

Figures of Merit for Optical Systems

Optical performance

- Geometric image size
 - Rms image diameter
 - FWHM
- Fractional encircled energy
- MTF at particular spatial frequencies
- RMSWE (root mean square wavefront error)
- Beam divergence
- Distortion
- Boresight

Two regimes for imaging systems

1. Geometric limit

Use simple ray trace to determine image quality

Rms spot size is most common FOM

Valid for wavefront errors $> 1 \lambda$

2. Near Diffraction limit

Must take the wave nature (interference and diffraction) into account.

Valid for wavefront errors $< \lambda/4$

Rms wavefront error is most common FOM

Aberrations - definitions

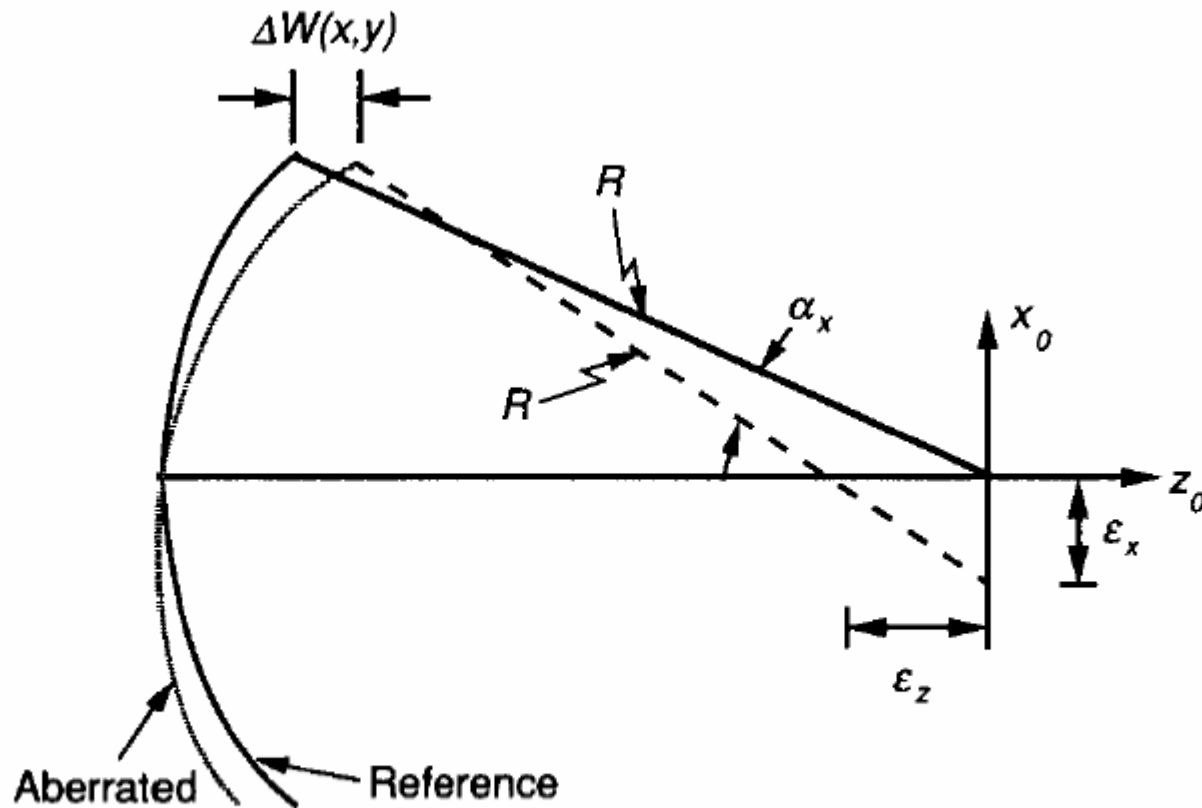
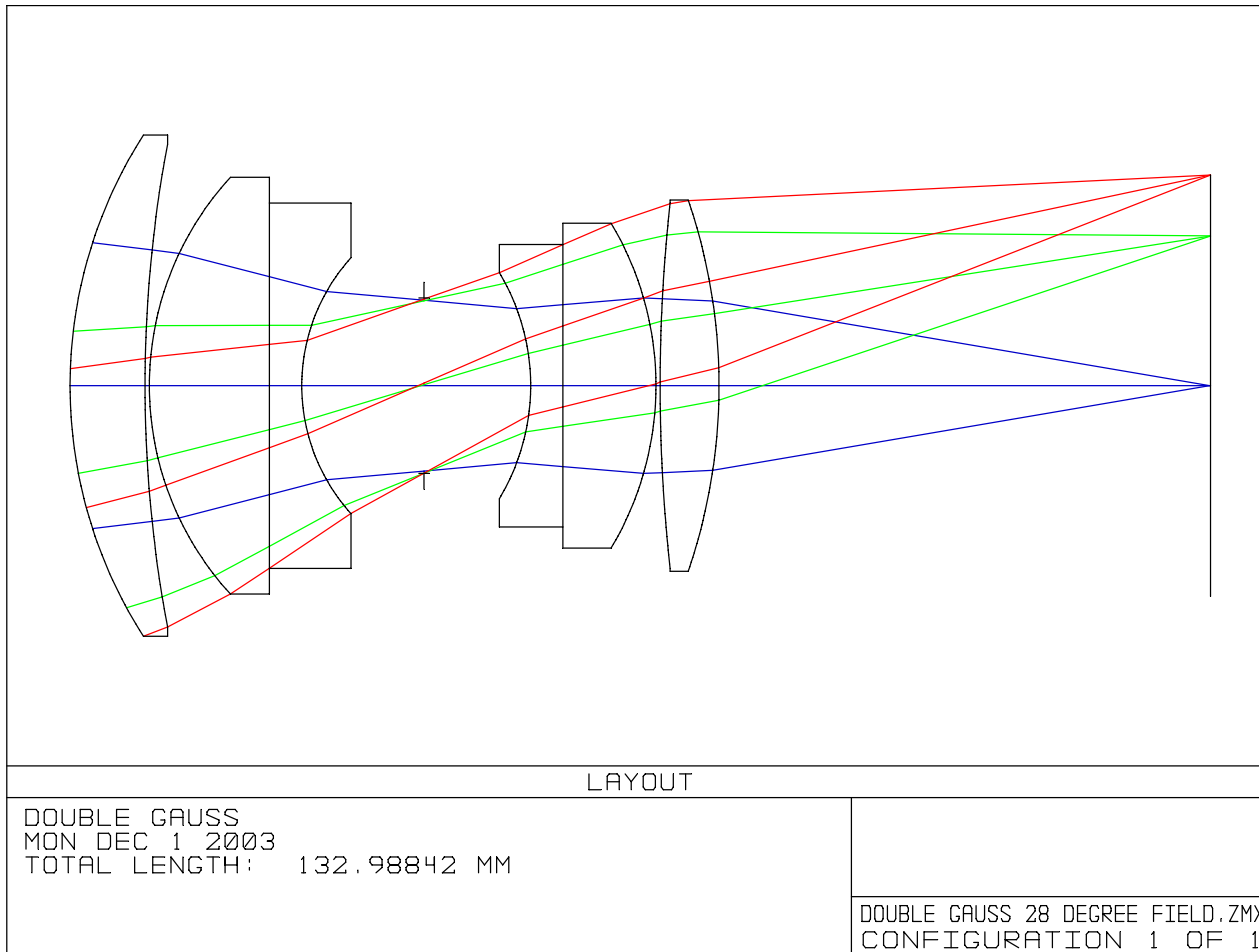
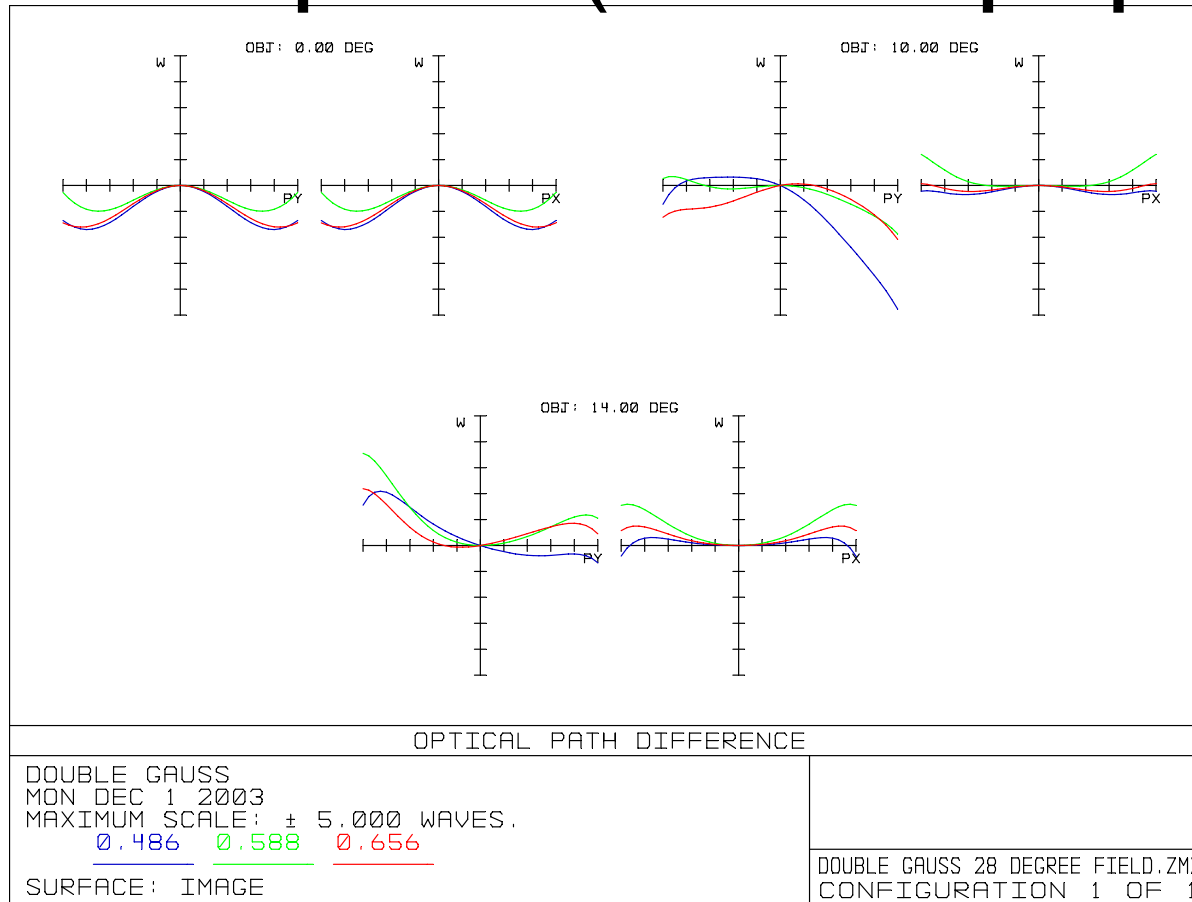


FIG. 15. Transverse and longitudinal aberrations. $\Delta W(x, y)$ = distance between reference wavefront and aberrated wavefront, ϵ_x = transverse or lateral aberration, ϵ_z = longitudinal aberration.

