Student Name: ______________________________________
OPTI 201R
Homework 9--Grading Sheet _____ **TOTAL POINTS** (out of 100 points)

7.01 Locate the cardinal points for the Newport lenses measured in lab:

For the KBX058 bi-convex lens (+75.6mm efl):

(1) Trace a paraxial ray (parallel to the optical axis in object space) to locate F*, P*, N* relative to V_2. (the lens is bi-convex, i.e. symmetrical, so this one ray also locates F, P, and N relative to V_1).

Approach using a Paraxial Raytrace:

- From the Newport catalog: n_g=n′=1.517 ; R_1 = -R_2 = 77.265mm ; t = 5.102mm
- From the Newport catalog: BFD = -FFD = 73.89mm ; δ = - δ* = 1.70mm
- The object is at -∞ so the ray in object space is parallel to the optical axis; i.e. u=0
- The ray height is arbitrary, so choose y_1 = 1mm
- Refract the ray at surface 1.
- Transfer the ray to surface 2.
- Refract the ray at surface 2.
- Transfer the ray until the ray height = 0 (to locate F^*).
- Transfer the ray from surface 2 backwards to a height equal to the incoming ray height (this distance is equal to δ*, and locates P* and N*)

- Refract the ray at Surface 1:

$$n'u' = nu - y_1\phi_1$$

$$n'u' = 0 - (.001)\frac{1.517-1}{.077265}$$

$$\boxed{n'u' = -.006691}$$

(This makes sense—the ray angle is negative, so the ray bends towards the axis as it should for a surface with positive power.)

- Transfer the ray to Surface 2:

$$y_2 = y_1 + \frac{t}{n'}n'u'$$

$$y_2 = (.001m) + \frac{.005102m}{1.517}(-.006691)$$

$$\boxed{y_2 = .000977496\ m}$$

- Refract the ray at Surface 2:

$$n''u'' = n'u' - y_2\phi_2$$

$$n''u'' = (1)u'' = (-.006691) - (.000977496)\frac{1-1.517}{-.077265}$$

$$\boxed{n''u'' = -.0132317}$$

- Transfer the ray until the ray height = 0 (to locate F^*):

$$y_{F^*} = y_2 + \frac{t}{n''} n''u'' = y_2 + \frac{BFD}{1}(n''u'')$$
$$0 = (.000977496) + BFD(-.0132317)$$

$\boxed{BFD = 73.875mm}$ i.e. the distance $\overline{V_2F^*} = +73.875mm$
So F* is located 73.875mm to the right of the second surface.
This is the Back Focal Distance, BFD (because we traced a ray in from an object at -∞).
According to the Newport catalog, BFD = 73.89mm!

- Transfer the ray backwards from Surface 2 until the ray height = y_1 (to locate P^*):

$$y_{P^*} = y_2 + \frac{t}{n''} n''u'' = y_2 + \frac{\delta^*}{1}(n''u'')$$
$$.001 = (.000977496) + \delta^*(-.0132317)$$
$$\boxed{\delta^* = -1.7008mm}$$
According to the Newport catalog, δ* = -1.70mm!
The lens is in air, so this also locates N*.

The lens is a biconvex lens, so we know that δ = -δ*. At this point, we have located all 6 cardinal points:

$$\overline{V_1F} = -73.87mm \;;\; \overline{V_1P} = 1.70mm \;;\; \overline{V_1N} = 1.70mm$$
$$\overline{V_2F^*} = +73.87mm \;;\; \overline{V_2P^*} = -1.70mm \;;\; \overline{V_2N^*} = -1.70mm$$

(2) Find the locations of the 6 cardinal points using a raytrace table.

(see the attached solution sheet)

KBX058 Lens

(all dimensions in mm):

Find BFD:

$n'u' = nu - y\phi$

$y_2 = y_1 + \left(\frac{t}{n}\right) nu$

YNU Method

	0	0–1	1	1–2	2	2–3	3	3–4	4	4–5	5	5–6	6	6–7	7	7–8	8	8–9	9
C			.012942		−.012942														
t				5.102		73.87													
n		1.0		1.517		1.0													
φ			.006691		.006691														
t/n				3.363216		73.87													
y	1		1		.977496		0												
nu		0		−.006691		−.013231													
u		0																	
y																			
nu																			
u																			
y																			
nu																			
u																			

KBX058 Lens

Find δ^*:

