Student Name: ____________________________________
OPTI 201R
Homework 5—CORRECTED (10/5/11) Grading Sheet _____ **TOTAL POINTS** (out of 100 points)

5.2 A plastic rod of refractive index 1.45 has a spherical radius of 15 cm. A 2 cm high object is located (in air) 20 cm in front of the rod on the extension of the axis (down the length of the rod).

(a) What are the front and back focal lengths?

The front focal length is the distance from the vertex of the rod to the front focal point F. It is calculated as:

$$\phi = \frac{-n}{f} = \;\rightarrow\; \phi = \frac{n'-n}{R} = \frac{-n}{f} \rightarrow f = \frac{-nR}{(n'-n)} = \frac{(-1)(15cm)}{(1.45-1)}$$

$$\rightarrow \boxed{f = -33.33 \text{ cm}}$$

Because the front focal length is negative, the front focal point F lies to the left of the vertex.

The back focal length is the distance from the vertex of the rod to the back focal point F*. It is calculated as:

$$\phi = \frac{n'}{f^*} = \;\rightarrow\; \phi = \frac{n'-n}{R} = \frac{n'}{f^*} \rightarrow f^* = \frac{n'R}{(n'-n)} = \frac{(1.45)(15cm)}{(1.45-1)}$$

$$\rightarrow \boxed{f^* = 48.33 \text{ cm}}$$

Because the back focal length is positive, the back focal point F lies to the right of the vertex.

NOTE: f and f* have different values because the refractive indices of object and image space are not equal. For a more typical optical system in air, the object and image space indices are both equal, and f would equal f*.

As equation (5.38) points out, the ratio of the focal lengths equals the ratio of the indices:

$$\frac{f^*}{f} = -\frac{n'}{n} \rightarrow \quad \frac{f^*}{f} = \frac{48.33cm}{-33.33cm} = -1.45 \quad \textit{which does equal } -\frac{n'}{n} = \frac{-1.45}{1} = -1.45$$

(b) What is the transverse magnification?

The transverse magnification is the ratio of the image height to object height, or equivalently the ratio of the reduced image distance to the reduced object distance:

$$M_t = \frac{h'}{h} = \frac{z'/n'}{z/n}$$ so first use the Gaussian imaging equation to find z′:

$$\frac{n'}{z'}=\frac{n}{z}+\phi \quad\rightarrow\quad \frac{n'}{z'}=\frac{n}{z}+\frac{n'-n}{R} \quad\rightarrow\quad \frac{1.45}{z'}=\frac{1}{-20cm}+\frac{1.45-1}{15cm} \qquad\rightarrow z'=-72.5\text{ cm}$$

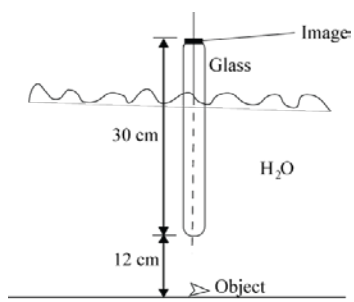
We get the curious result that z′ is negative, meaning that the image is located to the ***left*** of the vertex of the plastic rod….. This means that the image is a virtual image!

Now, calculate the transverse magnification:

$$M_t=\frac{z'/n'}{z/n}=\frac{-72.5cm/1.45}{-20cm/1} \qquad\rightarrow \boxed{M_t=2.5}$$

_____ (3) Correct answer for (a): f = -33.33 cm
_____ (3) Correct answer for (a): f* = 48.33 cm
_____ (4) Correct answer for (b): M_t = 2.5

5.3 Some algae are growing at the bottom of a lake, and a student would like to bring the image of the algae out of the water with a glass rod, such that the image would be formed on the end of the rod. The rod's refractive index is 1.52 with a convex radius on its end. The algae are 1 mm high, located in water about 12 cm away from the rod, and the rod is 30 cm long, extending above the water.



