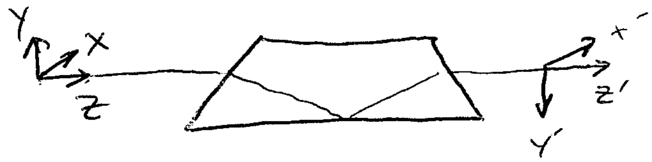
Solutions.

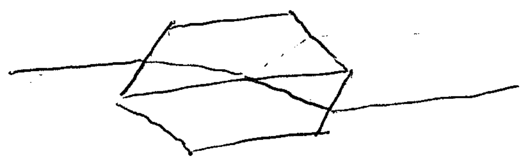
Optical Engineering 421/521 – Fall 2007

Midterm 1 Closed book, closed notes, 50 minutes.

1.) (10) Sketch a dove prism. Define three axes and write the mirror matrix for this prism. Draw its tunnel diagram.



$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



2.) (5) For the prism above, describe what happens if the prism is rotated by a small angle about each of the three axes.

θ_x : direction changes by $2\theta_x$

θ_y : no effect

θ_z : image rotates by $2\theta_z$

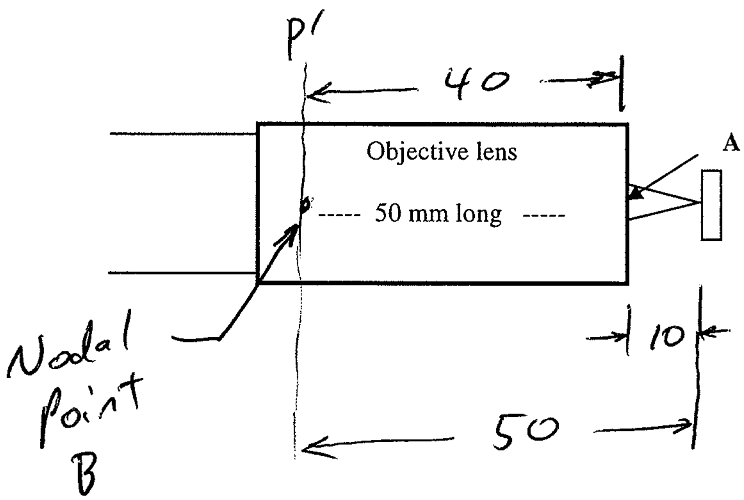
/15

3.) (5) For the following case with an objective lens making an image of a distant object, determine the position of the nodal point and show it on the drawing. Show the position of the nodal point relative to point A:

D_{EP} 10 mm aperture
0.1 NA
10 mm BFD
5° FOV

$F_n = 1/2NA = \frac{1}{2(.1)} = 5$

$EFL = F_n \cdot D_{EP} = 5 \cdot 10\ mm = 50\ mm$



4.) (5) What happens to the image if the lens above is tilted about point A by 1°.

$1° = 17.5\ mrad$

Rotation about A = Rotation about B + translation of B

(Rotation about B: no image motion; translation of B: image shifts by amount of translation)

B $\Delta Y_B = \theta_A \cdot 40\ mm = (17.5\ mrad)(40\ mm) = 700\ \mu m$ image shift

5.) (5) A 25 mm diameter mirror has a requirements of R = 1000 +/- 10 mm. Calculate the tolerance ~~in terms of~~ the sag of the surface. Is this tight?

$Sag = r^2/2R = \frac{D^2}{8R}$

$\Delta Sag = \frac{D^2}{8R^2}\Delta R = \frac{1}{8}\left(\frac{25}{1000}\right)^2 \cdot 10 = 0.78\ \mu m$

Yes, this is tight.

___/15

6.) (5) An wavefront with 1 μm P-V spherical aberration has a shape error described by the equation:

$S(\rho) = (1\mu m)\left(4\rho^4 - 4\rho^2 + \frac{2}{3}\right)$ where ρ is the normalized pupil radial coordinate ($\rho = 1$ at the edge)

Show how to calculate the rms wavefront deviation for this case. You do not need to evaluate the integrals.