Example Lens layout



OPD plots (ΔW vs pupil)



$$\Delta W = \text{OPD} \gg 1 \lambda$$

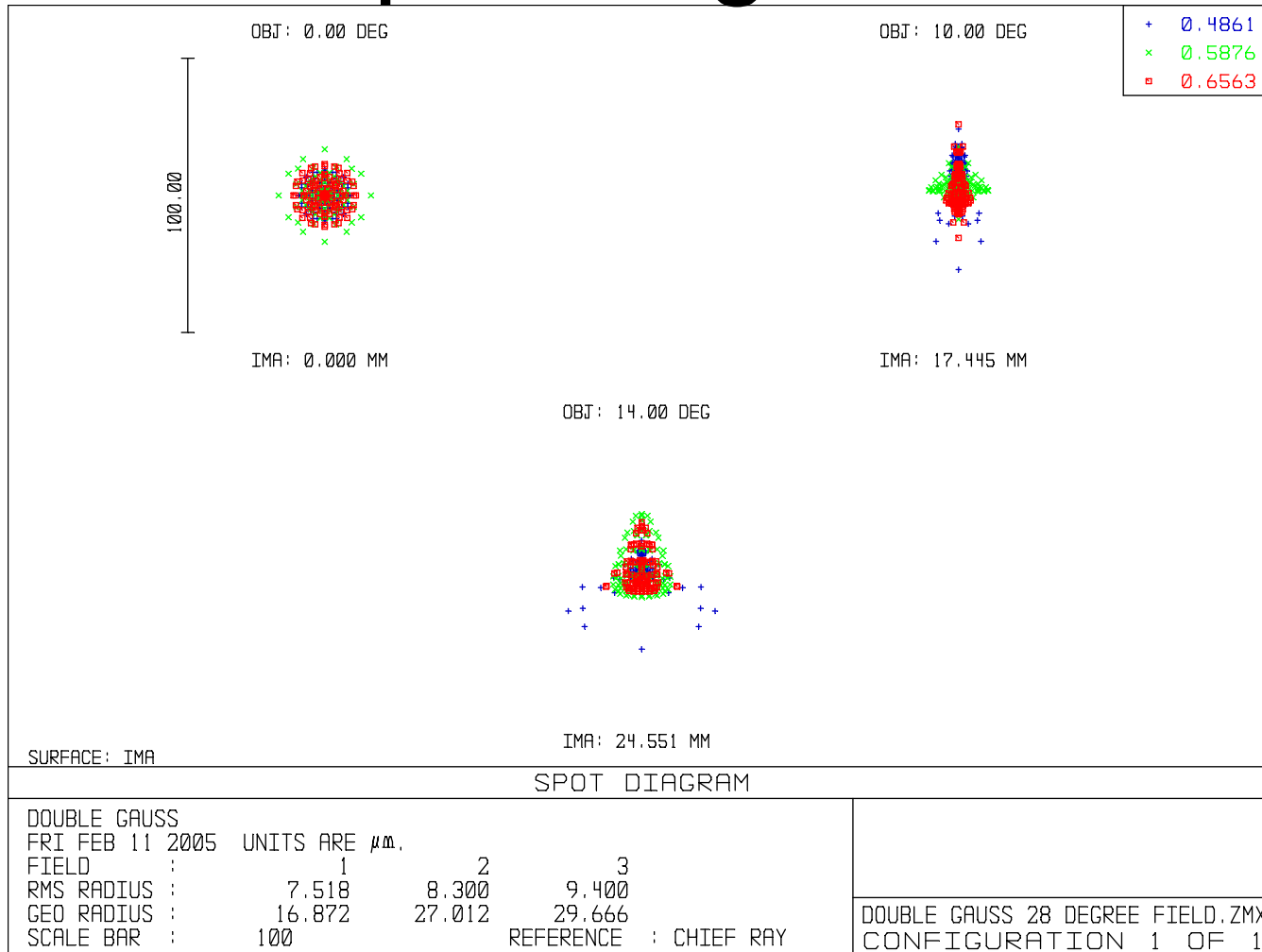
This system is in the geometric limit

J. H. Burge *Practical Optics* OPD (Optical Path Difference) is not a useful metric for this system

Image quality – point sharpness

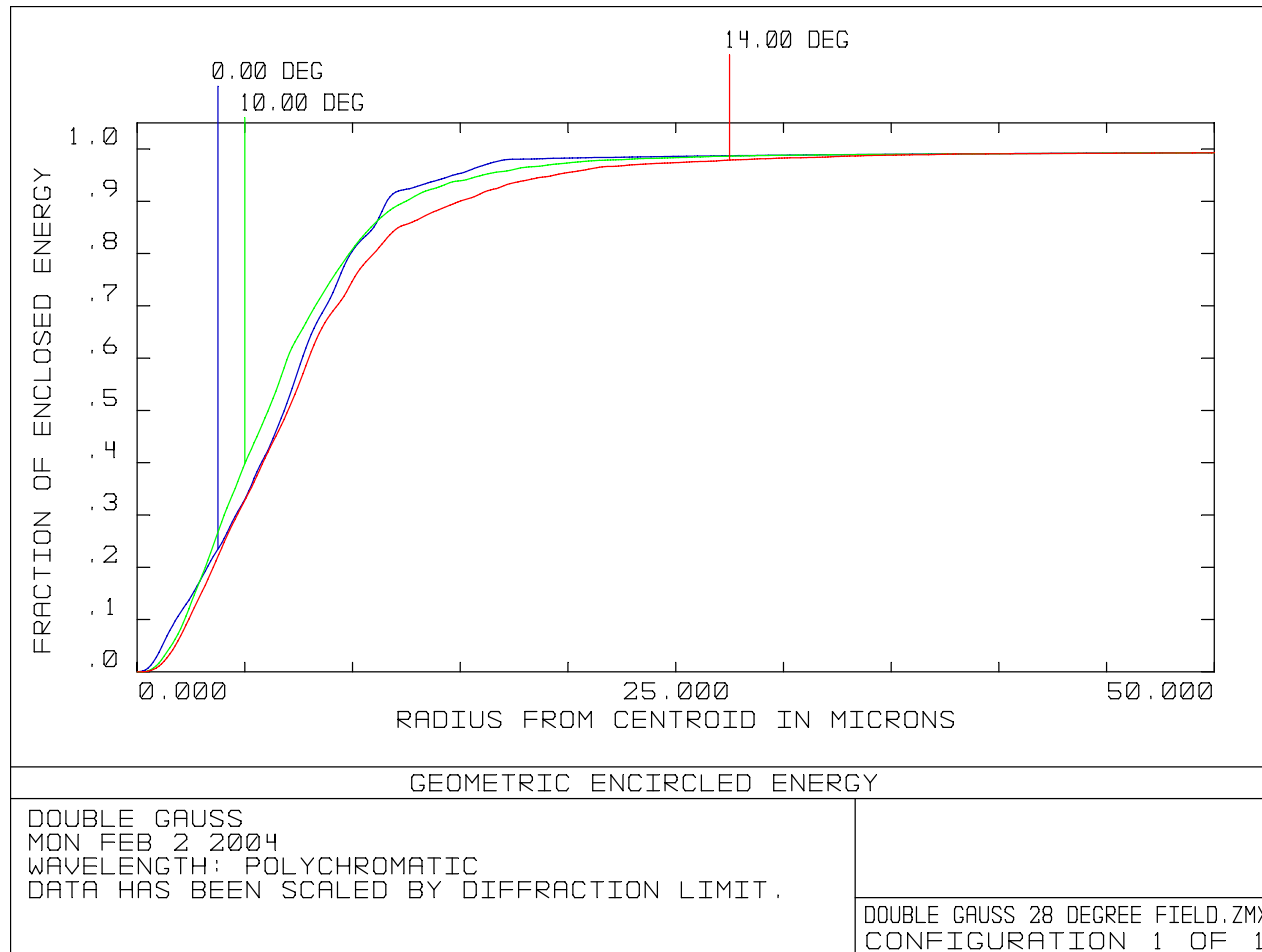
- Look at the image of points
- In the geometric limit:
 - rms diameter or radius (half-diameter)
 - Easily calculated using raytrace programs by tracing a bunch of rays: This only makes sense for geometric limit
 - FWHM
 - 80% encircled energy

Spot diagrams



In software, send a bunch of rays through the system and see where they intersect the image plane

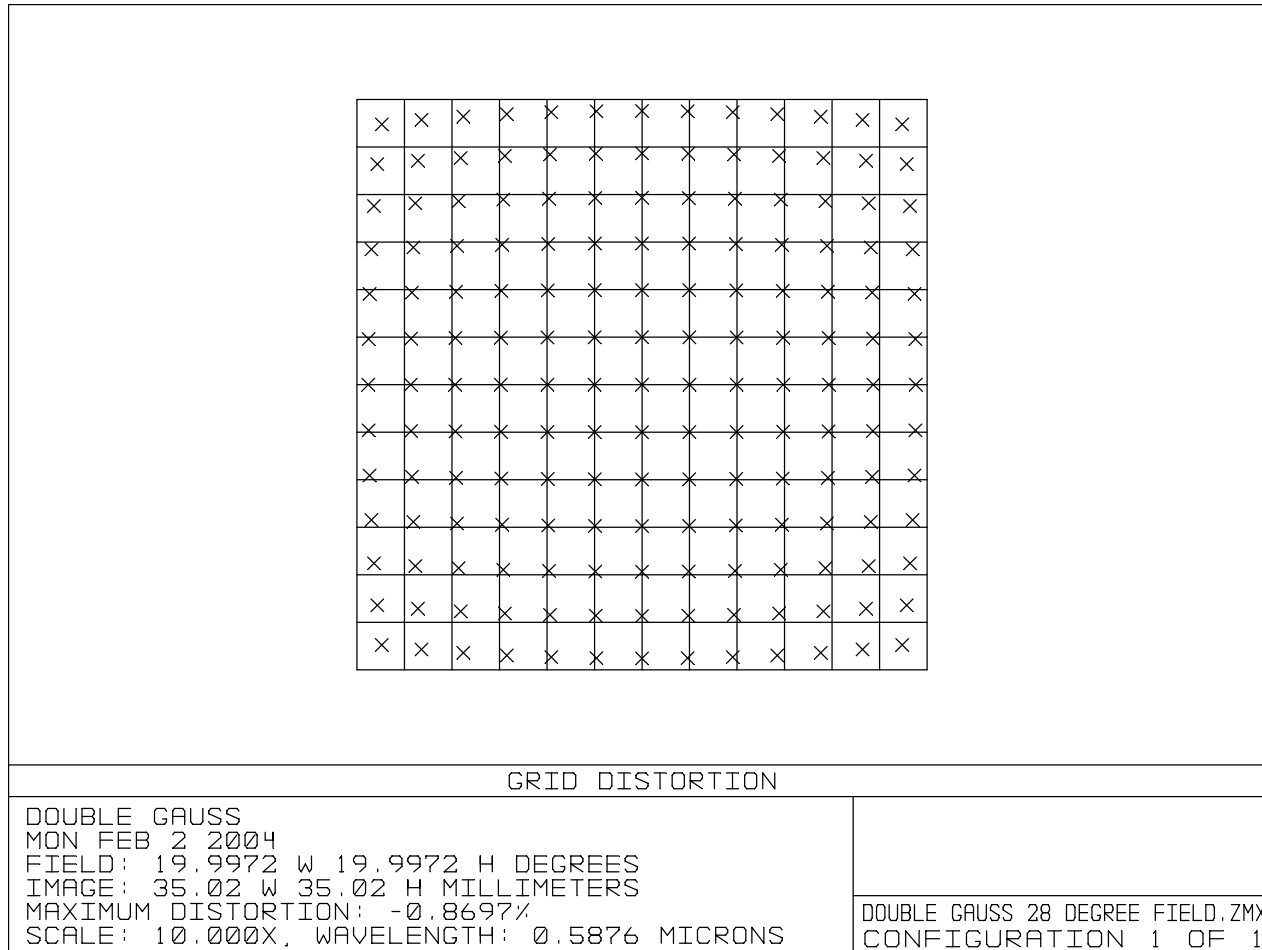
Geometric encircled energy EE



80% EE is often used as a threshold

Distortion

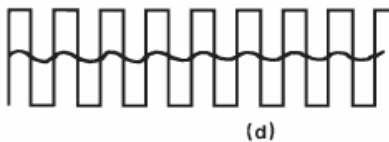
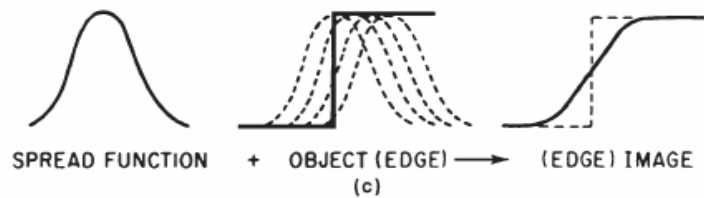
Mapping error for image, measured in %, $\mu\text{m}/\text{mm}$, or simply μm



