Student Name: ____
OPTI 201R
Homework 12--Solution Sheet _____ **TOTAL POINTS** (out of 100 points)

9.3 An object is 25 cm in front of a 60 cm focal length, $(F/\#)_\infty = 6$ lens. An aperture 10 cm in diameter is 2 cm behind the lens.

(a) Where is the entrance pupil located and what is its size?

NOTE: The aperture …"behind the lens" implies that it is to the right of the lens, not to the left of ("in front of") the lens, meaning the lens-aperture distance $\overline{LA} = +2\,cm$:

For this particular object, there are 2 possibilities for the entrance pupil:

(a) the image of the aperture in object space
(b) the lens itself

In other words, which one limits the bundle of rays in object space that leaves the on-axis object point and passes through the 'system'?

• ***Determine whether the lens or the aperture is the entrance pupil***:
Following what is discussed in section 9.1.3 and in Figure 9.6, image the aperture into object space, and then compare the marginal ray angles—the ray angles from the on-axis object point (a) to the edge of the image of the aperture, and (b) to the edge of the lens. The smallest MR angle will determine which edge limits the ray bundle from the object point in object space, and therefore whether the lens or the image of the aperture acts as the entrance pupil:

• Find the diameter of the lens:

$$F/\# = 6 = \frac{f}{d} = \frac{60cm}{d}$$

$$d = 10cm$$

• Find the diameter and location of the image of the aperture in object space:
"Unfold" the system, so light travels from L → R. This puts the aperture is in front of the lens, and

$\overline{LA} = z_A = -2\,cm$ becomes a negative object distance.

$$\frac{n'}{z'_A} = \frac{n}{z_A} + \phi$$

$$\frac{1}{z'_A} = \frac{1}{-2cm} + \frac{1}{+60cm}$$

$$z'_A\left(unfolded\right) = -2.0689\,cm$$

Now, "fold" the system back, putting the aperture A to the right of the lens. Then:

$z_{A'} = \overline{LA'} = +2.0689\,cm$ (the image of the aperture is to the right of the lens and therefore is virtual)

- Calculate the diameter of the image of the aperture:

$$M_t = \frac{z'/n'}{z/n} = \frac{+2.0689cm}{+2cm} = 1.034$$

$$d_{A'} = (1.034)(10cm) \qquad\qquad d_{A'} = 10.34\,cm$$

- Find the MR angle to the lens:

$$u_{lens} = \frac{y_{lens}}{\overline{OL}} = \frac{d_{lens}/2}{25cm} = \frac{10cm}{2\cdot 25cm} = 0.2$$

- Find the MR angle to the image of the aperture:

$$u_{A'} = \frac{y_{A'}}{\overline{OA'}} = \frac{d_{A'}/2}{(25cm + 2.0689cm)} = \frac{10.34cm/2}{27.068cm} = 0.19$$

So the smallest MR angle is subtended by the image of the aperture and not the lens.
Therefore:

- ***<u>the image of the aperture is the entrance pupil</u>***
- ***<u>the aperture is, in fact, the aperture stop of the system.</u>***

So as already calculated, the answers to part (a) are:

$\overline{LE} = 1.935\,cm$ and $E = 9.68\,cm$

The entrance pupil, located to the right of the lens, is in the virtual part of object space.
The entrance pupil is therefore a virtual pupil.

(b) Where is the exit pupil located and what is its size?
… in other words, find the <u>location</u> and <u>diameter</u> of the <u>image of the aperture stop</u> in <u>*image*</u> space.

This is trivial: the aperture is the aperture stop, and nothing to the right of it further images it into image space. Therefore, the aperture is also the exit pupil!

$\boxed{\overline{LE'} = 2\,cm}$ and $\boxed{E' = 10\,cm}$

Extra insight into this problem:

■ Find the critical distance z_c in object space, where the MR angle is the same from the object point to the edge of the lens and to the edge of A':

$$u_{lens} = \frac{y_{lens}}{\overline{\overline{OL}}} = \frac{d_{lens}/2}{z_c} = \frac{5cm}{z_c}$$

$$u_{A'} = \frac{y_{A'}}{\overline{\overline{OA'}}} = \frac{d_{A'}/2}{(z_c + 1.935cm)} = \frac{5cm}{(z_c + 1.935cm)}$$

Setting these 2 MR angles equal gives: $u_{lens} = \frac{5cm}{z_c} = u_{A'} = \frac{5cm}{(z_c + 1.935cm)}$

but this can only be true in the limit when $z_c \to \infty$

In other words, we conclude that for all real objects, the aperture will always be the aperture stop and also the entrance pupil. (this turns out to be true only because the lens and the aperture have the same diameters…..think about it.)

