OPTI 507 - Solid-State Optics
HW 3 Solutions

Jason M. Auxier

September 9, 2004

Homework 3 consists of problem 3.1 in the text: *Introduction to Semiconductor Optics* by Peyghambarian *et al* [1].

1 Problem 3.1 - 10 Points

Compute the reflectance $R(\omega)$ of an electromagnetic wave in vacuum with complex dielectric constant $\epsilon(\omega) = \epsilon'(\omega) + i\epsilon''(\omega)$, and express $R(\omega)$ in terms of the refractive index and absorption coefficient. Show that when $n(\omega) \to 0$, the reflection approaches 1.

1.1 Solution

From the Fresnel equations, the reflectance at normal incidence at an interface (air and index n) is given by [2]

$$R(\omega) = \left| \frac{n_c(\omega) - 1}{n_c(\omega) + 1} \right|^2, \tag{1}$$

where $n_c(\omega)$ is the complex index of refraction. Now, using Eqs. (3.18), (3.21), and (3.28) in the text [1], you can write $n_c(\omega)$ as

$$n_c(\omega) = n + i\frac{\alpha c}{2\omega}. \tag{2}$$

Putting Eq. 2 into Eq. 1, you can write the reflectance as

$$\boxed{R(\omega) = \frac{(n(\omega) - 1)^2 + \left(\frac{\alpha c}{2\omega}\right)^2}{(n(\omega) + 1)^2 + \left(\frac{\alpha c}{2\omega}\right)^2}.} \tag{3}$$

The index of refraction and the absorption coefficient are given by Eqs. (3.18) and (3.28) in the text [1], respectively:

$$n(\omega) = \sqrt{\frac{\epsilon' + \sqrt{(\epsilon')^2 + (\epsilon'')^2}}{2}} \tag{4}$$

and

$$\alpha(\omega) = 2\kappa(\omega) = \frac{\omega}{cn(\omega)}\epsilon''(\omega). \tag{5}$$

So, if $\epsilon''(\omega) \neq 0$, then as $n(\omega) \to 0$, $\alpha(\omega) \to \infty$. Thus, using Eqs. 4 and 5, you can write Eqs. 3 as

$$R(\omega) = \lim_{n\to 0}\left[\frac{(n-1)^2 + \left(\frac{\epsilon''}{2n}\right)^2}{(n+1)^2 + \left(\frac{\epsilon''}{2n}\right)^2}\right]. \tag{6}$$

Using L'Hospital's rule,

$$R(\omega) = \lim_{n\to 0}\left[\frac{2(n-1) - 2\left(\frac{\epsilon''}{2}\right)^2 n^{-3}}{2(n+1) - 2\left(\frac{\epsilon''}{2}\right)^2 n^{-3}}\right]. \tag{7}$$

Using L'Hospital's rule again,

$$R(\omega) = \lim_{n\to 0}\left[\frac{2 + 6\left(\frac{\epsilon''}{2}\right)^2 n^{-4}}{2 + 6\left(\frac{\epsilon''}{2}\right)^2 n^{-4}}\right]. \tag{8}$$

Simplifying,

$$\boxed{R(\omega) = 1.} \tag{9}$$

References

[1] Nasser N. Peyghambarian, Stephan W. Koch, and Andre Mysyrowicz. *Introduction to Semiconductor Optics*. Prentice Hall, Englewood Cliffs, NJ, 1993.

[2] Eugene Hecht. *Optics*. Addison-Westley, Reading, MA, 3rd ed., pp. 109-121, 1993.