Modulation Transfer Function

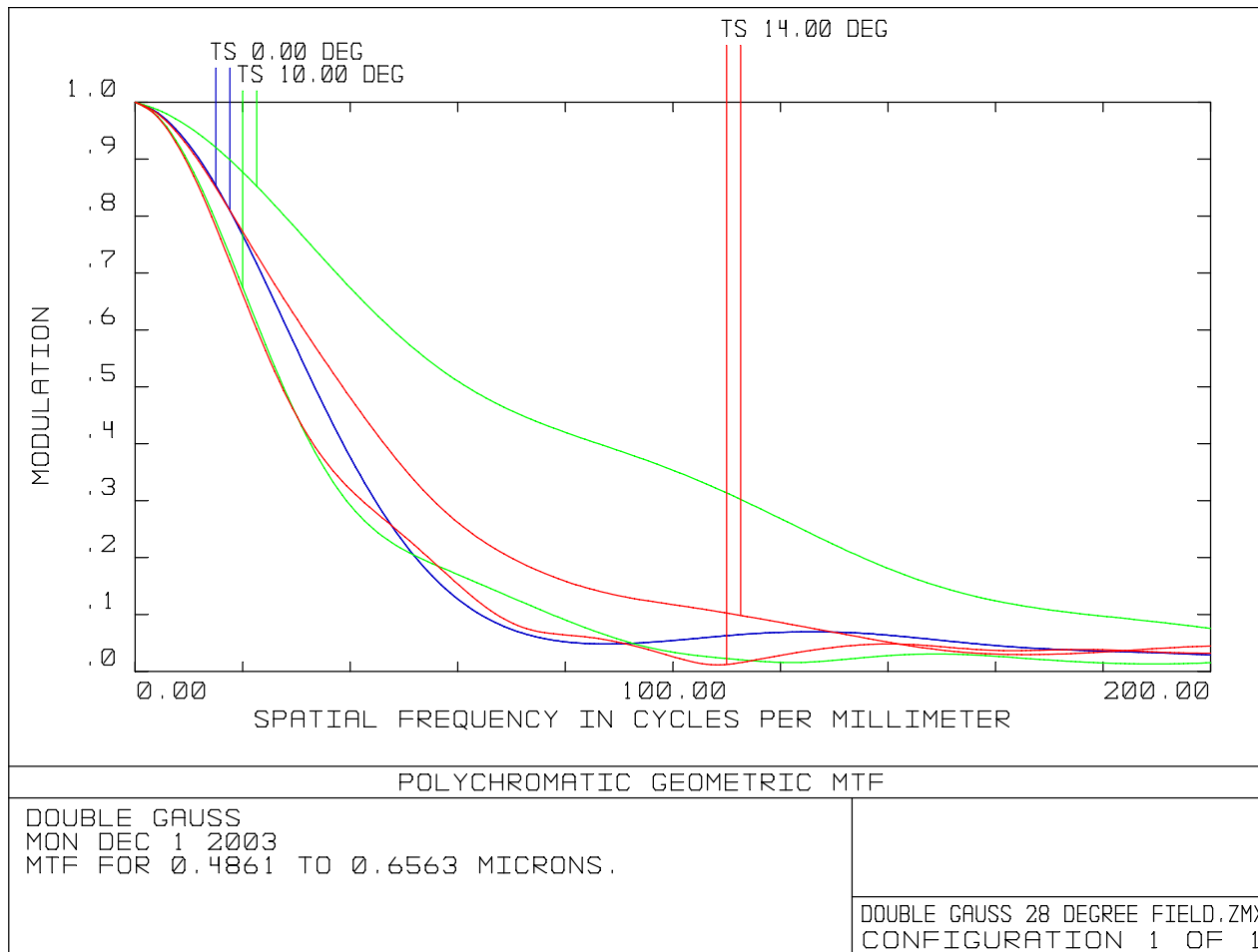
- Rather than using the blur size for image points – use the contrast reduction for high-frequency (small scale) features.
- MTF is plot of contrast (or modulation) vs. spatial frequency
- Has nice linear properties – system MTF = product of MTF for subsystems.

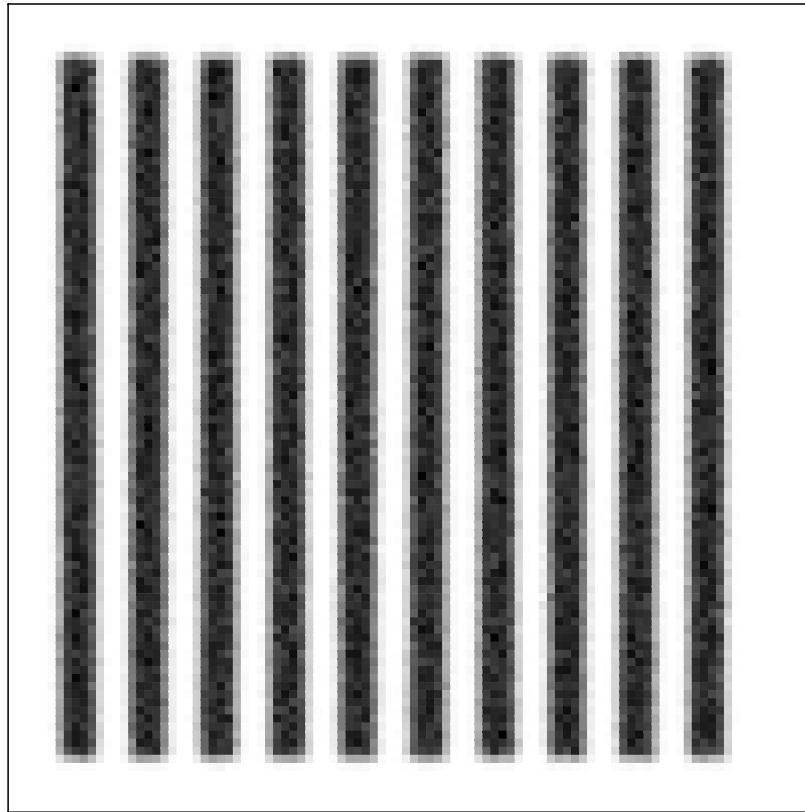
Definition of Modulation



$$\text{Modulation} = \frac{\text{max.} - \text{min.}}{\text{max.} + \text{min.}}$$

Modulation Transfer Function





3.1494
 2.8344
 2.5195
 2.2046
 1.8896
 1.5747
 1.2597
 0.9448
 0.6299
 0.3149
 0.0000

IMAGE DIAGRAM

DOUBLE GAUSS
 MON DEC 1 2003
 IMAGE WIDTH = 1.0000 MILLIMETERS, 100 X 100 PIXELS
 FIELD POSITION: 0.00 DEG
 PERCENT EFFICIENCY: 100.000%, 1.000E+000 WATTS
 SURFACE: 12. UNITS ARE WATTS PER MILLIMETERS SQUARED.

DOUBLE GAUSS 28 DEGREE FIELD.ZMX
 CONFIGURATION 1 OF 1

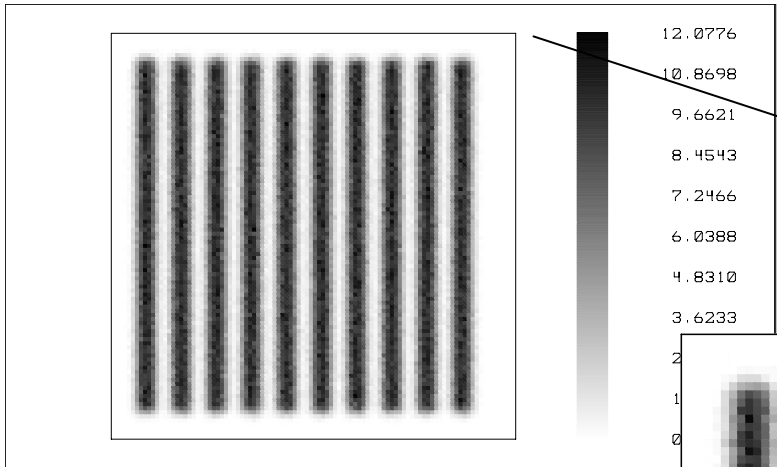
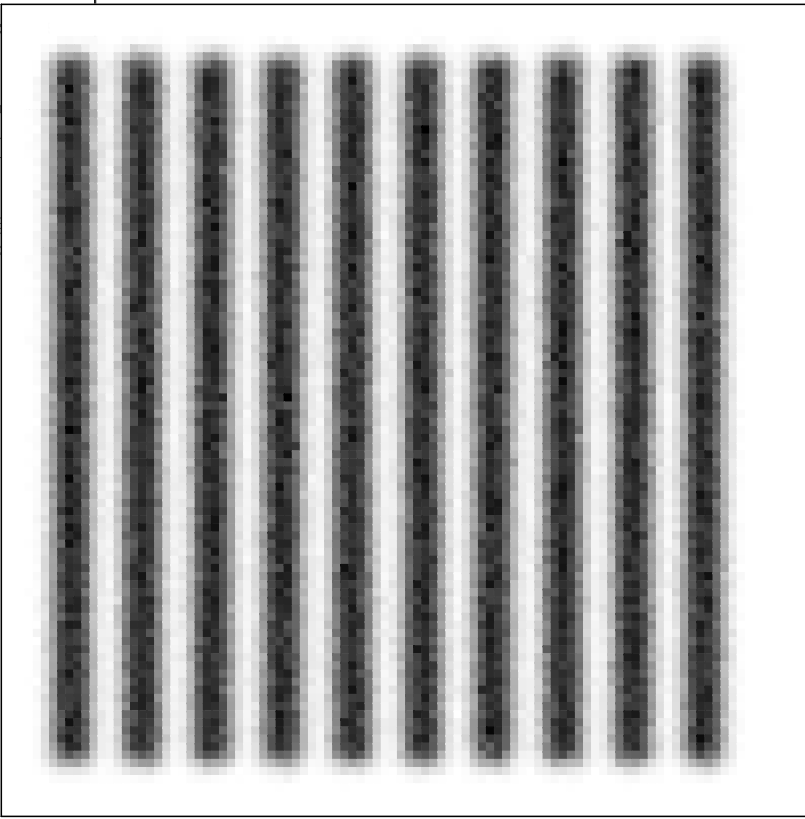
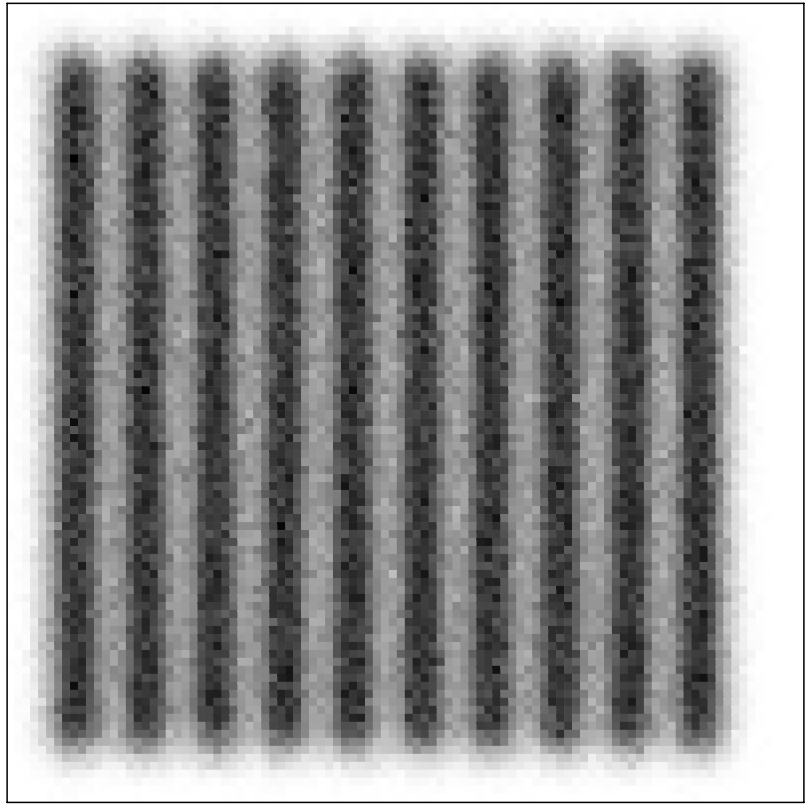
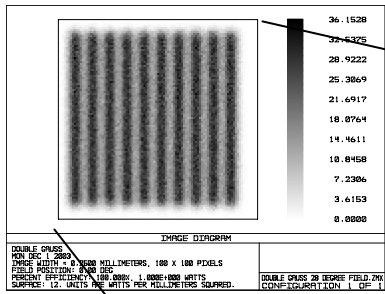