■ Move the same aperture to the left of the lens:

(a) Determine whether the aperture or the lens acts as the aperture stop:

- Find the MR angle to the lens:

$$u_{lens} = \frac{y_{lens}}{\overline{\overline{OL}}} = \frac{d_{lens}/2}{25cm} = \frac{10cm}{2 \cdot 25cm} = 0.2$$

- Find the MR angle to the aperture:

$$u_A = \frac{y_A}{\overline{\overline{OA}}} = \frac{d_A/2}{(25cm - 2cm)} = \frac{5cm}{23cm} = 0.217$$

So the smallest MR angle is subtended by the lens, and not the aperture.
Therefore, for this particular object:

- ***the lens is the entrance pupil***
- ***the lens is also the aperture stop of the system.***

■ Find the critical distance z_c in object space, where the MR angle is the same from the object point to the edge of the lens and to the edge of the aperture:

$$u_{lens} = \frac{y_{lens}}{\overline{OL}} = \frac{d_{lens}/2}{z_c} = \frac{5cm}{z_c}$$

$$u_A = \frac{y_A}{\overline{OA}} = \frac{d_A/2}{(z_c - 2cm)} = \frac{5cm}{(z_c - 2cm)}$$

Setting these 2 MR angles equal gives: $u_{lens} = \frac{5cm}{z_c} = u_A = \frac{5cm}{(z_c - 2cm)}$

Again, this can only be true in the limit as $z_c \to \infty$

■ Reduce the diameter of the aperture to 7cm and find the critical distance z_c in object space:

Now, the aperture has a smaller diameter than the lens, so we would expect to find a finite value for the critical distance:

$$u_{lens} = \frac{y_{lens}}{\overline{OL}} = \frac{d_{lens}/2}{z_c} = \frac{5cm}{z_c}$$

$$u_A = \frac{y_A}{\overline{OA}} = \frac{d_A/2}{(z_c - 2cm)} = \frac{3.5cm}{(z_c - 2cm)}$$

Setting these 2 MR angles equal gives: $u_{lens} = \frac{5cm}{z_c} = u_A = \frac{3.5cm}{(z_c - 2cm)}$

$$\frac{5cm}{z_c} = \frac{3.5cm}{(z_c - 2cm)}$$

$$(3.5cm) z_c = 5cm (z_c - 2cm)$$

$$(3.5cm) z_c = (5cm) z_c - 10cm^2$$

$$10cm^2 = (1.5cm) z_c$$

$$z_c = 6.66cm$$

So for an object located 6.66cm to the left of the lens (we've ignored the minus sign thus far), the MR angle to the edge of the aperture is equal to the MR angle to the edge of the lens.

This implies that:

- for objects located to the left of z_c, the lens acts as the aperture stop.
- for objects located to the right of z_c, the aperture acts as the aperture stop.

***** IF you understand this, you are well on your way to understanding section 9.1.3 ☺

***** IF not……

9.5 Two thin lenses (*L_1* and *L_2*) separated by 30 cm have effective focal lengths of 20 cm and 15 cm, and diameters of 12 cm and 10 cm, respectively. An aperture with a diameter of 8 cm is placed 10 cm in front of the 20 cm efl lens (*L_1*).

(a) For an object 50 cm in front of lens 1, locate the entrance and exit pupils.
“Your choice” as to how you approach this part.

For starters, “organize” the numbers to make sure you work the correct problem:

<u>Lens 1</u> f = 20 cm d = 12 cm
Distance between the lenses = 30 cm
<u>Lens 2</u> f = 15 cm d = 10 cm
Aperture located 10 cm to the <u>left</u> of Lens 1 d = 8 cm

Start by finding the entrance pupil for this particular object. As we’ve discussed in class, this is the “opening” in object space of the system for which the marginal ray (MR) from this object has the smallest value. The 3 possible “openings” are:

- the aperture itself
- Lens 1
- L_2' (the image of Lens 2, as imaged by Lens 1 into object space of the system)

So, image all openings (*A* and L_2 in this problem) back into object space. L_1 is already in object space.