YNU Method

	0	0–1	1	1–2	2	2–3	3	3–4	4	4–5	5	5–6	6	6–7	7	7–8	8	8–9	9
c			.012942		-.012942														
t				5.102		-1.70													
n		1.0		1.517		1.0													
ϕ			.006691		.006691														
t/n						-1.70													
y	1		1		.977496		1												
nu				-.006691		-.013231													
u																			
y																			
nu																			
u																			
y																			
nu																			
u																			

(3) Gaussian-reduce the lens to a pair of principal planes to locate all 6 cardinal points:
locate F*, P*, N* relative to V_2 ; locate F, P, and N relative to V_1

Approach using Gaussian-reduction:

- From the Newport catalog: n_g=n′=1.517 ; R_1 = -R_2 = 77.265mm ; t = 5.102mm
- From the Newport catalog: BFD = -FFD = 73.89mm ; δ = - δ* = 1.70mm
- Gaussian-reduce the 2 surfaces to locate all 6 cardinal points:
  - calculate the power of each of the two surfaces.
  - calculate the total power of the two surfaces.
  - find the location of P (by calculating δ)
  - find the location of P* (by calculating δ*)
  - calculate the front and back focal lengths.
  - calculate the location of F
  - calculate the location of F*

$$\phi_1 = \frac{n'-1}{R_1} = \frac{1.517-1}{.077265m} \qquad \boxed{\phi_1 = +6.691D}$$

$$\phi_2 = \frac{1-n'}{R_2} = \frac{1-1.517}{-.077265m} \qquad \boxed{\phi_2 = +6.691D}$$

$$\phi_{12} = \phi_1 + \phi_2 - \frac{t}{n'}\phi_1\phi_2 = (2)(6.691) - \frac{.005102}{1.517}(6.691)^2 \qquad \boxed{\phi_{12} = 13.2314D}$$

$$\delta = \frac{t}{n'}\frac{\phi_2}{\phi_{12}}\cdot n = \frac{.005102}{1.517}\left(\frac{6.691}{13.2314}\right)(1) \qquad \boxed{\delta = +1.700mm}$$

$$\delta^* = -\frac{t}{n'}\frac{\phi_1}{\phi_{12}}\cdot n'' = \frac{-.005102}{1.517}\left(\frac{6.691}{13.2314}\right)(1) \qquad \boxed{\delta^* = -1.700mm}$$

$$f = \frac{-n}{\phi_{12}} = \frac{-1}{13.2314m^{-1}} \qquad \boxed{f = -75.57mm}$$

$$f^* = \frac{n''}{\phi_{12}} = \frac{1}{13.2314m^{-1}} \qquad \boxed{f^* = 75.57mm}$$

$$FFD = f + \delta = -75.57mm + 1.7mm \qquad \boxed{\overline{V_1F} = FFD = -73.87mm}$$

$$BFD = f^* + \delta^* = 75.57mm - 1.7mm \qquad \boxed{\overline{V_2F^*} = BFD = 73.87mm}$$

So the locations of the 6 cardinal points are:

$$\overline{V_1F} = -73.87mm \ ; \quad \overline{V_1P} = 1.70mm \ ; \quad \overline{V_1N} = 1.70mm$$

$$\overline{V_2F^*} = +73.87mm \ ; \quad \overline{V_2P^*} = -1.70mm \ ; \quad \overline{V_2N^*} = -1.70mm$$

Compare your results from the raytrace and the Gaussian reduction.

The results from the two approaches match almost exactly (there is a negligible 80 micron difference in the locations of P,N from V_1 *and P*,N* from* V_2 *in the two approaches).*

For the KPX088 plano-convex lens (+75.6mm efl):

(4) "Orient" the lens so the curved surface is to the left. Trace a paraxial ray (parallel to the optical axis in object space) to locate F*, P*, N* relative to V_2. "Rotate" the lens so the flat side is to the left. Trace another parallel ray to locate the cardinal points to the right of the lens, which are now F, P, and N relative to V_1.

Approach using a Paraxial Raytrace:

- From the Newport catalog: n_g=n′=1.517 ; R_1 = 39.070mm ; $R_2 = \infty$; t = 5.122mm
- From the Newport catalog: FFD = -75.6mm ; BFD 72.22mm ; δ = 0 ; δ* = -3.37mm
- The object is at -∞ so the ray in object space is parallel to the optical axis; i.e. u=0
- The ray height is arbitrary, so choose y_1 = 1mm
- Refract the ray at surface 1.
- Transfer the ray to surface 2.
- Refract the ray at surface 2.
- Transfer the ray until the ray height = 0 (to locate F^*).
- Transfer the ray from surface 2 backwards to a height equal to the incoming ray height (this distance is equal to δ*, and locates P* and N*)
- Turn the lens around and repeat this process, to locate F, P, and N.