(a) What radius is needed on the end of the rod?

$$n=1.333 \quad ; \quad n'=1.52 \quad ; \quad z=-12cm \quad ; \quad z'=+30cm \quad ; \quad R=?$$

$$\frac{n'}{z'}=\frac{n}{z}+\frac{n'-n}{R}$$

$$\frac{1.52}{30cm}=\frac{1.333}{-12cm}+\frac{1.52-1.333}{R}$$

$$\frac{1.52}{30cm}+\frac{1.333}{12cm}=\frac{.187}{R}$$

$$\boxed{R=1.156\,cm}$$

(b) What is the transverse magnification?

$$M_t = \frac{z'/n'}{z/n} = \frac{30cm/1.52}{-12cm/1.333}$$

$$\boxed{M_t = -2.192}$$

(c) What are the front and back focal lengths of the rod's surface of optical power?

$$\varphi = \frac{n'-n}{R} = \frac{-n}{f} = \frac{n'}{f^*}$$

$$\varphi = \frac{1.52-1.333}{.01156m} = 16.18 \textit{ Diopters}$$

$$so \quad f = \frac{-n}{\varphi} = \frac{-1.333}{16.18m^{-1}} = -.0824m = -82.4mm$$

$$f^* = \frac{n'}{\varphi} = \frac{1.52}{16.18m^{-1}} = .0939m = 93.9mm$$

$$\boxed{f = -82.4mm \quad ; \quad f^* = 93.9mm}$$

_____ (3) Correct answer for (a): R = 1.156 cm
_____ (3) Correct answer for (b): M_t = -2.192
_____ (2) Correct answer for (c): f = -82.4 mm
_____ (2) Correct answer for (c): f* = 93.9 mm

5.4 A spherical surface with a radius of 2.75 cm is on a glass rod of refractive index 1.5. Find the optical power in diopters of this rod when placed in:
(a) air;
(b) water (n = 4/3);
(c) oil (n = 1.63)

(a) $\phi = \frac{n'-n}{R} = \frac{1.5-1}{.0275m}$ $\boxed{\phi = 18.18\,D}$

(b) $\phi = \frac{n'-n}{R} = \frac{1.5-1.333}{.0275m}$ $\boxed{\phi = 6.07\,D}$

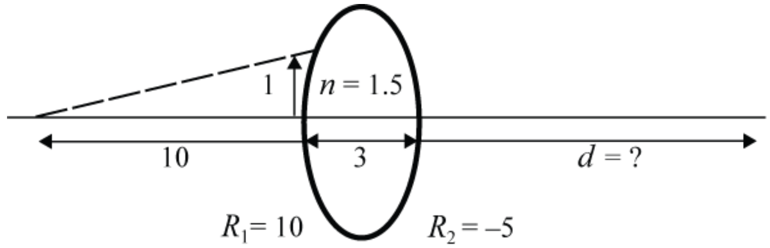
(c) $\phi = \frac{n'-n}{R} = \frac{1.5-1.63}{.0275m}$ $\boxed{\phi = -4.73\,D}$

The surface becomes a negative lens when placed in oil!

_____ (2) Correct answer for (a): ϕ = 18.18 D
_____ (2) Correct answer for (b): ϕ = 6.07 D

______ (2) Correct answer for (c): ϕ = -4.73 D

5.5 In the figure below, find the axial point (d) where the paraxial ray crosses the optical axis in image space by using refraction and transfer equations. (All dimensions are in centimeters).



("Use 15 for the object distance instead of 10"):

►Refract the ray at surface #1:

$$n'u' = nu - y_1\left(\frac{n'-n}{R}\right) \qquad u = \frac{1}{15} = .067 \;;\; n = 1 \;;\; n' = 1.5 \;;\; R = +10cm \;;\; y_1 = 1cm$$

$$u' = \frac{nu}{n'} - \frac{y_1(n'-n)}{n'R} = \frac{(1)(.067)}{1.5} - 1cm\left(\frac{1.5-1}{(1.5)10cm}\right)$$

$\boxed{u' = +.01133}$ (positive, so the ray bends upwards from the optical axis)