- Solve for the location of L_2':

Lens 2 is to the right of Lens 1, so first “unfold” the system, so light travels from L → R. This puts Lens 2 to the left of Lens 1, and now $\overline{L_1L_2} = z_{L2} = -30cm$

$$\frac{n'}{z'_{L2}} = \frac{n}{z_{L2}} + \phi$$

$$\frac{1}{z'_{L2}} = \frac{1}{-30cm} + \frac{1}{+20cm}$$

$$z'_{L2}\left(unfolded\right) = 60\,cm$$

Now, “fold” the system back, putting Lens 2 to the right of the Lens 1. Therefore:
$z'_{L2} = -60\,cm$ (the image of Lens 2 is now to the left of Lens 1 and also to the left of the object!)

- Solve for the diameter of L_2':

Calculate the transverse magnification to find the diameter of L_2':

$$M_t = \frac{z'/n'}{z/n} = \frac{-60cm}{30cm} = -2$$

$$d_{L2'} = \left|M_t\right| d_{L2} \qquad d_{L2'} = 20cm$$

- Find the MR angle to Lens 1:

$$u_{L1} = \frac{y_{L1}}{\overline{OL_1}} = \frac{d_{L1}/2}{50cm} = \frac{6cm}{50cm} = 0.12$$

- Find the MR angle to the aperture:

$$u_A = \frac{y_A}{\overline{OA}} = \frac{d_A/2}{\left(50cm - 10cm\right)} = \frac{4cm}{40cm} = 0.1$$

- Find the MR angle to L_2':

$$u_{L2'} = \frac{y_{L2'}}{\overline{OL_2'}} = \frac{d_{L2'}/2}{\left(50cm - 60cm\right)} = \frac{10cm}{-10cm} = -1$$

Because the image of Lens 2 is located to the left of the object, no rays could pass through this "opening" and this image cannot, therefore, be the entrance pupil.

So the smallest MR angle is subtended by the aperture itself.
Therefore, for this particular object:

- ***the aperture is the entrance pupil***
- ***therefore, the aperture is also the Aperture Stop of the system.***

- To locate the exit pupil, find the image of the aperture, as formed by the "system" (Lenses 1 and 2):

→ Sequential Imaging Method

■ Find the image of the aperture formed by Lens 1:

$$\frac{n'}{z'_{L1}} = \frac{n}{z_{L1}} + \phi$$

$$\frac{1}{z'_{L1}} = \frac{1}{-10cm} + \frac{1}{+20cm}$$

$$z'_{L1} = -20\,cm$$

■ Find the image of L_1' formed by Lens 2:

$$\frac{n'}{z'_{L2}} = \frac{n}{z_{L2}} + \phi$$

$$\frac{1}{z'_{L2}} = \frac{1}{\left(-20cm - 30cm\right)} + \frac{1}{+15cm} = \frac{1}{\left(-50cm\right)} + \frac{1}{+15cm}$$

$$z'_{L1} = 21.43\,cm$$

So the exit pupil is located 21.43 cm to the right of Lens 2. It is in the real part of system image space, and is therefore a real pupil (not a virtual one).

→ Raytrace Method— Find the A_{STOP}

■ Trace a "pseudo" MR through the system, and then scale it:

Let $u = 0.05$ (just an arbitrary angle to start with):

- Transer to *A*: $y_A = y_O + \frac{t}{n}(nu) = 0 + (50 - 10)(.05)$ $y_A = 2cm$
- Transer to L_1: $y_{L1} = y_A + \frac{t}{n}(nu) = 2 + (10)(.05)$ $y_{L1} = 2.5cm$
- Refract at L_1: $n'u' = nu - y_{L1}\phi_1 = .05 - (2.5)\left(\frac{1}{20}\right)$ $n'u' = -.075$
- Transer to L_2: $y_{L2} = y_{L1} + \frac{t}{n'}(n'u') = 2.5 + (30)(-.075)$ $y_{L2} = 0.25cm$
- Refract at L_2: $n'u' = nu - y_{L2}\phi_2 = (-.075) - (.25)\left(\frac{1}{15}\right)$ $n'u' = -.0916$
- Transer to *I*: $y_I = 0 = y_{L2} + \frac{t}{n'}(n'u') = (.25) + t(-.0916)$ $t = +2.72cm$