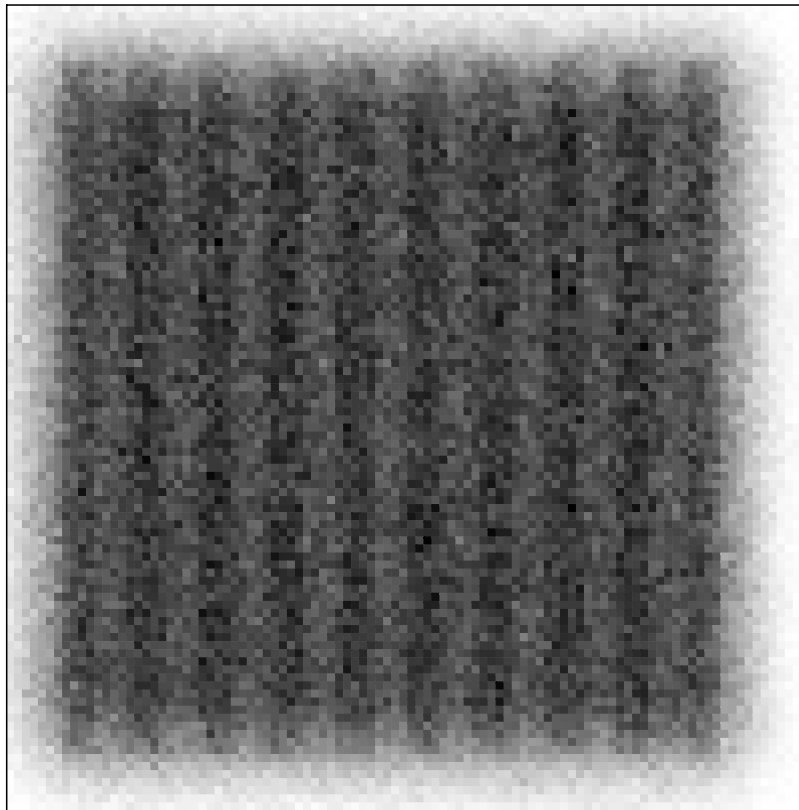
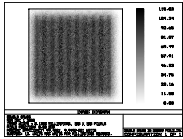
IMAGE DIAGRAM

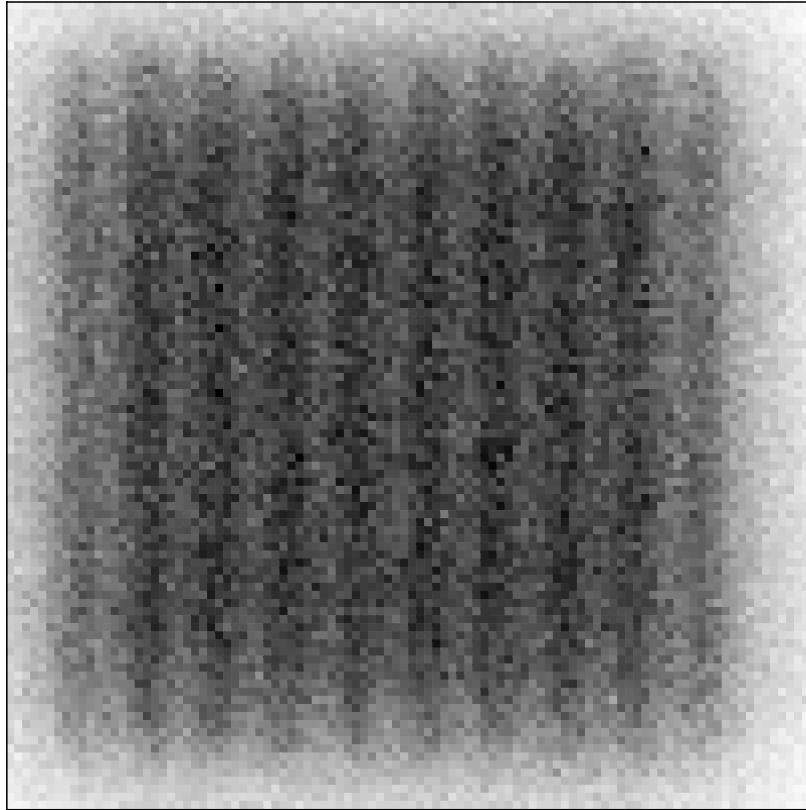
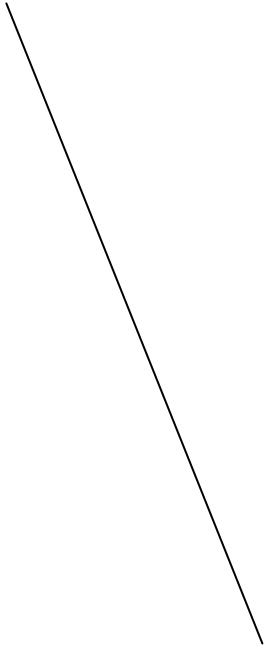
DOUBLE GAUSS
 MON DEC 1 2003
 IMAGE WIDTH = 0.5000 MILLIMETERS, 100 X 100 PIXELS
 FIELD POSITION: 0.00 DEG
 PERCENT EFFICIENCY: 100.000%, 1.000E+000 WATTS
 SURFACE: 12. UNITS ARE WATTS PER MILLIMETERS SQUARED.

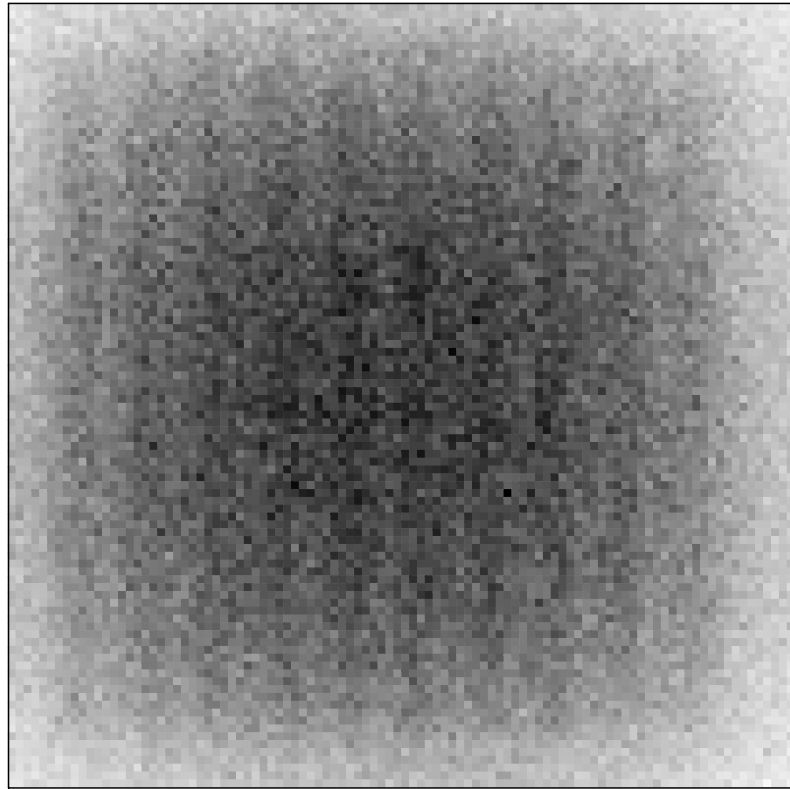
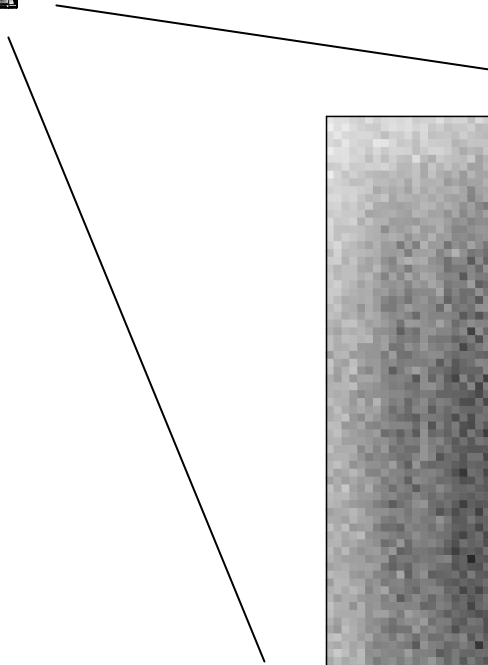
DOUBLE GAUSS 28 DEGR
 CONFIGURATION