►Transfer the ray from surface #1 to surface #2:

$$y_2 = y_1 + \frac{t'}{n'}n'u' = y_1 + t'u' = 1cm + (3cm)(.01133)$$

$\boxed{y_2 = 1.034cm}$ (> 1cm, because the ray is traveling upwards after surface 1)

► Refract the ray at surface #2:

$$n'u' = nu - y_2\left(\frac{n'-n}{R}\right) \qquad u = .01133 \;;\; n = 1.5 \;;\; n' = 1 \;;\; R = -5cm$$

$$u' = \frac{nu}{n'} - \frac{y_2(n'-n)}{n'R} = \frac{(1.5)(.01133)}{1} - 1.034cm\left(\frac{1-1.5}{(1)(-5cm)}\right)$$

$\boxed{u' = -.08641}$ (negative, so the ray is now bent downwards, towards the optical axis)

►Transfer the ray from surface #2 to the image:

At the image, $y=y_2=0$:

$$y_2 = y_1 + \frac{t'}{n'}n'u' \;;\; 0 = y_1 + t'u' = 1.034cm + (t')(-.08641)$$

$$0 = 1.034cm + (-.08641)t'$$

$\boxed{t' = d = 11.97cm}$ (positive, so the image lies to the right of the second surface, and is real)

_____	(1)	Correct answer for the refracted ray angle at surface #1:	u' = .01133
_____	(1)	Correct answer for the ray height at surface #2:	y_2 = 1.034 cm
_____	(1)	Correct answer for the ray angle at surface #2:	u' = -.08641
_____	(3)	Correct answer for the distance to the image:	t'= d = 11.97 cm

5.7 What is the sag of a spherical surface (R = 25 cm) for a ray 2 cm from the optical axis?

(a) exact sag: $z_s = R - \sqrt{R^2 - y^2} = 25cm - \sqrt{(25cm)^2 - (2cm)^2}$

$$\boxed{z_s(exact) = 0.080128\ cm}$$

(a) approximate sag: $z_s \approx \dfrac{y^2}{2R} = \dfrac{(2cm)^2}{2(25cm)}$

$$\boxed{z_s(approx.) = 0.08\ cm}$$

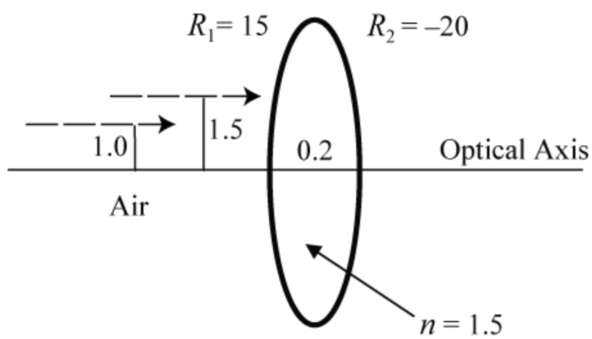
NOTE: The difference between the exact and approximate sag is:

$$\Delta = \frac{z_s(approx.) - z_s(exact)}{z_s(exact)} \times 100\% = \frac{.08 - .080128}{.080128} \times 100\% = -.159\%$$

so the approximation <u>underestimates</u> the exact value by about one-sixth of a percent.

______ (5) Correct answer for the exact sag: z_S (exact) = .080128 cm
______ (5) Correct answer for the approx. sag: z_S (approx) = .08 cm

5.10 Set up a numerical (algebraic) paraxial ray trace for the following lens (all dimensions are in centimeters).



(a) Trace two paraxial rays from an infinite object point at heights of 1.5 and 1.0.

Because each ray comes from an object at -∞, each ray is parallel to the optical axis, so u=0 for each ray.

►Refract ray "1.5" at surface #1:

$$n'u' = nu - y_1\left(\frac{n'-n}{R}\right) \quad u = 0\ ;\ n = 1\ ;\ n' = 1.5\ ;\ R = +15cm\ ;\ y_1 = 1.5cm$$

$$u' = \frac{nu}{n'} - \frac{y_1(n'-n)}{n'R} = 0 - 1.5cm\left(\frac{1.5-1}{(1.5)15cm}\right)$$

$\boxed{u' = -.03333}$ (Because u' is negative, the ray is refracted downwards, towards the optical axis).