Now, find the correct scaling factor:

- At opening *A*: $\frac{CA/2}{y} = \frac{8cm/2}{2cm} = 2$
- At opening L_1: $\frac{CA/2}{y} = \frac{12cm/2}{2.5cm} = 2.4$
- At opening L_2: $\frac{CA/2}{y} = \frac{10cm/2}{.25cm} = 20$

***** The ratio closest to 1.0 is at opening *A*, so *A* (for this particular object) the A_{STOP}.

And finally, (although the problem doesn't require this), scale (multiply) all of the "pseudo" ray heights and angles by 2.0 to get the true MR values:

$u_{MR} = 0.05 \times 2 = 0.1$

$y_A = 2cm \times 2 = 4cm$ *****Note that this is, in fact, the true radius of the aperture ☺

***** This isn't "by chance" but the result of this scaling process.

$y_{L1} = 2.5cm \times 2 = 5cm$

$n'u' = -.075 \times 2 = -.15$

$y_{L2} = 0.25cm \times 2 = .5cm$

$n'u' = -.0916 \times 2 = -.1832$

→ <u>Raytrace Method</u>—Find the Entrance Pupil, E:
….but we don’t need to do this. We’ve already determined that the opening *A* <u>is</u> the A_{STOP}, and because it is located in the object space of the system, it is also the Entrance pupil.

→E is located 10 cm to the left of lens 1
→the diameter of E = 8 cm (both numbers as stated in the original problem).

→ <u>Raytrace Method</u>—Find the <u>location</u> of the Exit Pupil, E′:

■ Find the image of the aperture *A* formed by Lens 1 and Lens 2, by tracing a ray from the on-axis point of *A* forwards (to the right) into image space of the system. Where this ray crosses the axis will locate E′:

$u = +0.1$ (arbitrary angle).

- Transer to *L₁*: $y_{L1} = y_A + \frac{t}{n}(nu) = 0 + (10)(.1)$ $y_{L1} = 1cm$
- Refract at *L₁*: $n'u' = nu - y_{L1}\phi_1 = .1 - (1)\left(\frac{1}{20}\right)$ $n'u' = .05$
- Transer to *L₂*: $y_{L2} = y_{L1} + \frac{t}{n'}(n'u') = .05 + (30)(.05)$ $y_{L2} = 2.5cm$
- Refract at *L₂*: $n'u' = nu - y_{L2}\phi_2 = (.05) - (2.5)\left(\frac{1}{15}\right)$ $n'u' = -.1166$
- Transer to *E′*: $y_{E'} = 0 = y_{L2} + \frac{t}{n'}(n'u')$ $\boxed{t = \overline{L_2E'} = +21.43cm}$

$$0 = (2.5) + t(-.1166)$$

→ <u>Raytrace Method</u>—Find the ***diameter*** of the Exit Pupil, E′:

■ Trace a ray from the edge of the A_{STOP} to the location of the exit pupil. The ray height at the exit pupil will be the radius of the pupil. (If we consider the A_{STOP} to be our object, we are in fact tracing a chief ray from the edge of this object to the edge of its image, the exit pupil).

Let $\overline{u} = 0$ (arbitrary angle—this chief ray can be parallel to the axis, for example).

- Transer to *L₁*: $\overline{y}_{L1} = \overline{y}_A + \frac{t}{n}(n\overline{u}) = 4 + (10)(0)$ $\overline{y}_{L1} = 4cm$
- Refract at *L₁*: $n'\overline{u'} = n\overline{u} - \overline{y}_{L1}\phi_1 = 0 - (4)\left(\frac{1}{20}\right)$ $n'\overline{u'} = -.2$
- Transer to *L₂*: $\overline{y}_{L2} = \overline{y}_{L1} + \frac{t}{n'}(n'\overline{u'}) = 4 + (30)(-.2)$ $\overline{y}_{L2} = -2cm$

- Refract at L_2: $\overline{n'u'} = n\bar{u} - \bar{y}_{L2}\phi_2 = (-.2) - (-2)\left(\frac{1}{15}\right)$ $\overline{n'u'} = -.0666$
- Transer to E': $\bar{y}_{E'} = \bar{y}_{L2} + \frac{t}{n'}\left(\overline{n'u'}\right) = (-2cm) + (21.43cm)(-.0666)$ $\bar{y}_{E'} = -3.427cm$

So the diameter of the exit pupil is: $\boxed{E' = 2 \times \left|\bar{y}_{E'}\right| = 6.85cm}$

(b) For an object 25 cm in front of lens 1, locate the entrance and exit pupils.