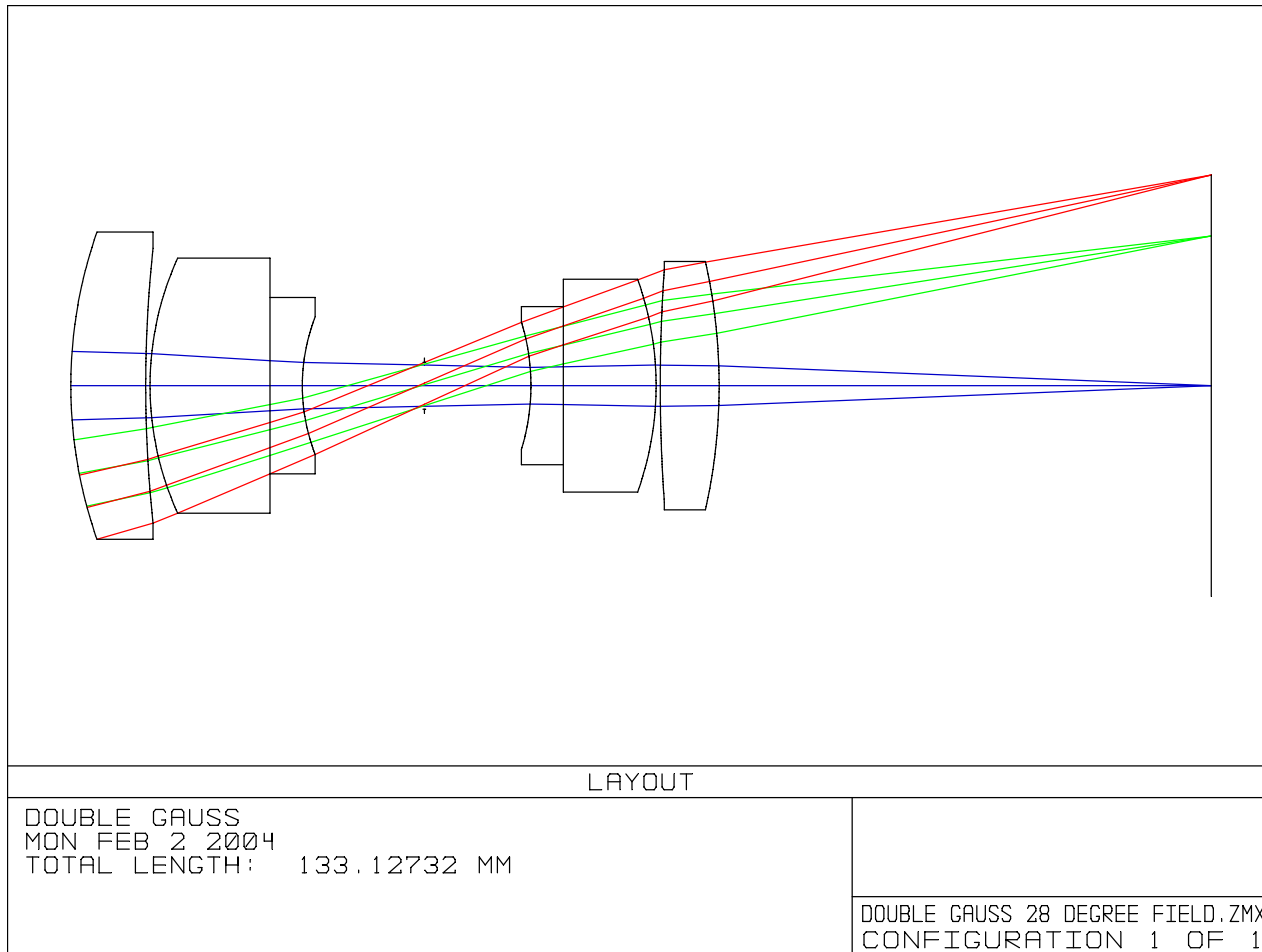




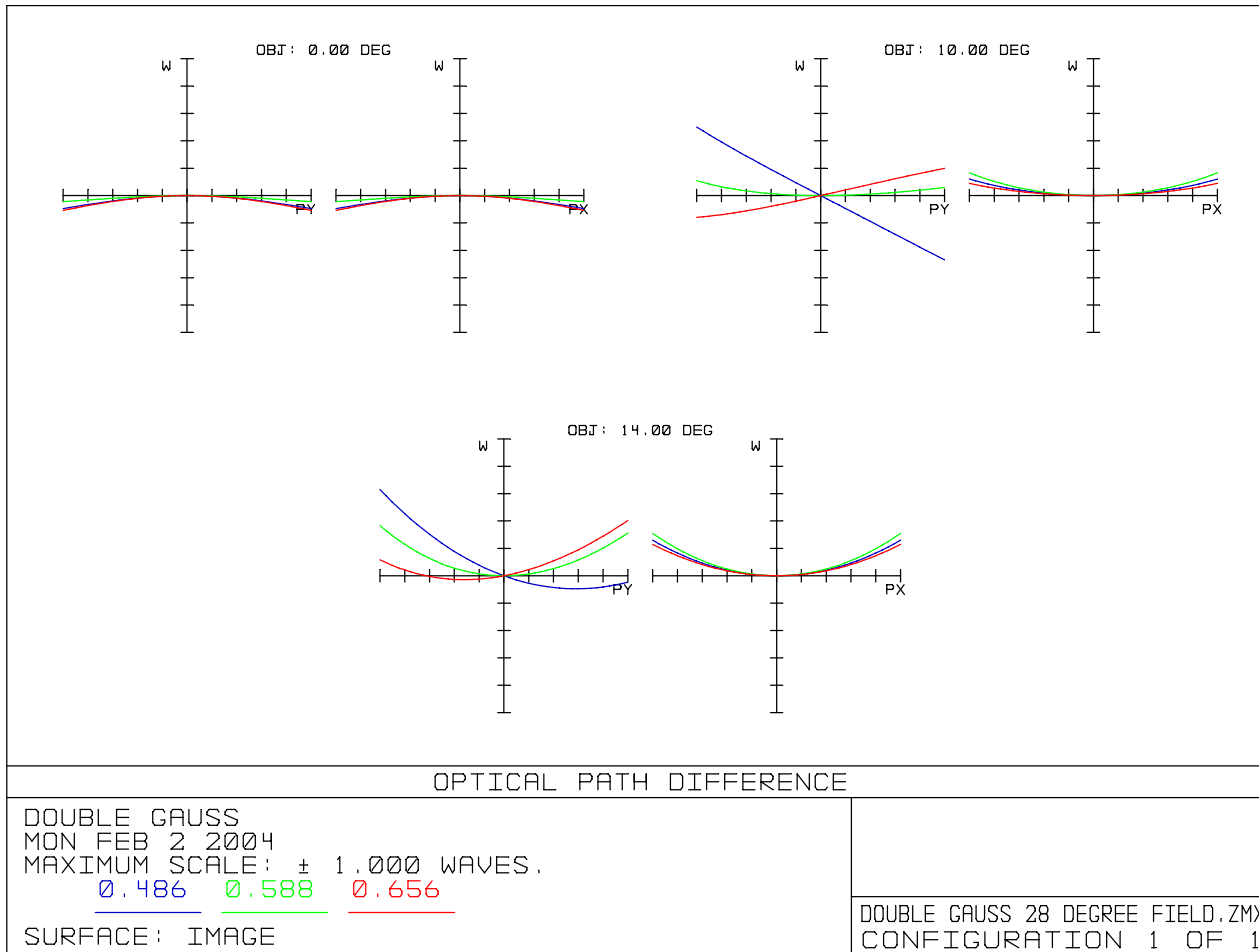


Diffraction Limit

Stop the above system down



OPD < 1 wave



Spot diagrams

- Now these are meaningless

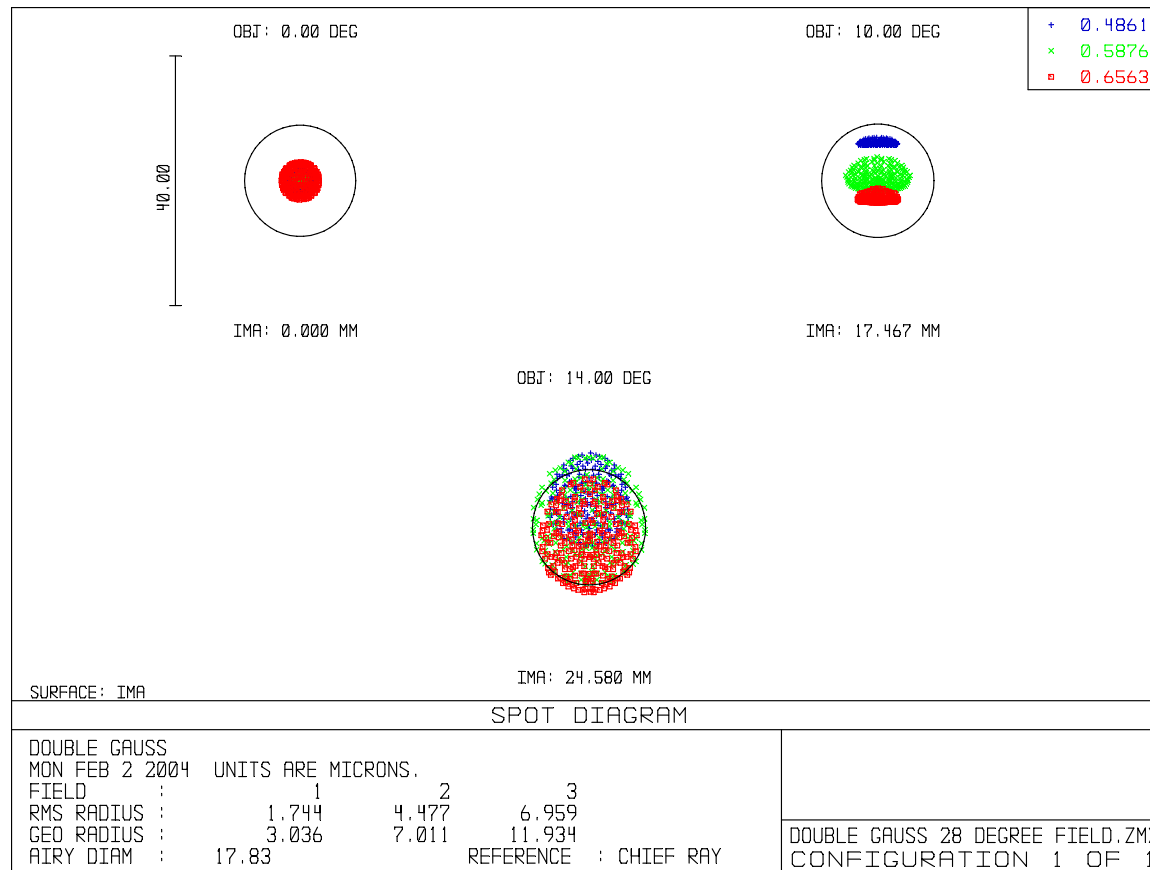
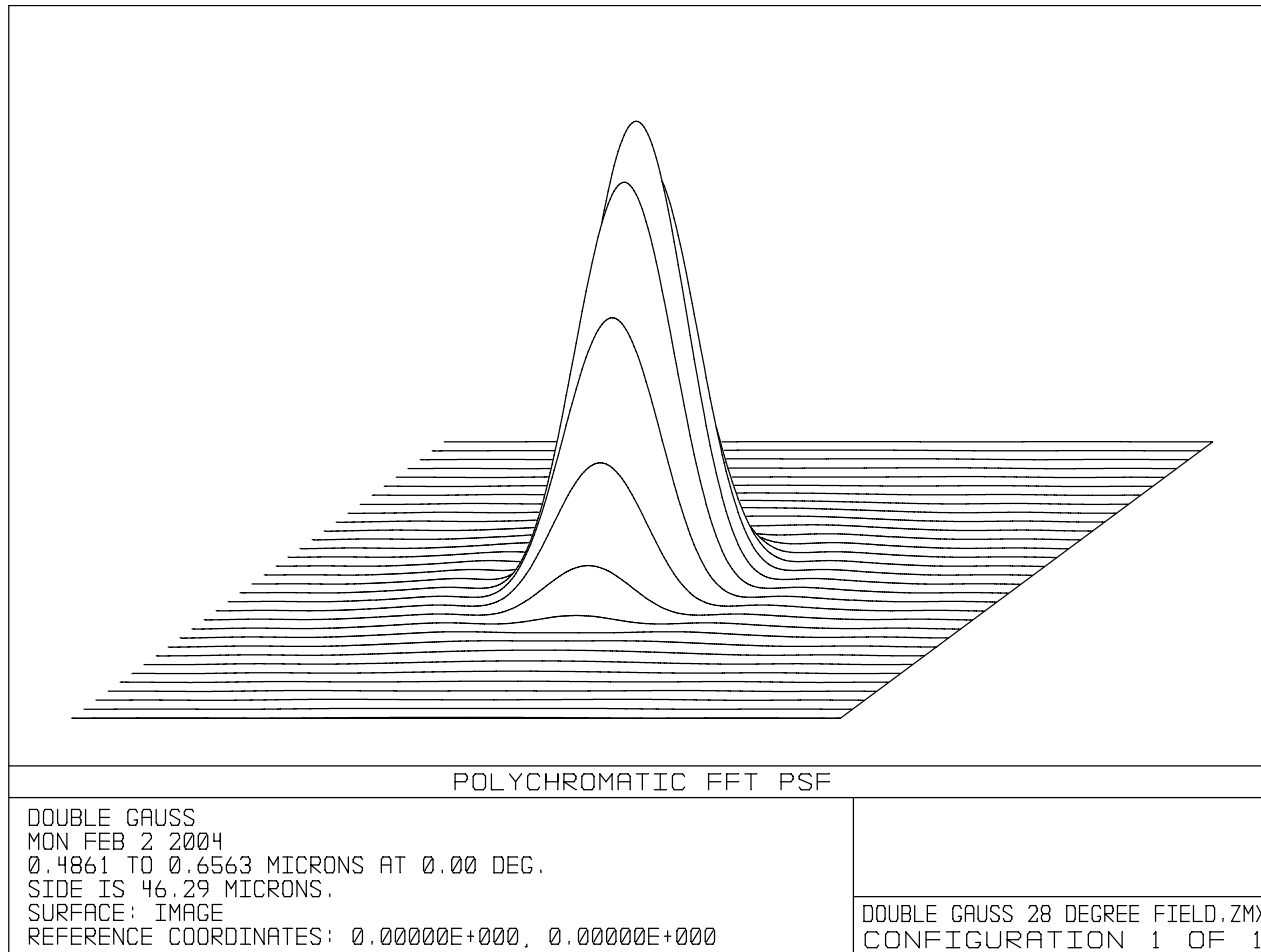