►Transfer ray "1.5" from surface #1 to surface #2:

$$y_2 = y_1 + \frac{t'}{n'} n'u' = y_1 + t'u' = 1.5cm + (.2cm)(-.03333)$$

$\boxed{y_2 = 1.4933cm}$ (The ray is heading downwards towards the axis, so the ray height at surface 2 should be less than the ray height at surface 1, and it is!).

► Refract ray "1.5" at surface #2:

$$n'u' = nu - y_2\left(\frac{n'-n}{R}\right) \quad u = -.03333 \ ; \ n = 1.5 \ ; \ n' = 1 \ ; \ R = -20cm$$

$$u' = \frac{nu}{n'} - \frac{y_2(n'-n)}{n'R} = \frac{(1.5)(-.03333)}{1} - 1.49333cm\left(\frac{1-1.5}{(1)(-20cm)}\right)$$

$\boxed{u' = -.08733}$ (The ray angle is more negative than after the first surface, so it is now bent even further towards the optical axis.)

►Transfer ray "1.5" from surface #2 to the image (where the ray crosses the optical axis):
At the image, $y=y_2=0$:

$$y_2 = y_1 + \frac{t'}{n'} n'u' \ ; \ 0 = y_1 + t'u' = 1.49333cm + (t')(-.08733)$$

$$0 = 1.49333cm + (-.08733)t'$$

$\boxed{t' = d = 17.10cm}$

Now trace the other ray:

►Refract ray "1.0" at surface #1:

$$n'u' = nu - y_1\left(\frac{n'-n}{R}\right) \quad u = 0 \ ; \ n = 1 \ ; \ n' = 1.5 \ ; \ R = +15cm \ ; \ y_1 = 1.0cm$$

$$u' = \frac{nu}{n'} - \frac{y_1(n'-n)}{n'R} = 0 - 1.0cm\left(\frac{1.5-1}{(1.5)15cm}\right)$$

$\boxed{u' = -.02222}$ (Because u' is negative, the ray is refracted downwards, towards the optical axis. Also, u' is 'less negative' than for ray "1.5" because the new ray height is less.)

►Transfer ray "1.0" from surface #1 to surface #2:

$$y_2 = y_1 + \frac{t'}{n'} n'u' = y_1 + t'u' = 1.0cm + (.2cm)(-.02222)$$

$\boxed{y_2 = .99556cm}$ (The ray is heading downwards towards the axis, so the ray height at surface 2 should be less than the ray height at surface 1, and it is!).

► Refract ray "1.0" at surface #2:

$$n'u' = nu - y_2\left(\frac{n'-n}{R}\right) \quad u = -.02222 \;;\; n = 1.5 \;;\; n' = 1 \;;\; R = -20cm$$

$$u' = \frac{nu}{n'} - \frac{y_2(n'-n)}{n'R} = \frac{(1.5)(-.02222)}{1} - .99556cm\left(\frac{1-1.5}{(1)(-20cm)}\right)$$

$\boxed{u' = -.05822}$ (The ray angle is more negative than after the first surface, so it is now bent even further towards the optical axis.)

►Transfer ray “1.0” from surface #2 to the image (where the ray crosses the optical axis):
At the image, $y=y_2=0$:

$$y_2 = y_1 + \frac{t'}{n'}n'u' \;;\; 0 = y_1 + t'u' = .99556cm + (t')(-.05822)$$

$$0 = .99556cm + (-.05822)t'$$

$\boxed{t' = d = 17.10cm}$

This is the same answer as for the other ray, as it should be. Both rays are from the same object point (at -∞) and therefore should image to the same point on the optical axis.

(b) Find the distance from the last surface to the point at which the paraxial ray crosses the axis in each case (BFD).

$\boxed{t' = d = 17.10cm}$ for each ray.