***** (I'll post this on Monday….)

$\boxed{9,10}$ (a) O at $-\infty$:

Entrance Pupil Approach! "Image all openings into object space":

Find A': ↳ $\frac{1}{z'} = \frac{1}{\ominus 20} + \frac{1}{100}$; $z' = -25 \Rightarrow z' = +25$.

$d_{A'} = (M_t)(80) = \left|\frac{25}{20}\right| \cdot 80 = 100$

Find L_2': ↳ $\frac{1}{z'} = \frac{1}{-40} + \frac{1}{100}$; $z' = -66.\overline{6} \Rightarrow z' = +66.6$

$d_{L_2'} = (M_t)(90) = \left(\frac{66.6}{40}\right)90 = 150$

$u=0$ so the opening in object space w/ the smallest diameter is the entrance pupil:

$d_{L_1} = \textcircled{100}$, $d_{A'} = \textcircled{100}$, $d_{L_2'} = 150$

= OR =

?

* So L_1 is the entrance pupil because it is before A' in object space (?)

Trace the MR that has $u=0$ + strikes the edge of L_1:

$\textcircled{R}_{L_1}$ $\boxed{u_1 = 0 \atop Y_1 = 50}$ $n'u' = nu - Y_1\phi_1 = 0 - (50)\left(\frac{1}{100}\right)$; $n'u' = -.5$

$\textcircled{T}_A$ $Y_A = Y_1 + \frac{t}{n}n'u' = (50) + (20)(-.5)$ $\boxed{Y_A = 40}$ → So which surface is the A_{STOP}?

$\textcircled{T}_{L_2}$ $Y_{L_2} = Y_A + \frac{t}{n}(n'u') = 40 + (20)(-.5)$ $Y_{L_2} = 30$

$\textcircled{R}_{L_2}$ $n''u'' = n'u' - Y_{L_2}\phi_2 = (-.5) - (30)\left(\frac{1}{100}\right)$ $n''u'' = -0.8$

$\textcircled{T}_I$ $Y_I = 0 = Y_2 + \left(\frac{t}{n''}\right)(n''u'') = (30) + (t)(-.8)$ $t = 37.5\text{cm}$

☆ Must know the A_{STOP} to find E':

(b) O at 200mm in front of L_1

calculate the MR angle subtended by L_1, A'_1 + L_2'

$$u_{L_1} = \frac{50}{200} = .25$$

$$u_{A'} = \frac{100/2}{(200+25)} = \textcircled{.22}$$ AE ∴ A is the A_{STOP}

$$u_{L_2}' = \frac{150/2}{(200+66.6)} = .28$$

Trace a pseudo MR:

let $u = .1$

(T-L_1)	$y_1 = 0 + \left(\frac{t}{n}\right) nu = (200)(.1) = 20$		$\left(\frac{CA/2}{Y}\right)$ = 2.5
(R-L_1)	$n'_1u'_1 = n_1u_1 - y_1\phi_1 = (.1) - (20)\left(\frac{1}{100}\right)$	$n'u'_1 = -.1$	
(T-A)	$y_A = y_1 + \frac{t}{n'}(n'u'_1) = (20) + (20)(-.1)$	$y_A = 18$	= $\textcircled{2.2}$
(T-L_2)	$y_2 = y_A + \left(\frac{t}{n'}\right)(n'u') = (18) + (20)(-.1)$	$y_2 = 16$	= 2.8
(R-L_2)	$n''u'' = n'u' - y_2\phi_2 = (-.1) - (16)\left(\frac{1}{100}\right)$	$n''u'' = -.26$	
(T-I)	$y_I = 0 = y_2 + \left(\frac{t}{n''}\right)(n''u'') = (16) + (t)(-.26)$ $\boxed{t = 61.5}$		