Image size comes from diffraction



Strehl Ratio

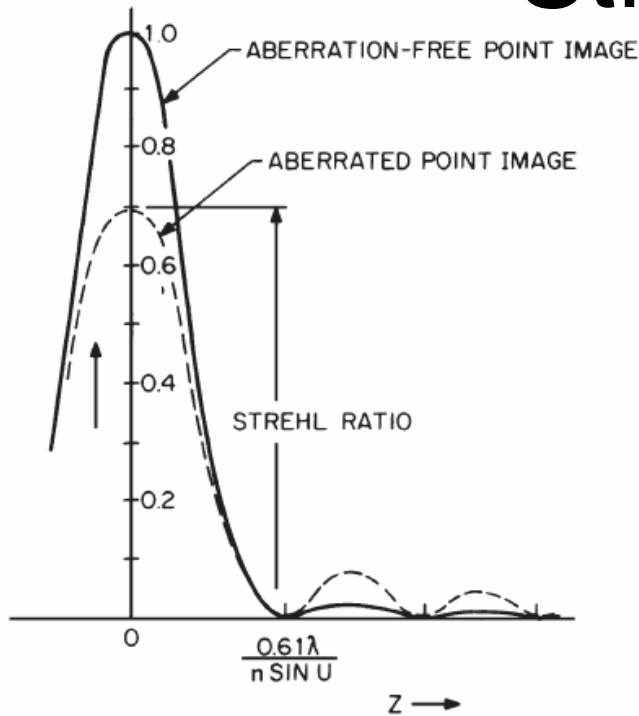


Figure 11.5

$$SR \cong e^{-\sigma^2} \cong 1 - \sigma^2$$

Where σ is RMS wavefront error in radians

$$SR \cong e^{-(2\pi W_{rms} / \lambda)^2} \cong 1 - (2\pi W_{rms} / \lambda)^2$$

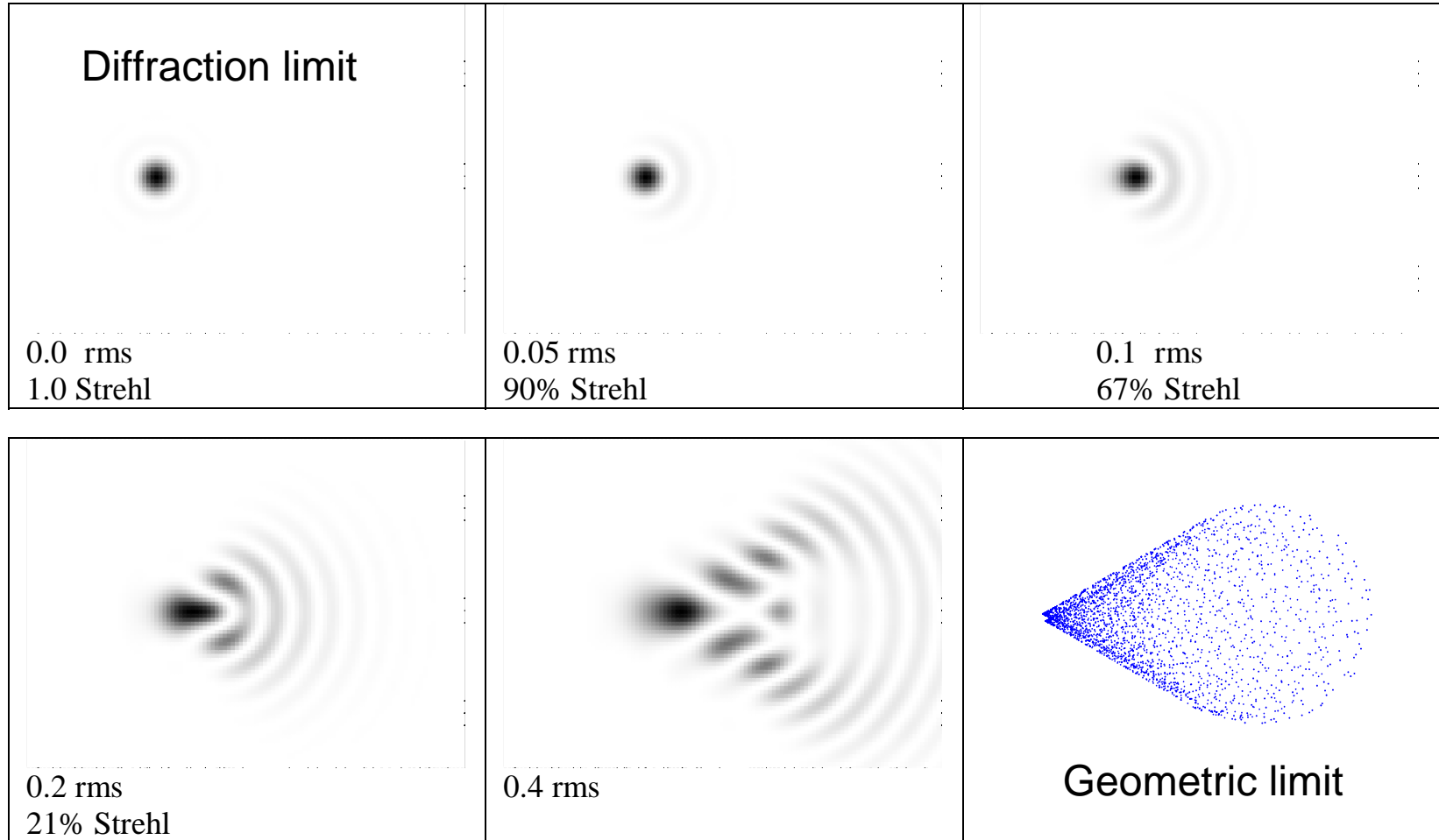
Where W_{rms} is RMS wavefront error in μm (assuming λ in μm)

Relation of Image Quality Measures to OPD

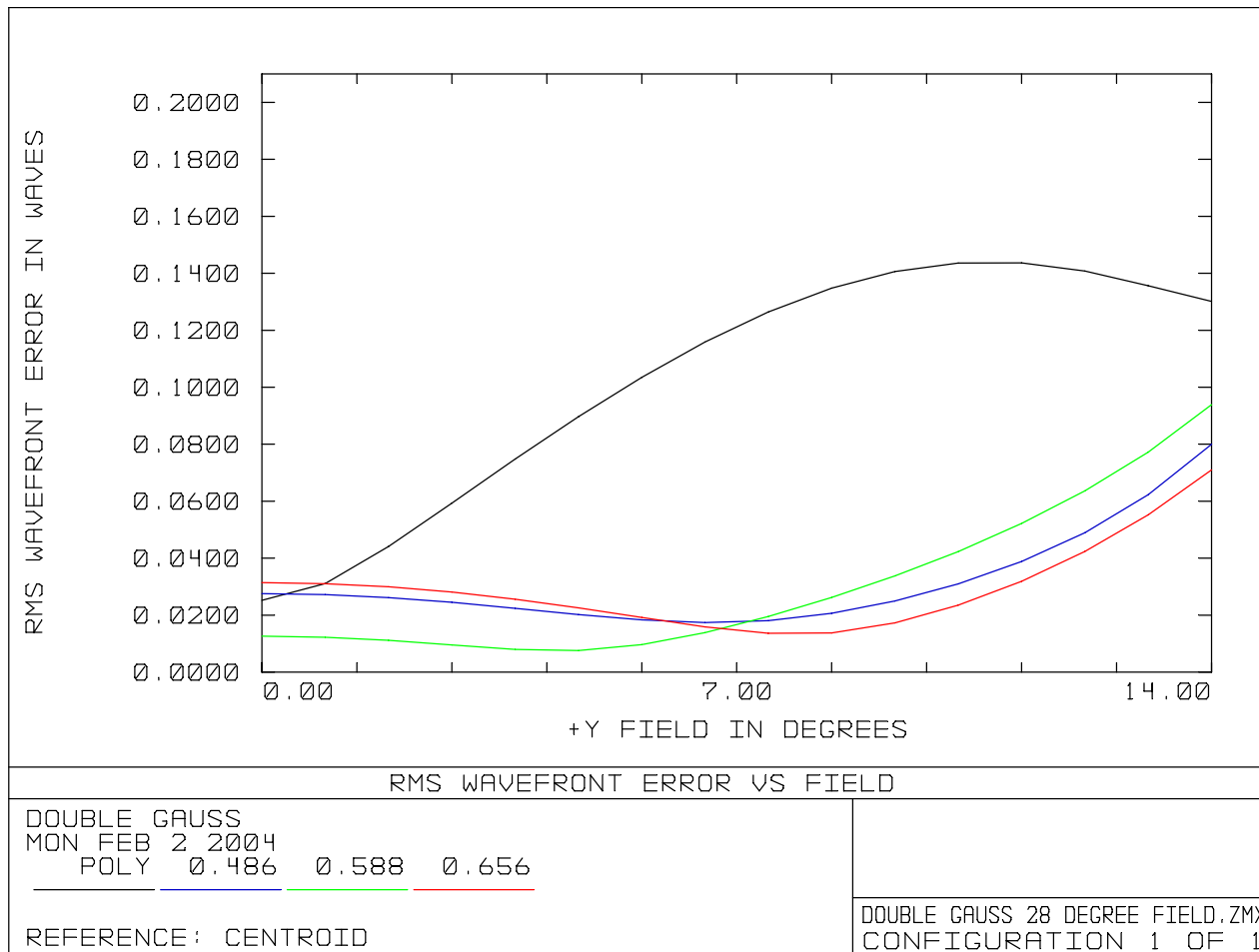
P-V OPD	RMS OPD	Strehl ratio	% energy in	
			Airy disk	Rings
0.0	0.0	1.00	84	16
0.25RL = $\lambda/16$	0.018 λ	0.99	83	17
0.5RL = $\lambda/8$	0.036 λ	0.95	80	20
1.0RL = $\lambda/4$	0.07 λ	0.80	68	32
2.0RL = $\lambda/2$	0.14 λ	0.4*	40	60
3.0RL = 0.75 λ	0.21 λ	0.1*	20	80
4.0RL = λ	0.29 λ	0.0*	10	90

*The smaller values of the Strehl ratio do not correlate well with image quality.