<u>Curved surface to the left:</u>

- Refract the ray at Surface 1:

$$n'u' = nu - y_1\phi_1$$

$$n'u' = 0 - (.001)\frac{1.517-1}{.03907m}$$

$$\boxed{n'u' = -.013233}$$

(This makes sense—the ray angle is negative, so the ray bends towards the axis as it should for a surface with positive power.)

- Transfer the ray to Surface 2:

$$y_2 = y_1 + \frac{t}{n'}n'u'$$

$$y_2 = (.001m) + \frac{.005122m}{1.517}(-.013233)$$

$$\boxed{y_2 = .00095532\ m}$$

- Refract the ray at Surface 2:

$$n''u'' = n'u' - y_2\phi_2$$

$$n''u'' = (1)u'' = (-.013233) - (.00095532)(0)$$

$$\boxed{n''u'' = -.013233}$$

- Transfer the ray until the ray height = 0 (to locate F^*):

$$y_{F^*} = y_2 + \frac{t}{n''} n''u'' = y_2 + \frac{BFD}{1}(n''u'')$$
$$0 = (.00095532) + BFD(-.013233)$$

$$\boxed{BFD = 72.19\,mm} \quad \text{i.e. the distance } \overline{V_2F^*} = +72.19\,mm$$

So F* is located 72.19mm to the right of the second surface.
This is the Back Focal Distance, BFD (because we traced a ray in from an object at -∞).
According to the Newport catalog, BFD = 72.22mm!

- Transfer the ray backwards from Surface 2 until the ray height = y_1 (to locate P^*):

$$y_{P^*} = y_2 + \frac{t}{n''} n''u'' = y_2 + \frac{\delta^*}{1}(n''u'')$$
$$.001 = (.00095532) + \delta^*(-.013233)$$
$$\boxed{\delta^* = -3.376\,mm}$$

According to the Newport catalog, δ* = -3.37mm!
The lens is in air, so this also locates N*.

Turn the lens around to locate the front cardinal points:

Curved surface to the right:

- Refract the ray at Surface 1:

$$n'u' = nu - y_1\phi_1$$
$$n'u' = 0 - (.001)(0)$$
$$\boxed{n'u' = 0}$$

(This makes sense—the surface is planar and therefore has no power, so the ray does not bend.)

- Transfer the ray to Surface 2:

$$y_2 = y_1 + \frac{t}{n'} n'u'$$
$$y_2 = (.001m) + \frac{.005122m}{1.517}(0)$$
$$\boxed{y_2 = .001\ m} \quad \text{same ray height, obviously……}$$

- Refract the ray at Surface 2:

$$n''u'' = n'u' - y_2\phi_2$$
$$n''u'' = (1)u'' = (0) - (.001)\left(\frac{1-1.517}{-.03907}\right)$$
$$\boxed{n''u'' = -.0132326}$$

- Transfer the ray until the ray height = 0 (to locate F^*):

$$y_{F^*} = y_2 + \frac{t}{n''} n''u'' = y_2 + \frac{FFD}{1}(n''u'')$$
$$0 = (.001) + FFD(-.0132326)$$

$\boxed{FFD = -75.57\,mm}$ i.e. the distance $\overline{V_1F} = -75.57\,mm$

So F is located 75.57mm to the left of the first (curved) surface.

This is the Front Focal Distance, FFD (because we had turned the lens around, hence the change in sign).

According to the Newport catalog, FFD = -75.6mm!

- Transfer the ray backwards from Surface 2 until the ray height = y_1 (to locate P):

$$y_P = y_2 + \frac{t}{n''} n''u'' = y_2 + \frac{\delta}{1}(n''u'')$$
$$.001 = (.001) + \delta(-.0132326)$$
$$\boxed{\delta = 0\,mm}$$

According to the Newport catalog, δ = 0 mm (the front principal point is at the first vertex, just as you would expect for a plano-convex lens).

The lens is in air, so this also locates N.

At this point, we have located all 6 cardinal points:

$$\overline{V_1F} = -75.57\,mm \;\; ; \;\; \overline{V_1P} = 0\,mm \;\; ; \;\; \overline{V_1N} = 0\,mm$$
$$\overline{V_2F^*} = +72.19\,mm \;\; ; \;\; \overline{V_2P^*} = -3.376\,mm \;\; ; \;\; \overline{V_2N^*} = -3.376\,mm$$

(5) Find the locations of the 6 cardinal points using a raytrace table.