(c) What is the point called at which these rays cross the optical axis?
This is the back focal point, F*.

Ray “1.5”—Correct numbers (to just 2 or 3 decimal places) for the following:

_____	(1)	nu (or u) in space 1:	nu = u = 0
_____	(1)	u (or nu) in space 2:	u = -.033 (or nu = -.049)
_____	(1)	y_2 at surface 2:	y_2 = 1.493 cm
_____	(1)	u (or nu) in space 3:	u = -.087 (or nu = -.087)
_____	(6)	distance t′ to the image	t′ = 17.10 cm

Ray “1.0”—Correct numbers (to just 2 decimal places) for the following:

_____	(1)	nu (or u) in space 1:	nu = u = 0
_____	(1)	u (or nu) in space 2:	u = -.022 (or nu = -.033)
_____	(1)	y_2 at surface 2:	y_2 = .99556 cm
_____	(1)	u (or nu) in space 3:	u = -.058 (or nu = -.058)
_____	(6)	distance t′ to the image	t′ = 17.10 cm

5.14 What is the optical power (in diopters) of the following surfaces made of 755276.479 material placed in different environments?

First realize that for this material: n = 1.755 (the first 3 digits of the Abbe number)

(a) air, radius = 25 cm
(b) water, radius = 25 cm
(c) oil (n=1.55), radius = 12.5 cm
(d) water, radius = 5 in
(e) air, radius = 0.1 ft

(a) $$\phi = \frac{n' - n}{R} = \frac{1.755 - 1}{.25m}$$ $$\boxed{\phi = 3.02\,D}$$

(b) $$\phi = \frac{n' - n}{R} = \frac{1.755 - 1.333}{.25m}$$ $$\boxed{\phi = 1.69\,D}$$

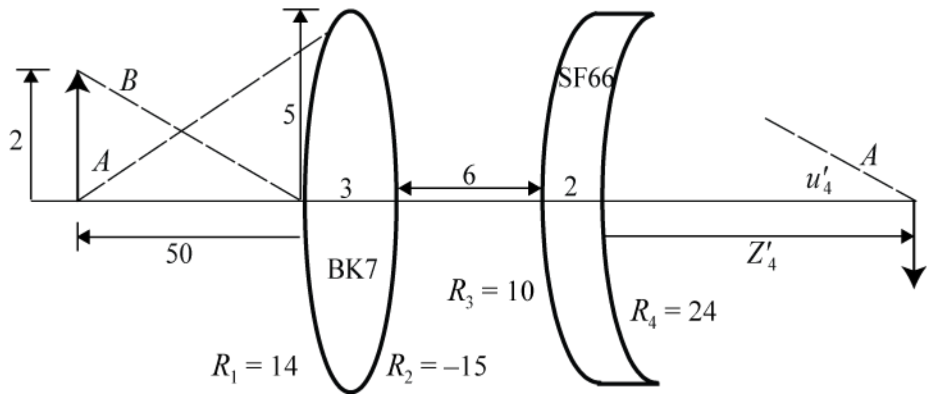
(c) $$\phi = \frac{n' - n}{R} = \frac{1.755 - 1.55}{.125m}$$ $$\boxed{\phi = 1.64\,D}$$

(d) $$\phi = \frac{n' - n}{R} = \frac{1.755 - 1.333}{(5in)\left(.0254m/in\right)}$$ $$\boxed{\phi = 3.32\,D}$$

(e) $$\phi = \frac{n' - n}{R} = \frac{1.755 - 1}{(1.2in)\left(.0254m/in\right)}$$ $$\boxed{\phi = 24.8\,D}$$

_____ (2) Correct answer for (a): ϕ = 3.02 D
_____ (2) Correct answer for (b): ϕ = 1.69 D
_____ (2) Correct answer for (c): ϕ = 1.64 D
_____ (2) Correct answer for (d): ϕ = 3.32 D
_____ (2) Correct answer for (e): ϕ = 24.8 D

5.16 Ray trace the rays shown (A and B) in the figure below from object to image space, where it crosses the optical axis, (N-BK7 = 517642.251; SF66 = 923209.602)
(To find z'_4 you only need to TRACE RAY A, for full credit)



Use the paraxial refraction and transfer equations:

(1) n′u′ = nu −yϕ   to find the angle in the next space.
(2) y′ = y + (t/n′) n′u′   to find the height at the next surface.