Transition from Diffraction limit to Geometric Limit



Diffraction limited



Diffraction Limited MTF

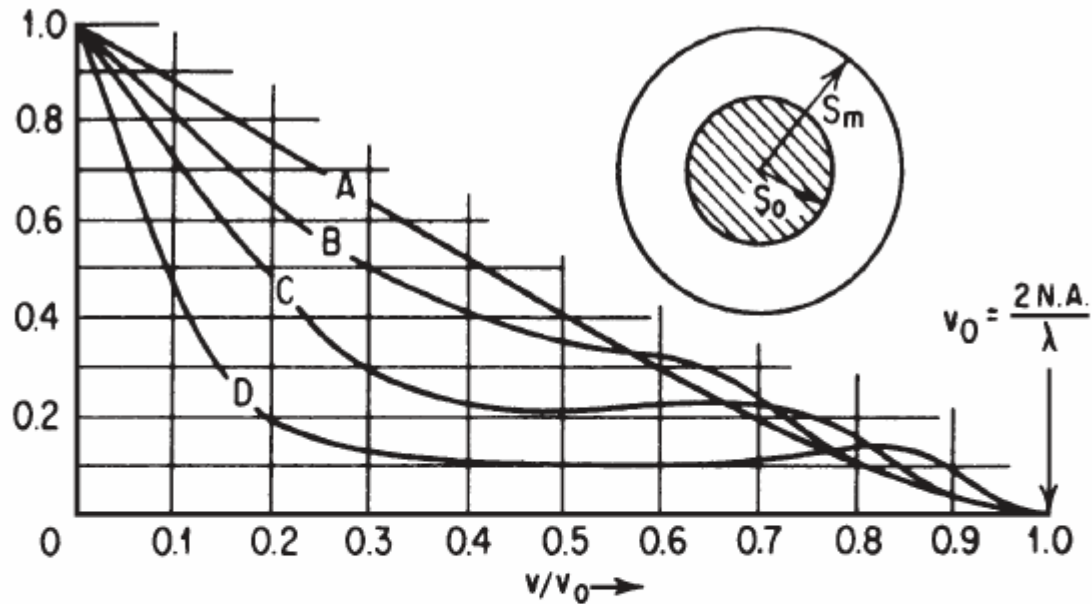
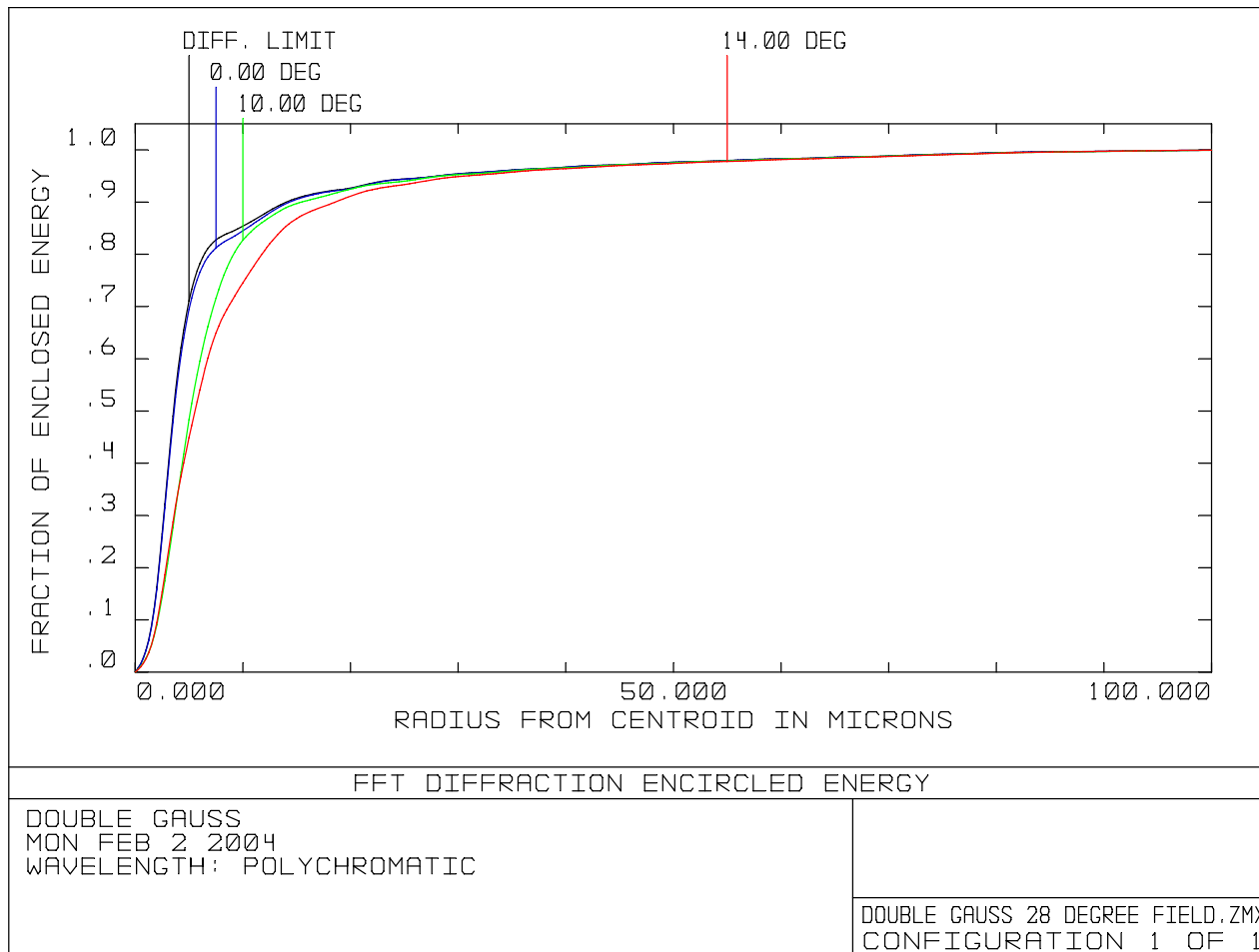


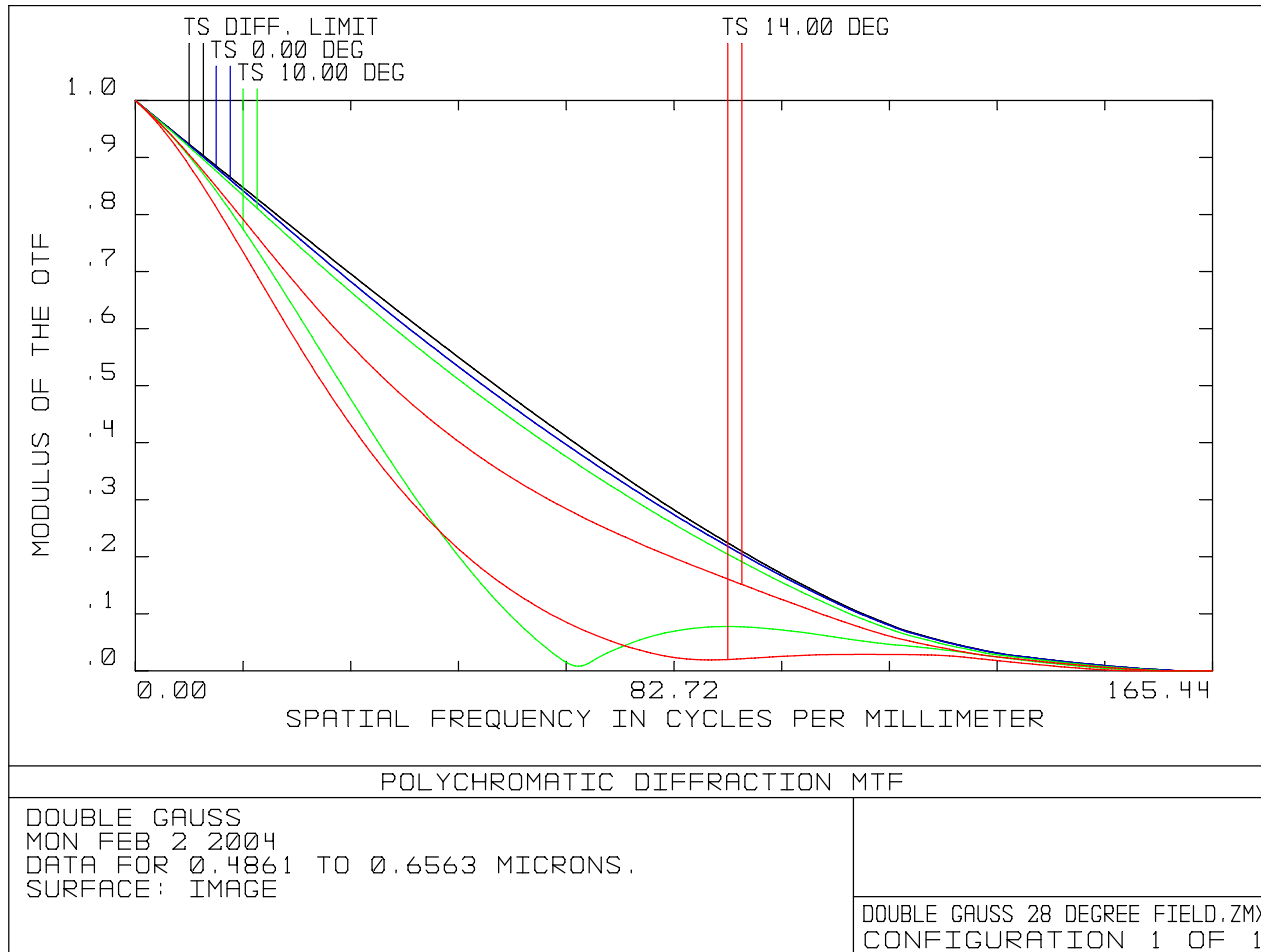
Figure 11.19 The effect of a central obscuration on the modulation transfer function of an aberration-free system.

- (a) $s_0/s_m = 0.0$
- (b) $s_0/s_m = 0.25$
- (c) $s_0/s_m = 0.5$
- (d) $s_0/s_m = 0.75$

