulation for each of the 2 different values of thickness, t , used in lab:

$t =$ (the thickness of the thin plate, in mm), $n = 1.517$

$t =$ (the thickness of the thick plate, in mm), $n = 1.517$

NOTE: *The appearance of the actual plots will (initially) depend on how the plot area is sized, and the aspect ratio of your computer monitor. Resize each plot so that on your monitor it looks square. Only then will the “dark area” look like a true circle.*

Save a picture of each plot somewhere in your spreadsheet (Right-click to choose the entire plot, then click Copy. Then, somewhere else in your spreadsheet, right-click and choose Paste Special\Picture (JPEG).) This will create a picture copy of the plot, that won't change to a different plot when you change the thickness, t . Label each plot with the corresponding thickness, t .

- (b) Study the effect of refractive index on ring diameter:

Set the plate thickness to the value for the thinner plate. Vary the refractive index and study how the diameter of the ring changes. In particular, vary n from 1.4-1.6, in steps of 0.2. Measure ('estimate') the ring diameter in the scatter plot, for each value of n .

Create and label 2 columns (or a table) in your spreadsheet, to record n and the resulting ring diameter.

Make an x-y plot of ring diameter vs. n . Comment on your results.

- (c) Study the effect of plate thickness on ring diameter:

Set the index to a value of 1.517. Vary the plate thickness from the thin value to the thick value, picking intermediate values of integer numbers (all in mm). Measure ('estimate') the (circular!) ring diameter in the scatter plot, for each value of t .

Create and label 2 columns (or a table) in your spreadsheet, to record t and the resulting ring diameter.

Make an x-y plot of ring diameter vs. t . Comment on your results. Do your computer-traced rays agree with your actual lab observations? Comment.

- (d) Email a working copy of your spreadsheet to Dr. Mike (nofziger@optics.arizona.edu) showing all numbers, and the 4 plots.