In order to keep track of variables, set up a table.
FOR RAY "A":

	Space 1	**Surface 1**	**Space 2**	**Surface 2**	**Space 3**	**Surface 3**	**Space 4**	**Surface 4**	**Space 5**
R	-	14	x	-15	X	10	X	24	x
ϕ	x	0.0369	x	0.0345	X	0.0923	X	-0.0385	x
t	50	x	3	x	6	x	2	x	6.048
n	1	x	1.517	x	1	x	1.923	x	1
y	x	5	x	4.832	x	3.320	x	2.740	0
nu	0.100	x	-0.056	x	-0.252	x	-0.558	x	-0.453

→ The bottom two rows of the table (y, nu) are calculated based on the raytrace equations.

Start with ray A:

In (object) Space 1:

$$u = \frac{5}{-(-50)} = .1\ ;\ nu = .1$$

►Refract ray A at surface #1:

$$n'u' = nu - \frac{y_1(n'-n)}{R_1} = (.1) - 5cm\left(\frac{1.517-1}{14cm}\right)$$

$$n'u' = -.085\ ;\ u' = -.056$$

►Transfer ray A from surface #1 to surface #2:

$$y_2 = y_1 + \frac{t'}{n'}n'u' = 5cm + \frac{3cm}{1.517}(-.085)$$

$$y_2 = 4.832cm$$

▶Refract ray A at surface #2:

$$n'u' = nu - \frac{y_2(n'-n)}{R_2} = (-.085) - 4.832cm\left(\frac{1-1.517}{-15cm}\right)$$

$n'u' = -.252$; $u' = -.252$

▶Transfer ray A from surface #2 to surface #3:

$$y_3 = y_2 + \frac{t'}{n'}n'u' = 4.832cm + \frac{6cm}{1}(-.252)$$

$y_3 = 3.32cm$

▶Refract ray A at surface #3:

$$n'u' = nu - \frac{y_3(n'-n)}{R_3} = (-.252) - 3.32cm\left(\frac{1.923-1}{10cm}\right)$$

$n'u' = -.558$; $u' = -.290$

▶Transfer ray A from surface #3 to surface #4:

$$y_4 = y_3 + \frac{t'}{n'}n'u' = 3.32cm + \frac{2cm}{1.923}(-.558)$$

$y_4 = 2.740cm$

▶Refract ray A at surface #4:

$$n'u' = nu - \frac{y_4(n'-n)}{R_4} = (-.558) - 2.740cm\left(\frac{1-1.923}{24cm}\right)$$

$n'u' = -.453$; $u' = -.453$

▶Transfer ray A from surface #4 to the image (surface #5):

$$y_5 = 0 = y_4 + \frac{t'}{n'}n'u' = 2.740cm + \frac{t'cm}{1}(-.453) = 0$$

▶ $t' = Z_4' = 6.0485cm$ ◀

Now trace ray B:

In order to keep track of variables, set up a table.

FOR RAY "B":

	Space 1	**Surface 1**	**Space 2**	**Surface 2**	**Space 3**	**Surface 3**	**Space 4**	**Surface 4**	**Space 5**
R	-	14	x	-15	X	10	X	24	x
ϕ	x	0.0369	x	0.0345	X	0.0923	X	-0.0385	x
t	50	x	3	x	6	x	2	x	6.048
n	1	x	1.517	x	1	x	1.923	x	1
y	x	0	x	-.03955	x	-.15133	x	-.15617	-.22064
nu	-.02	x	-0.02	x	-0.01863	x	-0.00466	x	-0.01066

"Use an object height = 1, instead of 2)"