Find E': • Trace the on-axis ray at the Astop into image space:

(pseudo-MR) $u_A = -.1$

$y_2 = y_A + \left(\frac{t}{n}\right)(nu) = 0 + (20)(-.1)$ $\quad y_2 = -2$

$n'u'_2 = nu_2 - y_2\phi_2 = (-.1) - (-2)\left(\frac{1}{100}\right)$ $\quad n'u' = -.08$

$y_I = y_{E'} = 0 = y_{L2} + \left(\frac{t}{n''}\right)(n''u'') = (-2) + (t)(-.08)$ $\boxed{t = L_2E' = -25}$

• Trace the true MR:

$u_A = (-.1)(2.2) = (-.22)$

$y_2 = y_A + \left(\frac{t}{n}\right)(nu) = 0 + (20)(-.22)$ $\quad y_2 = -4.4$

$n'u_2' = nu_2 - y_2\phi_2 = (-.22) - (-4.4)\left(\frac{1}{100}\right)$ $\quad n'u_2' = -.1782$

$y_I = y_{E'} = 0 = y_{L2} + \left(\frac{t}{n''}\right)(n''u'') = (-4.4) + (t)(-.17822)$ $\boxed{t = -24.68}$

• Image:

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$= \frac{1}{-20} + \frac{1}{100}$$

$$z' = -25$$

$d_{E'} = (M_E)(80) = \left(\frac{25}{20}\right)(80) = \boxed{100}$

Trace a ray leaving the edge of the Astop: let $u = 0$, $y_A = 40$

• (T-L_2) $y_2 = 40$

(R-L_2) $n'u' = nu - y_2\phi_2 = 0 - (40)\left(\frac{1}{100}\right)$ $\quad n'u' = -.4$

(T-E') ~~$y_{E'} = 0 = y_2 + \left(\frac{t}{n'}\right)(n'u') = (40) + t(-.4)$ $t =$~~

$y_{E'} = y_2 + \left(\frac{t}{n'}\right)(n'u') = (40) + (-25)(-.4)$; $y_{E'} = 50$ → $\boxed{d_{E'} = E' = 100}$

$\boxed{9.11}$ (a) $FOV = (2)\tan^{-1}\left(\dfrac{36.5\text{mm}/2}{100\text{mm}}\right) = 20.68^0$

(b) $FOV_1 = (2)\tan^{-1}\left(\dfrac{24.5\text{mm}/2}{100\text{mm}}\right) = 13.96^0$

(c) $FOV_/ = (2)\tan^{-1}\left(\dfrac{(24.5^2 + 36.5^2)^{1/2}/2}{180}\right) = 24.79^0$

$\boxed{9.20}$

(a) What is the A_{STOP}?

- Trace a pseudo MR: let $u = .1$

	$\frac{CA/2}{y}$
$y_1 = 0 + \frac{t}{n} nu = (150)(.1) = 15$	1.6
$y_2 = y_1 + \frac{t}{n} nu = 15 + (50)(.1) = 20$	1.25
$y_3 = y_2 + \frac{t}{n} nu = 20 + (50)(.1) = 25$	$\textcircled{.5}$

The second hole is the A_{STOP}

- Compare openings in object space:

Find H_2': @ $\frac{1}{z'} = \frac{1}{-50} + \frac{1}{100}$ z' 200 5 $z' = +100$

$u_1 = \frac{25}{100} = .25$ / $u_2 = \frac{25}{200} = .125$ / $u_3 = \frac{12.5}{300} = \boxed{.0416}$ ∴ H_2' is the E
H_2 is the A_{STOP}

(b) E is located 100mm to the R of the lens: $\boxed{LE = 100\text{ mm}}$

$E = (M_t)\,25 = \left(\frac{100}{50}\right)25$ $\boxed{E = 50\text{mm}}$

(c) H_2 is the exit pupil, $\boxed{E' = 25\text{mm}}$ $\boxed{LE' = 50\text{ mm}}$

(d) $\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$

$\frac{1}{z'} = \frac{1}{-200} + \frac{1}{100} \rightarrow z' = 200\text{ mm}$

$M_t = \frac{z'}{z} = \frac{200\text{ mm}}{-200\text{mm}} = -1$

Image is <u>inverted</u>