$$\sigma^2 = \langle W^2 \rangle - \langle W \rangle^2 = \frac{\iint W^2 dA}{A} - \left[\frac{\iint W\, dA}{A}\right]^2$$

$$\frac{\iint W^2 dA}{A} = \frac{1}{\pi} \cdot 1^2 \int_0^1 \left(4\rho^4 - 4\rho^2 + \tfrac{2}{3}\right)^2 \cdot 2\pi\rho\, d\rho$$

$$= 2\cdot(1\mu m)^2 \int_0^1 \left(16\rho^9 - 32\rho^7 + \tfrac{16}{3}\rho^5 + 16\rho^5 - \tfrac{16}{3}\rho^3 + \tfrac{4}{9}\rho\right) d\rho$$

$$= 2\cdot 1\mu m^2 \left(\tfrac{16}{10} - \tfrac{32}{8} + \tfrac{16}{18} + \tfrac{16}{6} - \tfrac{16}{12} + \tfrac{4}{18}\right) = \tfrac{4}{45}$$

$$\frac{\iint W\, dA}{A} = \frac{1}{\pi}(1\mu m)\int_0^1 \left(4\rho^4 - 4\rho^2 + \tfrac{2}{3}\right)\cdot 2\pi\rho\, d\rho$$

$$= 2\cdot 1\mu m \left(\tfrac{4}{6} - \tfrac{4}{4} + \tfrac{2}{6}\right) = 0$$

$$\sigma^2 = \tfrac{4}{45} \qquad \sigma = 0.298\,\mu m$$

7.) (10) Consider a 25 mm beamsplitter with refractive index inhomogeneity of +/- 0.00001. Determine the P-V and rms wavefront variation due to the glass. For 0.5 μm wavelength, calculate the effect on Strehl ratio.

$\Delta n = \pm 10$ ppm $= 20$ ppm P.V

$\cong 5$ ppm rms   RoT: $\frac{PV}{rms} = 4$

$\Delta W = t \cdot \Delta n$

$= 25$ mm $\cdot$ 20 ppm P.V $= 500$ nm P.V

$= 125$ nm rms

$\Delta W = 125$ nm rms $\cdot \frac{1 \text{ wave}}{500 \text{nm}} = 0.25\,\lambda$ rms

$\times \frac{2\pi \text{ rad}}{1\,\lambda} = 1.57$ rad

$SR \cong 1 - \sigma^2$ for $\sigma < 1$

$SR \cong 0$ $e^{-\sigma^2} = 8\%$

___/15

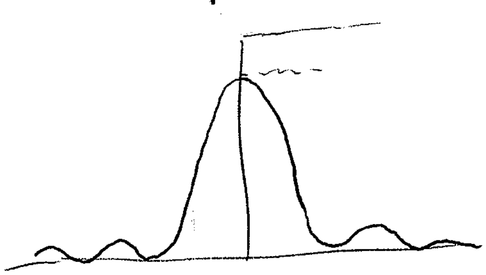
8.) (10) Consider an imaging system with 500 mm EFL and 10 mm circular entrance pupil, creating an image of a point at infinity with 500 nm wavelength light. For the case where the image plane is 2.5 mm out of focus:
a) Sketch the PSF intensity profile for the defocused system (show the image size as well as shape)

$f/50$ $\Delta W = \frac{\delta z}{8F^2} = 0.125\,\mu m = \lambda/4$

$rms \cong \frac{1}{4} \cdot \lambda/4 \cong \lambda/16 \cong \frac{2\pi}{16} \cong 0.4$ rad