In (object) Space 1:

$$-u = \frac{1}{-(-50)}; u = -.02 \; ; \; nu = -.02$$

▶Refract ray B at surface #1:

$$n'u' = nu - \frac{y_1(n'-n)}{R_1} = (-.02) - 0cm\left(\frac{1.517-1}{14cm}\right)$$

$n'u' = -.02$; $u' = -.01318$

▶Transfer ray B from surface #1 to surface #2:

$$y_2 = y_1 + \frac{t'}{n'}n'u' = 0cm + \frac{3cm}{1.517}(-.02)$$

$y_2 = -.03955cm$

▶Refract ray B at surface #2:

$$n'u' = nu - \frac{y_2(n'-n)}{R_2} = (-.02) - (-.03955cm)\left(\frac{1-1.517}{-15cm}\right)$$ $\boxed{n'u' = -.01863\ ;\ u' = -.01863}$

▶Transfer ray B from surface #2 to surface #3:

$$y_3 = y_2 + \frac{t'}{n'}n'u' = -.03955cm + \frac{6cm}{1}(-.01863)$$ $\boxed{y_3 = -.15133cm}$

▶Refract ray B at surface #3:

$$n'u' = nu - \frac{y_3(n'-n)}{R_3} = (-.01863) - (-.15133cm)\left(\frac{1.923-1}{10cm}\right)$$ $\boxed{n'u' = -.00466\ ;\ u' = -.00242}$

▶Transfer ray B from surface #3 to surface #4:

$$y_4 = y_3 + \frac{t'}{n'}n'u' = -.15133cm + \frac{2cm}{1.923}(-.00466)$$ $\boxed{y_4 = -.15617cm}$

▶Refract ray B at surface #4:

$$n'u' = nu - \frac{y_4(n'-n)}{R_4} = (-.00466) - (-.15617cm)\left(\frac{1-1.923}{24cm}\right)$$ $\boxed{n'u' = -.01066\ ;\ u' = -.01066}$

▶Transfer ray B from surface #4 to the image (surface #5):
($t' = 6.048cm$ is found by first tracing ray "A")

$$y_5 = y_4 + \frac{t'}{n'}n'u' = (-.15617cm) + \frac{Z_4'cm}{1}(-.01066)$$

$$y_5 = (-.15617cm) + \frac{6.0485cm}{1}(-.01066)$$ $\boxed{y_5 = -.22064cm}$

$$-u_4' = \frac{ray\ A\ height\ at\ surface\ 4}{Z_4'} = \frac{2.740cm}{+6.0485cm}$$ ▶ $\boxed{u_4' = -.4530\ rad = -25.95°}$ ◀

Raytrace for Ray "A"—Correct numbers (to just 2 decimal places) for the following:

_____ (1) n_d (N-BK7) = 1.517
_____ (1) n_d (SF66) = 1.923

_____ (1) nu (or u) in space 1: nu = u = .10

_____ (1) nu (or u) in space 2: nu = -.02 (or u = -.02)
_____ (1) y_2 at surface 2: y_2 = 4.832 cm

_____ (1) nu (or u) in space 3: nu = u = -.252
_____ (1) y_3 at surface 3: y_3 = 3.320 cm

_____ (1) nu (or u) in space 4: nu = -.558 (or u = -.290)
_____ (1) y_4 at surface 4: y_4 = 2.740 cm

______ (1) a. What is the Z'_4 distance? $t' = Z'_4 = 6.049$ cm

Raytrace for Ray "B"—Correct answers (to just 2 decimal places) for the following:

nu (or u) in space 1: nu = u = -.02

nu (or u) in space 2: nu = -.02 (or u = -.01318)
y_2 at surface 2: y_2 = -.03955 cm

nu (or u) in space 3: nu = u = -.01863
y_3 at surface 3: y_3 = -.15133 cm

nu (or u) in space 4: nu = -.00466 (or u = -.00242)
y_4 at surface 4: y_4 = -.15617 cm

nu (or u) in space 5: nu = u = -.01066

height of the image, y_5: y_5 = -.22064 cm

b. What is the u'_4 angle? u'_4 = -.4530 radians = -25.95°

5.18 A glass rod is used to form an image of the Moon inside the glass (n = 1.5). The radius of curvature on the end of the glass rod is 20 cm. The Moon is 2160 miles in diameter and 240,000 miles from the Earth. Find the image diameter (cm) and its location inside the glass rod (cm).