Encircled Energy

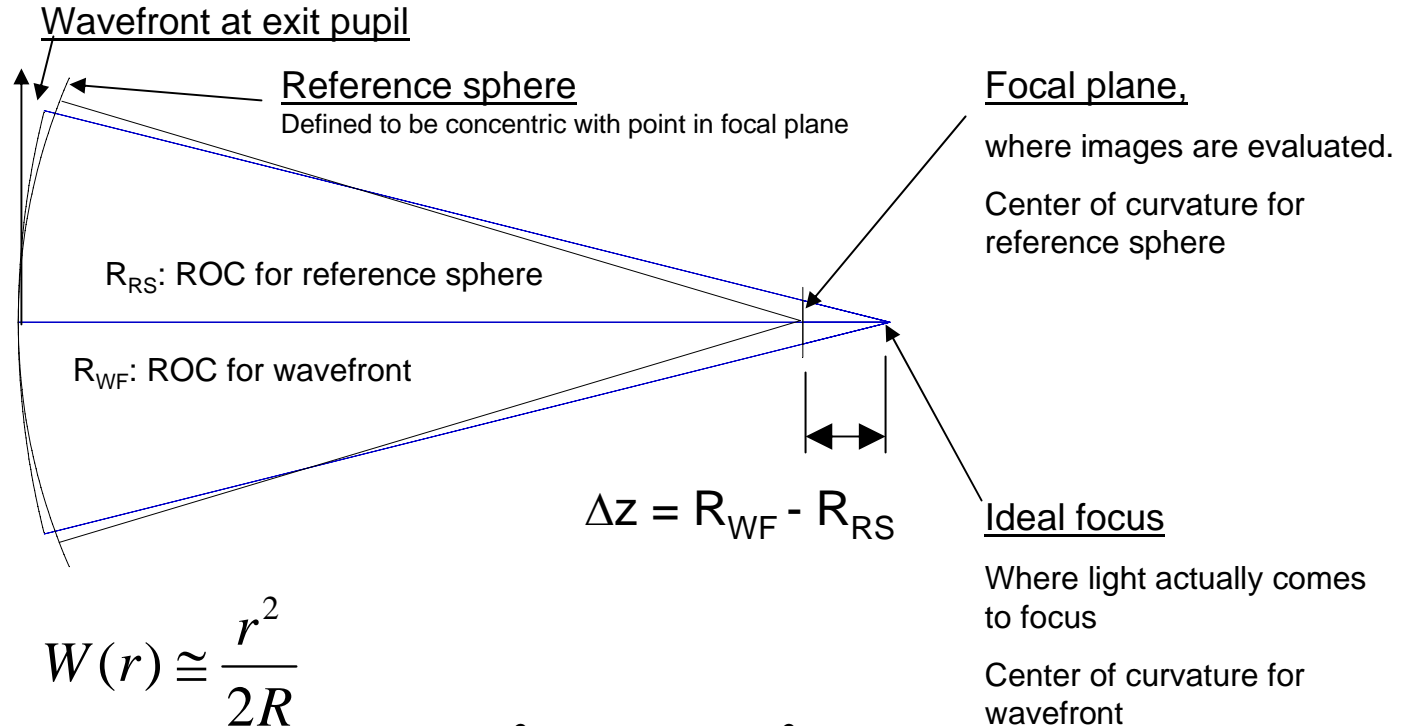


MTF



Wavefront error from defocus

- Shift in focus is exactly the same as the wavefront with the wrong radius



- Wavefront

$$W(r) \cong \frac{r^2}{2R}$$

- Differentiate

$$\Delta W \cong -\frac{r^2}{2R^2} \Delta R = -\frac{D^2}{8R^2} \Delta R$$

- But the change in radius ΔR is equivalent with a change of focus $-\Delta z$

$$\Delta W = \frac{D^2}{8R^2} \Delta z = \frac{\Delta z}{8F_n^2}$$

Practical Optics

Relate focus to RMSWE

A shift in focus is equivalent to wavefront error $W = a_2 \rho^2$

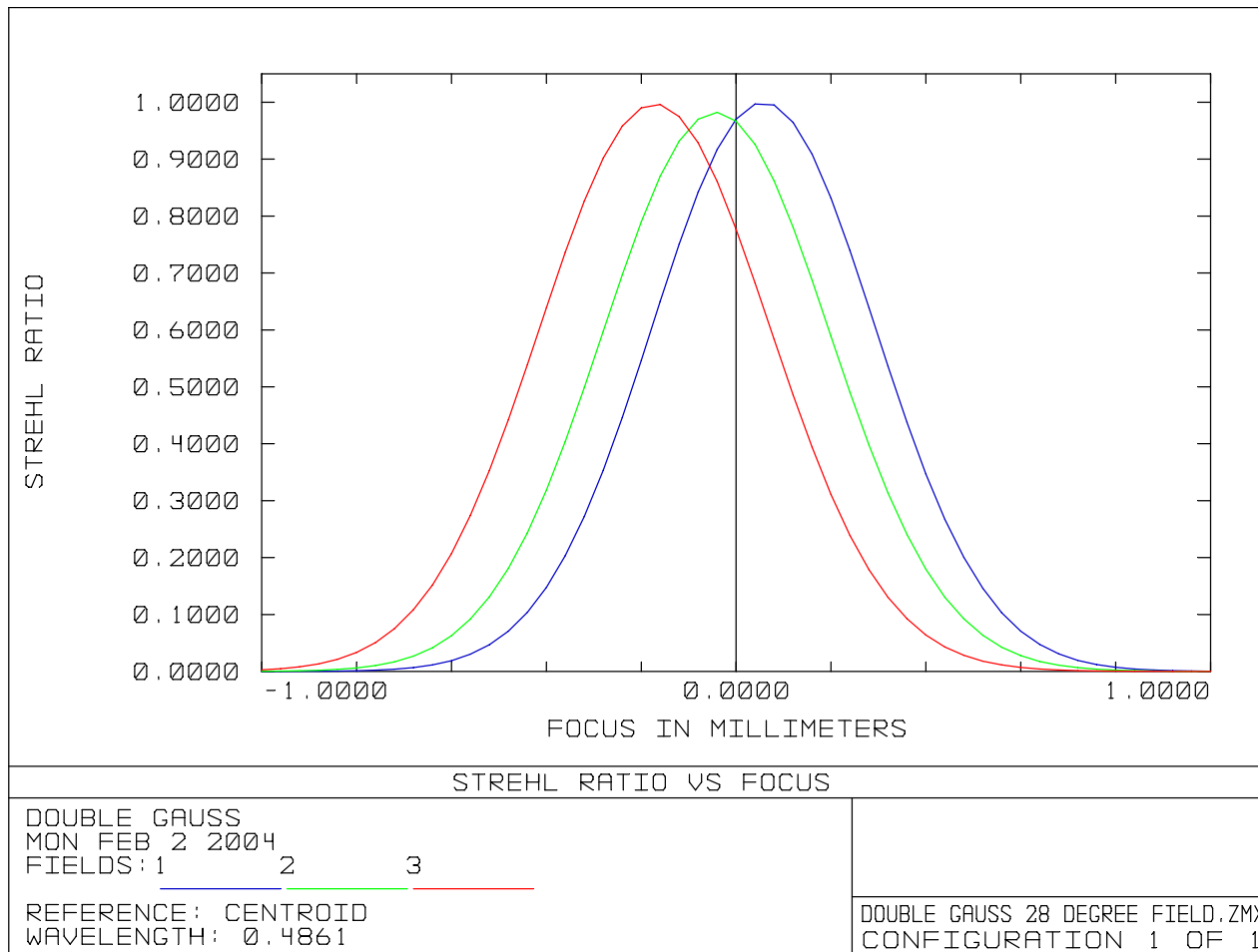
where
$$a_2 = \frac{\Delta z}{8F_n^2}$$

Remember diffraction limited, $a_2 = \lambda/4$ $\Delta z = 2\lambda F_n^2$

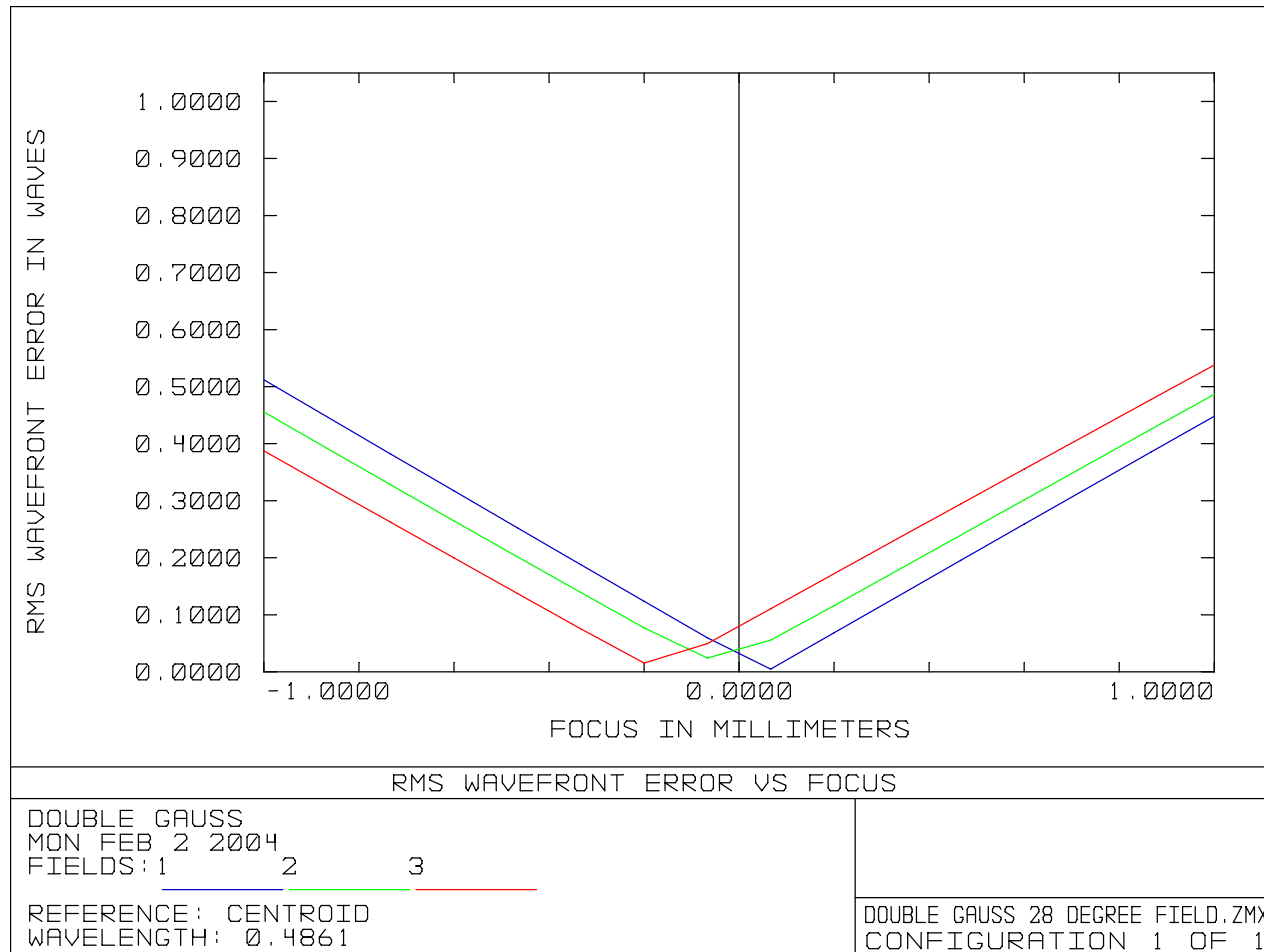
Convert to rms:
$$W_{rms} = 0.289a_2 = .0361 \frac{\Delta z}{F_n^2}$$

$$\sigma = \frac{2\pi W_{rms}}{\lambda} = 0.227 \frac{\Delta z}{\lambda F_n^2}$$

Effect of focus



Wavefront error effect of focus



Figures of Merit for optical systems

	Description	When to use it	<i>How to combine terms</i>
MTF Modulation transfer function	Gives the image contrast as a function of spatial frequency f	For imaging systems looking at extended objects	$MTF_{total}(f) = MTF_1(f) \times MTF_2(f) \times \dots$
RMS Wavefront Error	Gives magnitude of wavefront errors, relative to ideal	Diffraction limited systems looking for resolution of small objects (rms OPD < $\lambda/6$)	$rms_{total} = \sqrt{rms_1^2 + rms_2^2 + \dots}$
Image size	Usually this is given as the rms image diameter	Systems looking for resolution of small objects for the case when not diffraction limited (rms OPD > $\lambda/6$)	$rms_{total} = \sqrt{rms_1^2 + rms_2^2 + \dots}$
Strehl ratio	Defined as the central intensity of the PSF relative to a perfect system	Diffraction limited systems looking for resolution of small objects (rms OPD < $\lambda/6$)	$SR_{total} = SR_1 \times SR_2 \times \dots$
Boresight	Relative angular alignment between optical systems	When different systems must be pointed to the same thing	$\alpha_{total} = \sqrt{\alpha_1^2 + \alpha_2^2 + \dots}$
Image motion	Motion of the image. Jitter – in Hz Stability – in secs, days, ...	Jitter – increases image size Stability – affects calibration	$\alpha_{total} = \sqrt{\alpha_1^2 + \alpha_2^2 + \dots}$