(see the attached solution sheet)

KPX088 Lens

(all dimensions in mm):

Find BFD:

YNU Method

	0	1	2	3	4	5	6	7	8	9
C		.025595	0							
t		5.122	72.19							
n	1.0	1.517	1.0							
φ		.013232	0							
t/n		3.376400								
y	1	1	.955323	0						
nu	0	-.013232	-.013232							
u	0									
y										
nu										
u										
y										
nu										
u										

KPX088 Lens ""

Find δ*

YNU Method

	0	1	2	3	4	5	6	7	8	9
C		.025595	0							
t		5.122	-3.37							
n	1.0	1.517	1.0							
φ		.013232	0							
t/n		3.376400								
y	1	1	.955323	1						
nu	0	-.013232	-.013232							
u										
y										
nu										
u										
y										
nu										
u										

KPX088 "D"

Find BFD:

YNU Method

	0	1	2	3	4	5	6	7	8	9
c		0	.025395							
t		5.122	75.57							
n	1.0	1.517	1.0							
ϕ		0	-.013232							
t/n		3.376400								
y	1	1	1	0						
nu	0	0	-.013232							
u	0									
y										
nu										
u										
y										
nu										
u										

KPX088 "D"

Find δ:

YNU Method

	0	1	2	3	4	5	6	7	8	9
C		0	.025595							
t		5.122	0							
n	1.0	1.517	1.0							
φ		0	-.013232							
t/n		3.376400								
y	1	1	1	1						
nu	0	0	-.013232							
u	0									
y										
nu										
u										
y										
nu										
u										

(6) Gaussian-reduce the lens to a pair of principal planes to locate all 6 cardinal points:
locate F*, P*, N* relative to V_2 ; locate F, P, and N relative to V_1

Approach using Gaussian-reduction:

- From the Newport catalog: n_g=n′=1.517 ; R_1 = 39.070mm ; R_2 = ∞ ; t = 5.122mm
- From the Newport catalog: FFD = -75.6mm ; BFD 72.22mm ; δ = 0 ; δ* = -3.37mm
- The object is at -∞ so the image will be located at F^* so find the location of F^*:
    - calculate the power of each of the two surfaces.
    - calculate the total power of the two surfaces.
    - find the location of P (by calculating δ)
    - find the location of P* (by calculating δ*)
    - calculate the front and back focal lengths.
    - calculate the location of F
    - calculate the location of F*

$$\phi_1 = \frac{n'-1}{R_1} = \frac{1.517-1}{.039070m} \qquad \boxed{\phi_1 = +13.23266\,D}$$

$$\phi_2 = \frac{1-n'}{R_2} = \frac{1-1.517}{\infty} \qquad \boxed{\phi_2 = 0\,D}$$

$$\phi_{12} = \phi_1 + \phi_2 - \frac{t}{n'}\phi_1\phi_2 = \phi_1 \qquad \boxed{\phi_{12} = 13.23266\,D}$$

$$\delta = \frac{t}{n'}\frac{\phi_2}{\phi_{12}}\cdot n = \frac{.005122}{1.517}\left(\frac{0}{13.2314}\right)(1) \qquad \boxed{\delta = 0\,mm}$$

$$\delta^* = -\frac{t}{n'}\frac{\phi_1}{\phi_{12}}\cdot n'' = \frac{-.005122}{1.517}\left(\frac{13.23266}{13.23266}\right)(1) \qquad \boxed{\delta^* = -3.376\,mm}$$

$$f = \frac{-n}{\phi_{12}} = \frac{-1}{13.23266\,m^{-1}} \qquad \boxed{f = -75.57\,mm}$$

$$f^* = \frac{n''}{\phi_{12}} = \frac{1}{13.23266\,m^{-1}} \qquad \boxed{f^* = 75.57\,mm}$$

$$FFD = f + \delta = -75.57\,mm + 0\,mm \qquad \boxed{\overline{V_1F} = FFD = -75.57\,mm}$$

$$BFD = f^* + \delta^* = 75.57\,mm - 3.376\,mm \qquad \boxed{\overline{V_2F^*} = BFD = 72.19\,mm}$$

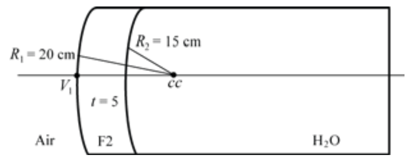
So the locations of the 6 cardinal points are:

$$\overline{V_1F} = -75.57\,mm \quad ; \quad \overline{V_1P} = 0\,mm \quad ; \quad \overline{V_1N} = 0\,mm$$
$$\overline{V_2F^*} = +72.19\,mm \quad ; \quad \overline{V_2P^*} = -3.376\,mm \quad ; \quad \overline{V_2N^*} = -3.376\,mm$$

Compare your results from the raytrace and the Gaussian reduction.