This is an imaging problem, so use the Gaussian imaging equation!

$$\frac{n'}{z'} = \frac{n}{z} + \phi \quad \rightarrow \quad \frac{n'}{z'} = \frac{n}{z} + \frac{n'-n}{R} \quad \rightarrow \quad \frac{1.5}{z'} = \frac{1}{(-240{,}000 miles)(5280\, ft/mi)(12 in)(2.54 cm/in)} + \frac{1.5-1}{20cm}$$

$$\frac{1.5}{z'} = -2.589x10^{-11} + \frac{1.5-1}{20cm} = 0 + \frac{1.5-1}{20cm} \qquad \boxed{z' = 60\,cm}$$

To find the image diameter (height), calculate the transverse magnification:

$$M_t = \frac{z'/n'}{z/n} = \frac{60cm/1.5}{-240{,}000mi\left(\frac{5280ft}{mi}\right)(12in/ft)(2.54cm/in)}$$

Image <u>diameter</u> (so use the object diameter, not height):

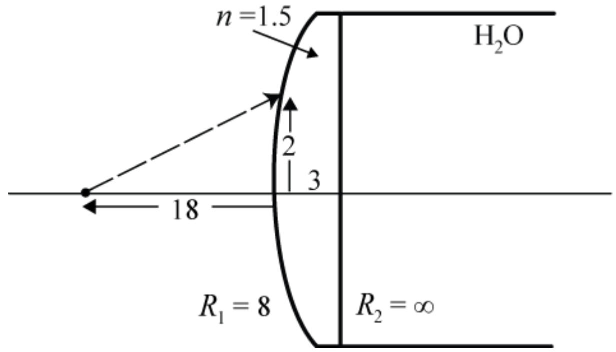
$$h' = h \cdot M_t = (2160mi)\cdot\left(\frac{5280ft}{mi}\right)(12in/ft)(2.54cm/in)\frac{60cm/1.5}{-240{,}000mi\left(\frac{5280ft}{mi}\right)(12in/ft)(2.54cm/in)}$$

$$h' = -0.36cm = -3.6mm$$

The minus sign indicates that the image is inverted.

_____ (4) Correct answer: z′ = 60 cm
_____ (4) Correct answer: h′ = -3.6mm

5.23 Using the refraction and transfer equations for paraxial ray tracing, find the ray angle in image space (water) shown below.



"Use an object distance of 25 (instead of 18)."

In the first (object) space: $nu = (1)\dfrac{2}{25} = .08$

Refract the ray at Surface #1:

$$n'u' = nu - y\frac{n'-n}{R}$$

$$n'u' = .08 - (2)\frac{1.5-1}{8} = -.045$$

Transfer the ray from Surface #1 to #2:

$$y_2 = y_1 + \left(\frac{t'}{n'}\right)n'u'$$

$$y_2 = 2 + \left(\frac{3}{1.5}\right)(-.045) = 1.91cm$$

Refract the ray at surface #2:

$$n'u' = nu - y\frac{n'-n}{R}$$

$$n'u' = -.045 - (1.91)\frac{1.333-1.5}{\infty} = -.045$$

$$u' = \frac{-.045}{n'} = \frac{-.045}{1.333} = -.03376$$

$$\boxed{u' = -.03376\,rad \;=\; -1.93°}$$

Correct answers for:

______	(2)	nu (or u) in space 1:	nu = u = .08
______	(2)	nu (or u) in space 2:	nu = -.045 (or u = -.03)
______	(2)	y_2 at surface 2:	y_2 = 1.91 cm
______	(4)	nu (or u) in image space 3:	nu = -.03376 (or u = -.02533)