$SR \cong 80\%$

PSF

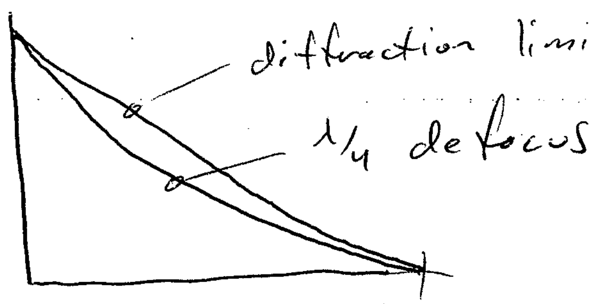


~20 energy taken from central lobe and put into ring

$|\leftarrow \rightarrow| \sim 2.4 \lambda F^{\#} = 60\,\mu m$

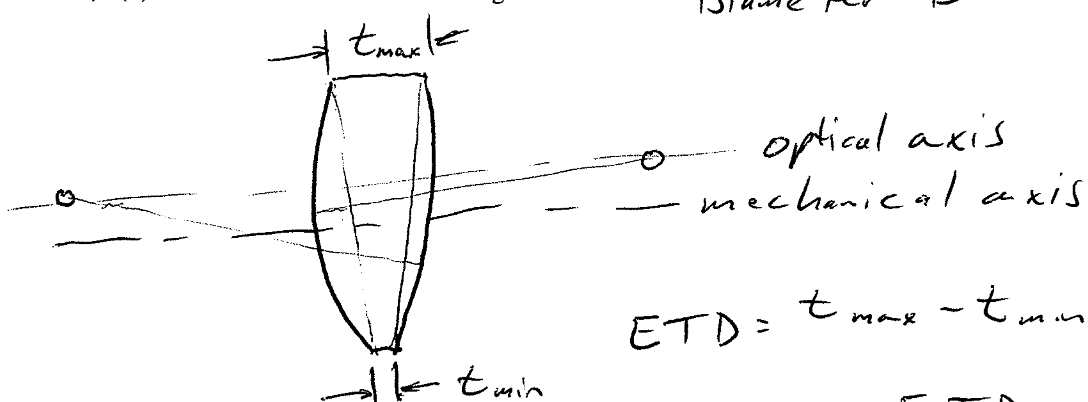
b) Sketch the approximate MTF for the defocused system (show the zero crossing as well as approximate shape)

MTF



$\frac{1}{\lambda F^{\#}}$ ~~cycles~~ $= 40 \frac{cycles}{mm}$

9.) (5) Use a sketch to define the "wedge" in a lens. Diameter D



$ETD = t_{max} - t_{min}$

$wedge = \frac{ETD}{D}$

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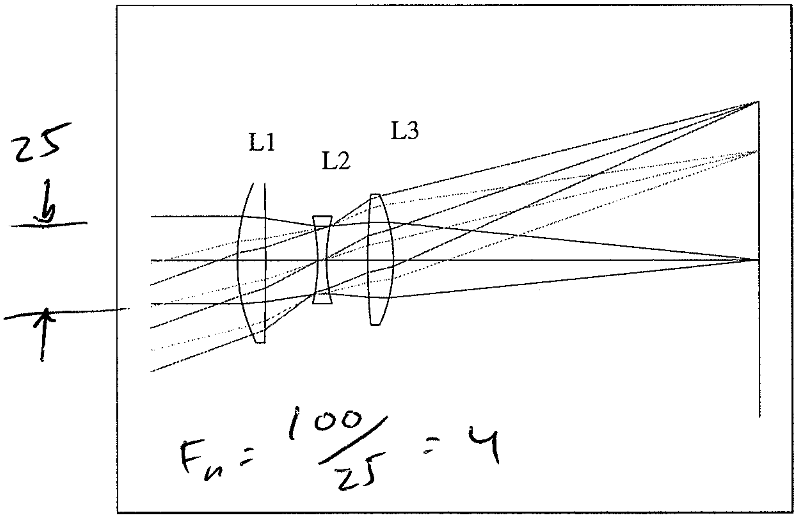
10.) (10) Consider the following imaging system. We are concerned about the effect of lens motion on image stability. For the case of **L1**, calculate the image motion for 1 µm of lens decenter for this element alone.

The system has 25 mm entrance pupil diameter, 21 mm pupil, and 100 mm EFL.

L1: 80 mm focal length
(50 mm diameter)
L2: -40 mm focal length
L3: 45 mm focal length



$F_n = 100/25 = 4$

$\Delta\theta_1 = \frac{\Delta x_1}{f_1} = \frac{1\mu m}{80 mm} = 12.5 \mu rad$

$\varepsilon_1 = D_1 \Delta\theta_1 F^{\#}$
$= 25 \cdot 12.5 \cdot 4$
$= 1250 \mu \cdot mm = 1.25 \mu m$

1 fringe $= \frac{\lambda}{2}$ surface $\cong .25 \mu m$ P-V surface
$\rightarrow$ RoT $0.25 \frac{\mu m}{radius}$ slope rms

11.) (10) Consider the same system above. The lens surfaces are specified to have surface irregularity less than 1 fringe over the full diameter as measured by a test plate. Estimate the degradation of image quality, defined as rms spot radius, due to each of the surfaces on **L1**.

Surface slope $0.25 \frac{\mu m}{radius} \cdot \frac{radius}{25 mm} = 0.01$ mrad rms
$= 10 \mu rad$ rms

WF slope $= \nabla W = \nabla S(n-1) = 5 \mu rad$ rms / surface

$\varepsilon_1 = D_1 \cdot \Delta\theta_1 \cdot F_n = 25 \cdot 5 \cdot 4 = 500 \mu \cdot mm = 0.5 \mu m$ rms

0.5 µm rms radius degradation per surface on L1
(RSS: 0.7 µm rms radius total)

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12.) (20) Sketch a 3-view orthographic projection of the following part, including dimensions. (One block shown = 1 mm)