The results are the same as for the ray trace.

7.7 A thick lens has two concentric surfaces R_1 = 20 cm, R_2 = 15 cm, with a thickness of 5 cm. It separates air from water.



a. What is the power (diopters)?

First, look up the index of refraction of F2 glass, in d-light. From page 48 of the Schott glass data sheets: $n_d = 1.62004$

$$\phi_{12} = \phi_1 + \phi_2 - \frac{t'}{n'}\phi_1\phi_2$$

$$= \frac{1.62004-1}{.2m^{-1}} + \frac{1.33333-1.62004}{.15m^{-1}} - \frac{.05m}{1.62004}\left(\frac{1.62004-1}{.2m^{-1}}\right)\left(\frac{1.33333-1.62004}{.15m^{-1}}\right)$$

$$\phi_{12} = 1.3719\,D \quad \text{so} \quad f = \frac{-1}{1.3719\,m^{-1}} = -72.89\,cm \quad ; \quad f^* = \frac{-1.333}{1.3719\,m^{-1}} = -97.18\,cm$$

b. ***Make a sketch*** of the cardinal points relative to the front vertex.

First calculate the power of each surface, then calculate δ and δ^*:

$$\phi_1 = \frac{1.62004-1}{.2m^{-1}} = 3.1002\,D$$

$$\phi_2 = \frac{1.33333-1.62004}{.15m^{-1}} = -1.9114\,D$$

$$\delta = \frac{t'}{n'}\frac{\phi_1}{\phi_{12}}\cdot n = \frac{.05m}{1.62004}\frac{-1.9114}{1.3719}(1)$$

$$\delta^* = -\frac{t'}{n'}\frac{\phi_2}{\phi_{12}}\cdot n'' = \frac{-.05m}{1.62004}\frac{3.1002}{1.3719}(1.33333)$$

$$\delta = -4.3\,cm$$

$$\delta^* = -9.29\,cm$$

Also, calculate the separation of the principal-nodal points:

$$\overline{PN} = \overline{P^*N^*} = f + f^* = \frac{-n}{\phi_{12}} + \frac{n''}{\phi_{12}} = \frac{-1}{1.3719\,D} + \frac{1.33333}{1.3719\,D} = 24.29\,cm$$

So relative to V_1, the cardinal points are located:

$$\overline{V_1P} = \delta = -4.3\,cm$$

$$\overline{V_1N} = \overline{V_1P} + \overline{PN} = -4.3\,cm + 24.29\,cm = 19.99\,cm$$

$$\overline{V_1F} = \overline{V_1P} + f = -4.3\,cm - 72.89\,cm = -77.19\,cm$$

$$\overline{V_1P^*} = \overline{V_1V_2} + \delta^* = 5\,cm - 9.29\,cm = -4.29\,cm$$

$$\overline{V_1N^*} = \overline{V_1P^*} + \overline{P^*N^*} = -4.29\,cm + 24.29\,cm = 20\,cm$$

$$\overline{V_1F^*} = \overline{V_1P^*} + f^* = -4.29\,cm - 97.18\,cm = -101.47\,cm$$

SHOW THIS IN A SKETCH!

7.8 You have two lenses. Lens A is a negative meniscus and Lens B is a positive meniscus. The lenses have the prescriptions shown in Table 7.6.

(a) What is the optical power of each lens?

$$\phi_A = \phi_1 + \phi_2 - \frac{t}{n}\phi_1\phi_2 = \left(\frac{n-1}{R_{a1}}\right) + \left(\frac{1-n}{R_{a2}}\right) - \frac{t}{n}\left(\frac{n-1}{R_{a1}}\right)\left(\frac{1-n}{R_{a2}}\right)$$

$$\phi_A = \left(\frac{1.45-1}{.07m}\right) + \left(\frac{1-1.45}{.03m}\right) - \frac{.005m}{1.45}\left(\frac{1.45-1}{.07m}\right)\left(\frac{1-1.45}{.03m}\right) \qquad \boxed{\phi_A = -8.23D}$$

$$\phi_B = \phi_1 + \phi_2 - \frac{t}{n}\phi_1\phi_2 = \left(\frac{n-1}{R_{b1}}\right) + \left(\frac{1-n}{R_{b2}}\right) - \frac{t}{n}\left(\frac{n-1}{R_{b1}}\right)\left(\frac{1-n}{R_{b2}}\right)$$

$$\phi_B = \left(\frac{1.55-1}{.03m}\right) + \left(\frac{1-1.55}{.07m}\right) - \frac{.005m}{1.55}\left(\frac{1.55-1}{.03m}\right)\left(\frac{1-1.55}{.07m}\right) \qquad \boxed{\phi_B = 10.94D}$$

(b) Determine their cardinal points in air and sketch the lenses.

For Lens "A":

First calculate the power of each lens surface:

$$\phi_{A1} = \frac{1.45-1}{.07m} = 6.42857D \qquad \phi_{A2} = \frac{1-1.45}{.03m} = -15D$$

$$\delta_A = \left(\frac{t}{n}\right)\frac{\phi_{A2}}{\phi_A} = \left(\frac{.5cm}{1.45}\right)\left(\frac{-15D}{-8.23D}\right) \qquad \boxed{\delta_A = .628cm = 6.28mm}$$

$$\delta_A^* = \left(\frac{-t}{n}\right)\frac{\phi_{A1}}{\phi_A} = \left(\frac{-.5cm}{1.45}\right)\left(\frac{6.42857D}{-8.23D}\right) \qquad \boxed{\delta_A^* = .269cm = 2.69mm}$$

$$\phi_A = \frac{-n}{f} = -8.23D \quad \text{so} \quad f_A = \frac{-1}{-8.23m^{-1}} = .1215m = 121.5mm$$

$$\phi_A = \frac{n}{f^*} = -8.23D \quad \text{so} \quad f_A^* = \frac{1}{-8.23m^{-1}} = -.1215m = -121.5mm$$

Lens A has negative power, so as expected, the focal points are "reversed."

$$\overline{PN} = \overline{P^*N^*} = f_A + f_A^* = 0$$

The nodal points N and N* are located at their respective principal points (because the object space and image space indices are the same—i.e. each lens is in air).

The 6 cardinal points for Lens "A" are located at:

$$\overline{V_1P} = \delta_A = 6.28mm \qquad \overline{V_2P^*} = \delta_A^* = 2.69mm$$

$$\overline{V_1N} = \delta_A = 6.28mm \qquad \overline{V_2N^*} = \delta_A^* = 2.69mm$$

$$\overline{V_1F} = \delta_A + f_A = 127.8mm \qquad \overline{V_2F^*} = \delta_A^* + f_A^* = -118.8mm$$

For Lens "B":

First calculate the power of each lens surface:

$$\phi_{B1} = \frac{1.55-1}{.03m} = 18.33333D \qquad \phi_{B2} = \frac{1-1.55}{.07m} = -7.85714D$$

$$\delta_B = \left(\frac{t}{n}\right)\frac{\phi_{B2}}{\phi_B} = \left(\frac{.5cm}{1.55}\right)\left(\frac{-7.85714D}{10.94D}\right) \qquad \boxed{\delta_B = -.232cm = -2.32mm}$$

$$\delta_B{}^* = \left(\frac{-t}{n}\right)\frac{\phi_{B1}}{\phi_B} = \left(\frac{-.5cm}{1.55}\right)\left(\frac{18.33333D}{10.94D}\right) \qquad \boxed{\delta_B = -.541cm = -5.41mm}$$

$$\phi_B = \frac{-n}{f_B} = 10.94D \text{ so } f_B = \frac{-1}{10.94m^{-1}} = -.0914m = -91.4mm$$

$$\phi_B = \frac{n}{f_B^*} = 10.94D \text{ so } f_B^* = \frac{1}{10.94m^{-1}} = .0914m = 91.4mm$$

Lens A has positive power, so as expected, the focal points are "<u>not</u> reversed."

$$\overline{PN} = \overline{P^*N^*} = f_B + f_B^* = 0$$

The nodal points N and N* are located at their respective principal points (because the object space and image space indices are the same—i.e. each lens is in air).

The 6 cardinal points for Lens "B" are located at:

$$\overline{V_1P} = \delta_B = -2.32mm \qquad \overline{V_2P^*} = \delta_B{}^* = -5.41mm$$

$$\overline{V_1N} = \delta_B = -2.32mm \qquad \overline{V_2N^*} = \delta_B{}^* = -5.41mm$$

$$\overline{V_1F} = \delta_B + f_B = -93.72mm \qquad \overline{V_2F^*} = \delta_B^* + f_B^* = 85.99mm$$

(c) Determine the optical power of an in-contact combination of these lenses in the following two cases:
(1) lens A to lens B
(2) lens B to lens A

Solution strategies:

- Gaussian-reduce the 3 surfaces ….. =*OR*=
- Recall that eqn. (7.51) IS the result for Gaussian-reducing 3 surfaces, so just use that ….. =*OR*=
- Raytrace the lenses to locate F^*, then backtrace the image-space ray to locate P^*. Calculate f^*.

What you can NOT do is to treat the surfaces as thin lenses and just add the 3 powers…

(1) Lens A to Lens B

- Gaussian-reduce the 3 surfaces:

$$\phi_1 = \phi_{A1} = \frac{1.45-1}{.07m} = 6.42857D$$

$$\phi_2 = \phi_{A2/B1} = \frac{n_B - n_A}{.03m} = \frac{1.55-1.45}{.03m} = 3.33333D$$

$$\phi_3 = \phi_{B2} = \frac{1-1.55}{.07m} = -7.85714D$$

$$\varphi_{12} = \varphi_1 + \varphi_2 - \frac{t_A}{n_A}\varphi_1\varphi_2 = \left(6.42857m^{-1}\right) + \left(3.33333m^{-1}\right) - \frac{.005m}{1.45}\left(6.42857m^{-1}\right)\left(3.33333m^{-1}\right)$$

$$\varphi_{12} = 9.68801D$$

$$\delta_{12}^* = \left(\frac{-t_A}{n_A}\right)\frac{\phi_1}{\phi_{12}} n_B = \left(\frac{-.5cm}{1.45}\right)\left(\frac{6.42857D}{9.68801D}\right)(1.55) = -.354661cm$$

$$t_{12,3} = t_3 - \delta_{12}^* = 0.5cm - (-.3546cm) = .8547cm$$

$$\varphi_{12,3} = \varphi_{12} + \varphi_3 - \frac{t_{12,3}}{n_B}\varphi_{12}\varphi_3 = \left(9.68801m^{-1}\right) + \left(-7.85714m^{-1}\right) - \frac{.008547m}{1.55}\left(9.68801m^{-1}\right)\left(-7.85714m^{-1}\right)$$

$$\boxed{\varphi_{12,3} = 2.251D}$$

- Use eqn. (7.51):

$$\varphi_{12,3} = \phi_1 + \phi_2 + \phi_3 - \frac{t_A\phi_1}{n_A}(\phi_2 + \phi_3) - \frac{t_B\phi_3}{n_B}(\phi_1 + \phi_2) + \frac{t_A t_B}{n_A n_B}\phi_1\phi_2\phi_3$$

$$\varphi_{12,3} = (6.42) + (3.33) + (-7.85) - \frac{(.005m)6.42}{1.45}(3.33 - 7.85) - \frac{(.005m)(-7.85)}{1.55}(6.42 + 3.33) + \frac{(.005m)^2}{(1.45)(1.55)}(6.42)(3.33)(-7.85)$$

$$\boxed{\varphi_{12,3} = 2.251D}$$

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(1) Lens B to Lens A

- Gaussian-reduce the 3 surfaces:

$$\phi_1 = \phi_{B1} = \frac{1.55-1}{.03m} = 18.33333D$$

$$\phi_2 = \phi_{B2/A1} = \frac{n_A - n_B}{.07m} = \frac{1.45-1.55}{.07m} = -1.42857D$$

$$\phi_3 = \phi_{A2} = \frac{1-1.45}{.03m} = -15D$$

$$\varphi_{12} = \varphi_1 + \varphi_2 - \frac{t_B}{n_B}\varphi_1\varphi_2 = \left(18.33333m^{-1}\right) + \left(-1.42857m^{-1}\right) - \frac{.005m}{1.55}\left(18.33333m^{-1}\right)\left(-1.42857m^{-1}\right)$$

$$\varphi_{12} = 16.98924D$$

$$\delta_{12}^* = \left(\frac{-t_B}{n_B}\right)\frac{\phi_1}{\phi_{12}} n_A = \left(\frac{-.5cm}{1.55}\right)\left(\frac{18.33333D}{16.98924D}\right)(1.45) = -.50477cm$$

$$t_{12,3} = t_3 - \delta_{12}^* = 0.5cm - (-.50477cm) = 1.00475cm$$

$$\varphi_{12,3} = \varphi_{12} + \varphi_3 - \frac{t_{12,3}}{n_A}\varphi_{12}\varphi_3 = \left(16.98924m^{-1}\right) + \left(-15m^{-1}\right) - \frac{1.00475m}{1.45}\left(16.98924m^{-1}\right)\left(-15m^{-1}\right)$$

$$\boxed{\varphi_{12,3} = 3.755D}$$

- Use eqn. (7.51):

$$\varphi_{12,3} = \phi_1 + \phi_2 + \phi_3 - \frac{t_B\phi_1}{n_B}(\phi_2 + \phi_3) - \frac{t_A\phi_3}{n_A}(\phi_1 + \phi_2) + \frac{t_B t_A}{n_B n_A}\phi_1\phi_2\phi_3$$

$$\varphi_{12,3} = (18.33) + (-1.43) + (-15) - \frac{(.005m)}{1.45}(3.33 - 7.85) - \frac{(.005m)(-7.85)}{1.55}(6.42 + 3.33) + \frac{(.005m)^2}{(1.45)(1.55)}(6.42)(3.33)(-7.85)$$

$$\boxed{\varphi_{12,3} = 3.755D}$$

7.11 A thick lens made of 755276.479 glass, R_1 = 4 cm, and R_2 = -2 cm, is 3 cm thick. This lens is placed at the end of a tank containing a transparent liquid with a refractive index of 1.42. R_2 is in contact with the liquid.

(a) What is the front focal length? What is the back focal length?
(b) What is the distance from the vertices to the focal points?
(c) What is the distance from the vertices to the principal points?
(d) What is the distance from the vertices to the nodal points?

***** In other words, where are the cardinal points located?!!

$$\phi_1 = \frac{n' - n}{R_1} = \frac{1.755 - 1}{.04m} = 18.875D$$

$$\phi_2 = \frac{n'' - n'}{R_2} = \frac{1.42 - 1.755}{-.02m} = 16.75D$$

$$\varphi_{12} = \varphi_1 + \varphi_2 - \frac{t}{n}\varphi_1\varphi_2 = \left(18.875m^{-1}\right) + \left(16.75m^{-1}\right) - \frac{.003m}{1.755}\left(18.875m^{-1}\right)\left(16.75m^{-1}\right)$$

$$\varphi_{12} = 30.2206D$$

$$\delta_{12} = \left(\frac{t}{n'}\right)\frac{\phi_2}{\phi_{12}} n = \left(\frac{3cm}{1.755}\right)\left(\frac{16.75D}{30.2206D}\right)(1) = .94745cm$$

$$\delta_{12}^* = \left(\frac{-t}{n'}\right)\frac{\phi_1}{\phi_{12}} n'' = \left(\frac{-3cm}{1.755}\right)\left(\frac{18.875D}{30.2206D}\right)(1.42) = -1.51606cm$$

(a) To calculate the front focal length:

$$\varphi_{12} = 30.2206m^{-1} = \frac{-n}{f} = \frac{-1}{f} \qquad \boxed{f = -3.31cm}$$

To calculate the back focal length, don't forget that the image-space index = 1.42:

$$\varphi_{12} = 30.2206m^{-1} = \frac{n''}{f^*} = \frac{1.42}{f^*} \qquad \boxed{f^* = 4.70cm}$$

(b) The distance from the front vertex to the front focal point is $\overline{V_1F} = f + \delta_{12}$

$$\overline{V_1F} = f + \delta_{12} = -3.31cm + .94745cm \qquad \boxed{\overline{V_1F} = -2.36cm}$$

The distance from the back vertex to the back focal point is $\overline{V_2F^*} = f^* + \delta_{12}^*$

$$\overline{V_2F^*} = f^* + \delta_{12}^* = 4.70cm - 1.51606cm \qquad \boxed{\overline{V_2F^*} = 3.18cm}$$

(c) The distance from the front vertex to the front principal point is δ_{12}: $\boxed{\delta_{12} = .947cm}$

The distance from the back vertex to the back principal point is δ_{12}^* : $\boxed{\delta_{12}^* = -1.52cm}$

(d) The distance from the front vertex to the front nodal point is :

$$\overline{V_1N} = \overline{V_1P} + \overline{PN} = \delta_{12} + \left(f + f^*\right) \qquad \boxed{\overline{V_1N} = 2.34cm}$$

The distance from the back vertex to the back nodal point is :

$$\overline{V_2N^*} = \overline{V_2P^*} + \overline{P^*N^*} = \delta_{12}^* + \left(f + f^*\right) \qquad \boxed{\overline{V_2N^*} = -.13cm}$$